INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

d - Lucky Face Labeling of Polytopes

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OPEN ACCESS

Received: 08-02-2023

Accepted: 11-08-2023

Published: 20-11-2023

Citation: Arockiamary ST, Sahayamary ARA (2023) *d* - Lucky Face Labeling of Polytopes. Indian Journal of Science and Technology 16(SP3): 39-43. https://doi.org/10.17485/IJST/v16iSP3.icrtam264

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives: Our main focus in this study is to apply the concept of d-lucky face labeling to a family of polytopes like D_n, S_n, T_n and U_n. **Methods:** The results were obtained by the method of application of formula to the graphs. **Findings:** We have found the d-lucky face number and proved the graphs are Euler lucky graphs. **Novelty:** The concept of d-lucky face labeling and Euler lucky graphs are new findings. The d-lucky face labeling differs from other face labeling by its formula.

2020 Mathematics Subject Classification: 05C78

Keywords: Euler lucky number; Polytopes; Lucky number; d-Lucky labeling; d-Lucky face labeling

1 Introduction

Graph theory has a wide range of applications with numerous research work. In graph labeling our concept has its origin in lucky labeling [1] and proper lucky labeling [2] was its extension. With add on condition it was named as d-lucky labeling [3,4]. This labeling was studied for various graphs by different authors and it was studied for edges as d-lucky edge labeling [5]. The d-lucky face labeling is a variant of d-lucky labeling introduced by us and the concept of lucky number defined by Euler is also applied [6]. We have now applied this concept for the graphs of certain polytopes to study their behaviour and have shown that the graphs are Euler lucky graphs. Some of the applications include image segmentation, image processing, etc. We introduce this type of face labeling in order to enhance the existing application to use it more effectively.

2 Methodology

We have used the method of applying the formula to prove that it satisfies the necessary condition of d-lucky face labeling to find the d-lucky face number. To begin with, the d-lucky labeling was defined as "Let l be a labeling of the vertices of a graph G by positive integers. Define $c(u) = d(u) + \Sigma_{v \in N(u)} l(v)$ where $d(u)$ denotes the u's degree. For each pair of vertices u and v adjacent in G, we define a labeling l as d-lucky if $c(u) \neq c(v)$. The least positive integer k such that a graph G has a d-lucky labeling with the set of labels {1, 2,..., k} is the d-lucky number for that graph, indicated by $\eta_{dl}(G)$".

We have extended the definition of d-lucky labeling to d-lucky face labeling by defining it as "A labeling f : $F(G) \rightarrow (1, 2, \ldots, r\}$ is said to be d-lucky face labeling such that $f(f_i) = d(f_i) + l(f_i) + \sum_{f_j \in N(f_i)} l(f_j) + (v_i|$, where f_i denotes the faces of the graph G, $d(f_i)$ denotes the degree of the faces, $l(f_i)$ denotes the label of the face considered, $l(f_j)$ denotes the labels of the adjacent faces of f_i and f_j belongs to the open neighbourhood of f_i and $|v_i|$ denotes the cardinality of the vertices forming the face f_i, satisfying the condition $f(f_i) \neq f(f_j)$ for every pair of adjacent faces. The least positive integer satisfied by the faces of the graph is the d-lucky face number denoted by $\eta_{df}(G)$". This formula $f(f_i)$ helps in proving the condition $f(f_i) \neq f(f_j)$.

Here, we are finding whether the graphs are Euler lucky for which we define "*A* graph *G* to be Euler lucky graph if its *d*-lucky face number $\eta_{df}(G)$ is the least positive integer satisfied by the graph *G* belonging to the Euler's lucky number set $\{1, 2, 3, 5, 11, 17, 41\}$". We have obtained this Euler lucky number set generated by Euler which states that "A lucky number of Euler is a number p such that the prime generating polynomial $n^2 - n + p$ is prime for $n = 1, 2, \ldots, p-1$. Such numbers are related to the imaginary quadratic field in which the ring of integers is factorable". The following results have been proved using the above definitions.

3 Results and discussion

Theorem 2. 1: