INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Check for updates

OPEN ACCESS

Received: 05-02-2023

Accepted: 11-08-2023

Published: 20-11-2023

Citation: Punitha MJ, Angeline EFB (2023) Dom-Chromatic Number o f Wrapped Butterfly & Bloom Graphs . Indian Journal of Science and Technology 16(SP3): 8-13. https://doi.org/10.17485/IJST/v16iSP3.icrtam211

* **Corresponding author**.

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Dom-Chromatic Number o f Wrapped Butterfly & Bloom Graphs

M Joice Punitha[1], E F Beulah Angeline[2]*

1 Department of Mathematics, Bharathi Women's College (Autonomous), Affiliated to the University of Madras, Chennai-108, Tamil Nadu, India
2 Department of Mathematics, Nazareth College of Arts and Science, Affiliated to the University of Madras, Chennai-62, Tamil Nadu, India

Abstract

Objectives: For a given graph G with proper coloring, the problem of selecting a dom-coloring set is to choose a dominating set having a property that it has a minimum of one vertex from every possible color class in G. Our aim is to determine the family of networks that allow dom-coloring and to find its dom-chromatic number denoted by $\gamma_{dc}(G)$. **Method:** We have applied the algorithmic method of choosing the dom-coloring set(dc-set). Here we have designed a coloring algorithm to yield the proper coloring for the vertices of the graph. D-set algorithm has been developed to determine the dominating set for the given graph. Then, the dc-set for the graph is obtained by applying the above two algorithms. **Findings:** In this study, we have established the study on finding the dc-set of wrapped butterfly network and bloom graphs. Further, we have found the dom-chromatic number of the above-mentioned graphs. **Novelty**: Dom-coloring is an extended variation of graph coloring and domination which has emerged as a result of the combination of the two broad concepts in graph theory namely, domination and coloring. A dominating set which includes a minimum of one vertex from all possible color classes of the graph forms a dom-coloring set. In this paper, a study on dom-coloring of wrapped butterfly and bloom graphs have been accomplished. These results may be generalized for butterfly derived networks to determine its dom-chromatic number.

Keywords: Dominating set; Domination; number; Coloring; Chromatic number; Domcoloring set; Domchromatic number 1

1 Introduction

In graph theory, the notion of domination along with coloring have found diversified applications across several areas of research. For any graph G, a non- empty subset D of the vertex set V is a dominating set if all the vertices in the complement of D and some vertex in D share a common edge. Such a set with the least number of vertices is a minimum dominating set whose cardinality is termed as the domination number which is represented by $\gamma(G)$. Coloring can be stated as the method of allotting colors to every single vertex included in the vertex set of G, in which vertices sharing a common edge are allotted mismatched colors. The lesser number of such colors used in coloring the graph is termed as the chromatic number, $\chi(G)$. Until 2016, the above two concepts were dealt separately by many researchers, whereas, the above two concepts were blended to form a new problem called dom-coloring, which was put forth by Chaluvaraju B. and Appajigowda C in 2016. For a graph G with minimum coloring whose vertex set is V, a dominating set of G can be regarded as a dom-coloring set ($dc - $ set) if it constitutes at the least of one vertex from every possible color class of G. The dc $-$ set with the least cardinality is the dc $-$ number of G which is symbolized as $\gamma_{dc}(G)$.

Interconnection networks help in communication of data and provide intermediate results in-between processors. These interconnection networks can be represented in the form of a graph by relating processors with vertices and communication links with edges. A plenty of such networks are available in graph theory. In this paper we have dealt with two such networks namely wrapped butterfly and bloom graphs. The butterfly graph has been extensively used in parallel computer architectures and bloom graphs play a vital role in broadcasting.

The study on domination and coloring in wrapped butterfly and bloom graphs are interesting areas of research in graph theory. In wrapped butterfly network, few studies like 1-harmonious coloring [1], T-coloring [2], ST-coloring [2], total domination [3] and domination of its digraph [4] were carried on. In the above research work, only one concept, either coloring or domination were applied, whereas in this paper both the concepts are applied together to determine the dom-chromatic number of wrapped butterfly graph. This work has been extended to bloom graph also.

2 Methodology

In this paper, the problem of dom-coloring has been solved by using algorithmic approach. The purpose of this work is to design general algorithms called coloring algorithm and D-set algorithm for any n. Using coloring algorithm, wrapped butterfly and bloom graphs are given proper coloring. The D-set algorithm is proposed to choose the dominating set in both the graphs, satisfying the condition that it contains a minimum of one vertex from each color class. The above two algorithms are applied to determine the dc-set which yields the dom-chromatic number.

3 Results and Discussion

3.1 Dom-chromatic number of wrapped butterfly network $WB(k)$

Butterfly networks are variations of hypercubes. These graphs are both Eulerian and planar. In this section we formulate a coloring algorithm for $WB(k)$ and a D-set algorithm for choosing the dominating set in $WB(k)$.

Definition 3.1.1 [3] The vertex set of an $n-$ dimensional butterfly network $BF(k)$ is $V = \{(y;j)/\ y = (y_1,\ y_2,\ \ldots,\ y_k),\ \ y_i = 0\ or\ 1,\ 1 \leq j \leq k\}$. An edge joins two vertices $(y;j)$ and $(z;m)$if and only if $m = j+1$ and either $y = z$ or y and z differ in the m^{th} bit.

A wrapped butterfly network $WB\,(k)\,, k \geq\ 3$ is the result of connecting the first and last levels of $BF(k)$. $WB\,(k)$ has $k-$ levels with $k.2^k$ vertices, each vertex with degree 4 and has a strong symmetry.

Algorithm 3 .1.1 Coloring algorithm for $WB\,(k)\,,\ k \geq 3$

Input: $WB\,(k) k \geq 3$.

Step 1: When k is even, vertices in levels $L_1,\ L_3,\ \ldots L_{k-1}$ receive color 1 and the remaining vertices receive color 2.

Step 2: When k is odd, vertices in levels $L_1,\ L_3,\ \ldots L_{k-2}$ receive color 1, vertices in levels $L_2,\ L_4,\ \ldots L_{k-1}$ receive color 2 and the remaining vertices receive color 3.

Output: Proper coloring of $WB\,(k) k \geq 3$.

Algorithm 3.1.2 $D-$ **set a lgorithm of** $WB\,(k)\,,\ k > 3$

The symmetric nature of $WB(k)$, enables to split the 2^k columns into two halves as B_1 and B_2 which represents the left half and the right half of $WB(k)$ respectively. The following algorithm determines the dominating set, D_c for $WB(k)$.

Input: $WB\,(k) k > 3$.

Step 1: Select the last level L_k of B_1.

Step 2: Split the successive vertices in L_k of B_1 into 4 groups each with 2^{k-3}vertices and include all the vertices of the $1^{st}$and 3^{rd} groups into D_c

Step 3: For $2 \leq r \leq k-3$, split the consecutive vertices in level L_r of B_1into 2^{k-r+2} groups each with 2^{r-3} vertices.

Step 4: Select the vertices in the 2^{k-r} groups among the 2.2^r groups which are not dominated by any of the vertices in level L_{r+1}. The vertices selected in level L_r is 2^{k-3}.

Step 5: In level L_3 choose 2^{k-3} vertices to D_c which are not dominated by any vertices in level L_4.

Step 6: Include to D_c the vertices in B_2 which are mirror images of the vertices included in B_1.

Output: Dominating set of $WB\,(k) k > 3$.

Note: The $D-$set obtained by the above algorithm is also an independent dominating set, as no two vertices in D_c have a common edge between them.

Theorem 3.1.1 [3] For any graph G of order m and maximum degree Δ, we have $m \geq \gamma(G) \geq \left[\frac{m}{\Delta+1}\right]$.

Theorem 3.1.2 [5] For any graph G, $max\{\gamma(G),\ \chi(G)\} \leq \gamma_{dc}(G) \leq \gamma(G) + \chi(G) - 1$.

Theorem 3.1.3 [3] If G is a 4-regular graph of order m, then $\gamma(G) \geq \left[\frac{m}{5}\right]$.

Theorem 3.1.4 [6] Let G be a connected undirected graph $WB\,(k)$, $k \geq 3$. Then $\gamma(G) = k.\ 2^{k-2}$.

Remark: The dominating set D_c, obtained by the $D-$set algorithm includes a minimum of one vertex from every disjoint color class of G. Hence, D_c is a dom-coloring set of G. Since the vertices in D_c are independent, it also forms an independent dom-coloring set.

Theorem 3.1.5 Let $WB(k)$ be a wrapped butterfly network $k \geq 2$. Then $\gamma_{dc}\,(WB(k)\,) = k.\ 2^{k-2}$.

Proof: Let G be a wrapped butterfly graph $WB\,(k)$, $k \geq 2$. For G, $\chi(WB\,(k)) = 2$, when k is even and $\chi(WB\,(k)) = 3$, for odd k.By Theorem 3.1.2, $max\,(\gamma(G),\ \chi\,(G)\} = max\,(k.\ 2^{k-2},\ \chi\,(G)\} = k.\ 2^{k-2}$. Hence, $\gamma_{dc}\,(G) \geq k.\ 2^{k-2}$. But by Theorem 3.1.4 we have $\gamma(G) = k.\ 2^{k-2}$. Since the D_c-set itself forms the dom-coloring set; we conclude that $\gamma(G) = \gamma_{dc}\,(G) = k.\ 2^{k-2}$.

Illustration 1

Consider the graph $WB\,(2)$. The network $WB(2)$ has 2 levels with 4 vertices in each level. The vertices of level 1 receive color 1 and that of level 2 receives color 2. The 1^{st} vertex of level 1 and the last vertex of level 2 forms the minimum dominating set with one vertex from each color. Refer Figure 1. Hence, the dom-chromatic number of $WB(2)$ is 2.

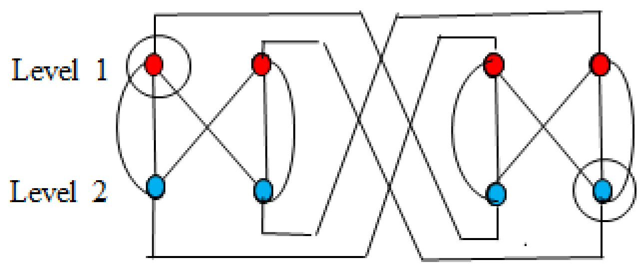

Fig 1. Encircled vertices form *a d c* set of $WB\,(2)$

Illustration 2

Consider the graph $WB\,(3)$. The network $WB(3)$ has 3 levels with 8 vertices in each level. The vertices of level 1, 2 and 3 receive colors 1, 2 and 3 respectively. Split the 2^3 columns into 2 halves, the left half has 2^2 columns and the right half has 2^2 columns. Select the left half and choose the 3^{rd}, 1^{st} and 4^{th} vertices from the successive levels. Due to symmetry, select the vertices in the right half which are mirror images of the vertices selected in the left half. These 6 vertices selected dominates all the vertices of $WB(3)$ and also includes a vertex of each color. So, these form the dc-set. The dom-chromatic number $\gamma(WB\,(3)) = 6$. Refer Figure 2.

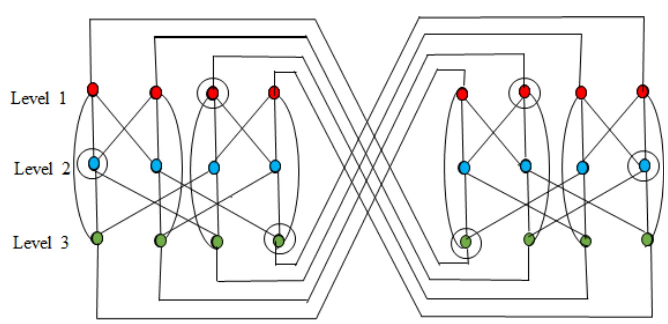

Fig 2. Encircled vertices form a$dc-$ set of WB(3)

Illustration 3 Let $WB(k)$ be a wrapped butterfly network $k = 4$. Then$\gamma_{dc}\,(WB(4)\,) = 4.\ 2^{4-2} = 16$. Refer Figure 3.

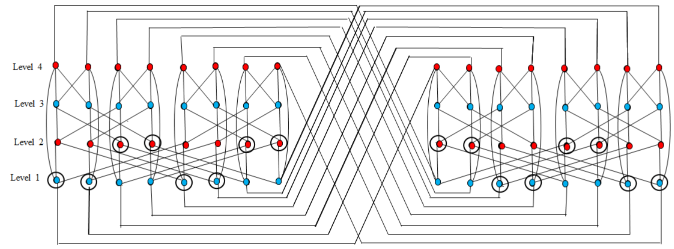

Fig 3. Encircled vertices form a$dc-$ set of WB (4)

3.2 Dom-chromatic number of bloom graph *B(u, v), u = v*

Bloom graphs are 4-regular, planar graphs which are also tripartite. In this section a coloring algorithm and the dom-chromatic number of $B(u,\ v)$ have been determined.

Definition 3.2[7] A bloom graph $B\,(u,\ v),\ \ u,\ v \geq 2$ has $V\,(B\,(u,\ v)) = \{(p,\ q) : 1 \leq p \leq u,\ 1 \leq q \leq v\}$ as the vertex set and an edge connects 2 distinct vertices $(p_1,\ q_1)$ and $(p_2,\ q_2)$ if the following conditions are satisfied.

i. $p_2 = p_1 + 1$ and $q_1 = q_2$

ii. $p_2 = p_1 = 0$ and $q_1 = q_1\ (mod\ v)$

iii. $p_2 = p_1 = 0$ and $q_1 + 1 = q_2\ (mod\ v)$

iv. $p_2 = p_1 + 1$ and $q_1 + 1 = q_2\ (mod\ v)$

The first condition is for vertical edges, the second and third conditions are for horizontal edges in the top most and the lower most rows respectively. Condition four is for slant edges.

Algorithm 3.2.1 $\chi-$ Coloring a lgorithm for $B(u,\ v)$

Input: $B\,(u,\ v) u = v\ \&\ u,\ v > 4$.

Case (i): When $u = v = k$ and k is even, vertices in

Step 1: row 1 receive color in the order 1212..

Step 2: row $3k-1,\ k = 1,\ 2,\ \ldots \frac{u-4}{2}$ receive 3.

Step 3: row $3k,\ k = 1,\ 2,\ \ldots \frac{u-4}{2}$ receive 1.

Step 4: row $3k+1,\ k = 1,\ 2,\ \ldots \frac{u-4}{2}$ receive 2.

Step 5: row u receive color in the order 1212.. if row $u-1$ has color 3.

Step 6: row u receive color in the order 3131.. if row $u-1$ has color 2.

Step 7: row u receive color in the order 2323... if row $u-1$ has color 1.

Case (ii): When $u = v = k$ and k is odd, vertices in

Step 1: column 1 receive color in the order 123123...

Step 2: column v receive color in the order 312312...

Step 3: from $(1,\ 2)$ to $(1,\ v-1)$ receive *color in the order* $2121\ldots$

Step 4: from $(2,\ k)$ to $(u-1,\ k),\ k = 2, 3, \ldots.\ v-1$ receive color in the order $312312\ldots$

Step 5: from $(u,\ 2)$ to $(u,\ v)$ receive *color in the order* $3131\ldots$ if $(u,\ 1) = 2$.

Step 6: from $(u,\ 2)$ to $(u,\ v)$ receive *color in the order* $1212\ldots$ if $(u,\ 1) = 3$.

Step 7: from $(u,\ 2)$ to $(u,\ v)$ receive *color in the order* $3131\ldots$ if $(u,\ 1) = 1$.

Output:χ-coloring of $(u,\ v)$.

Observation:

• Let G be the bloom graph $B\,(4,4)$. The vertices encircled in the Figure 4 is the dom-coloring set. Clearly, $\gamma(G) = \left[\frac{uv}{5}\right] = \left[\frac{16}{5}\right] = 4 = \gamma_{dc}(G)$.

• Let G be the bloom graph $B\,(5,\ 5)$. The vertices encircled in the Figure 5 forms the dom-coloring set. Here, $\gamma(G) = \left[\frac{uv}{5}\right] = \left[\frac{25}{5}\right] = 5$. But $= \gamma_{dc}\,(G) = 6 = \gamma(G) + 1$.

The following theorem yields the dom-chromatic number for $B\,(u,u),\ \ u > 5$.

Theorem 3. 2.1 Let $B(u,\ u)$ be a bloom graph, $u > 5$. Then $\left[\frac{u^2}{5}\right] \leq \gamma_{dc}\,(B(u,\ u)\,) \leq \left[\frac{u^2}{4}\right]$.

Theorem 3. 2.2 Let $B(u,\ u)$ be a bloom graph $u \equiv 1\ mod\ 3$. Then $\gamma_{dc}\,(B(u,\ u)\,) = \left[\frac{u^2}{5}\right] + m - 3 - i,\ where\ m = 3i + 4\ \&\ i = 1,\ 2,\ 3,\ \ldots.$

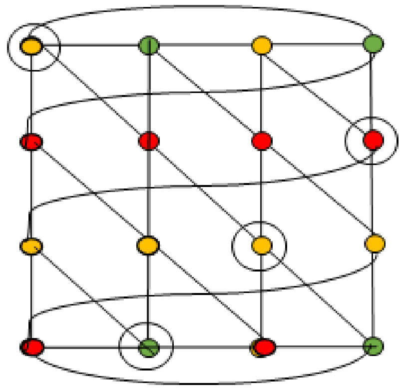

Fig 4. Encircled vertices form a $dc-$ **set of** $B(4,\ 4)$

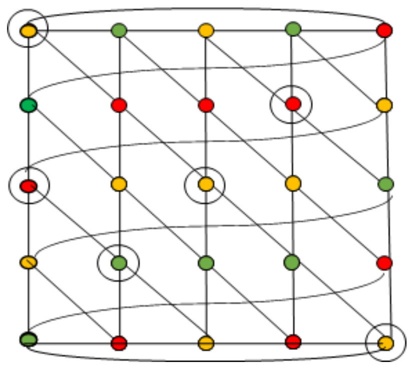

Fig 5. Encircled vertices form a $dc-$ **set of** $B(4,\ 4)$

Proof: The graph $B(u,\ u)$ with $u \equiv 1\ mod\ 3$ is 3-colorable. Also, $\left[\frac{u^2}{5}\right] + m - 3 - i$, where $m = 3i + 4\ \&\ i = 1,\ 2,\ 3,\ \ldots.$ vertices form a dominating set which includes a minimum of one vertex from each color forming a dc-set. Refer Figure 6.

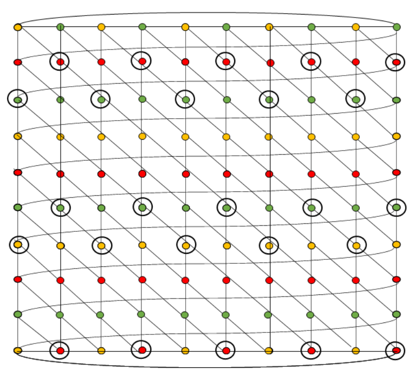

Fig 6. Encircled vertices form a $dc-$ **set of** $B(10,\ 10)$

4 Conclusion

In this study, we have initiated the study of dom-coloring to determine the $dc-$number of wrapped butterfly graph $WB(n)$ and bloom graphs $B(n,\ n)$ for any n. The dom-chromatic number has been obtained for both the graphs by applying the proposed algorithms namely the coloring algorithm and the D-set algorithm which yields the proper coloring and the dominating set respectively. This topic still has scope for further research. To determine the bounds for the bloom graph $B(u, v)$ is open to solve.

Also, the study can be extended to find the $dc-$number for larger networks like the butterfly and benes networks.

5 Declaration

Presented in "International Conference On Recent Trends In Applied Mathematics (ICRTAM 2023)" during 24th -25th February 2023, organized by Department Of Mathematics, Loyola College, Chennai, Tamil Nadu, India. The Organizers claim the peer review responsibility.

References

1) Liu JB, Arockiaraj M, Nelson A. Tight Bounds on 1-Harmonious Coloring of Certain Graphs. *Symmetry*. 2019;11(7):1–11. Available from: https://doi.org/10.3390/sym11070917.
2) Moran R, Bora N, Pegu A, Chamua M. Some results on T-coloring and ST-coloring of Generalized Butterfly Graphs. *Advances and Applications in Discrete Mathematics*. 2022;31:1–12. Available from: https://www.researchgate.net/publication/360319546_SOME_RESULTS_ON_T-COLORING_AND_ST-COLORING_OF_GENERALIZED_BUTTERFLY_GRAPHS.
3) Shalini V, Rajasingh I. Domination and Total domination in Wrapped Butterfly networks. *Procedia Computer Science*. 2020;172:66–70. Available from: https://doi.org/10.1016/j.procs.2020.05.010.
4) Liu J, Zhang X, Chen X. The domination number of wrapped butterfly digraphs. *Applied Mathematics and Computation*. 2021;409:126407. Available from: https://doi.org/10.1016/j.amc.2021.126407.
5) Chaluvaraju B, Appajigowda C. The dom-chromatic number of a graph. *Malaya Journal of Matematik*. 2016;4(01):1–7. Available from: https://www.malayajournal.org/articles/MJM16_1.pdf.
6) Shalini V, Rajasingh I. Domination and Total domination in Wrapped Butterfly networks. *Procedia Computer Science*. 2020;172:66–70. Available from: https://doi.org/10.1016/j.procs.2020.05.010.
7) Xavier DA, Deeni CJ. Bloom Graph. *International Journal of Computing Algorithm*. 2014;3(1):48–49. Available from: http://www.ijcoa.com/documents/V3_I1_2014_paper12.pdf.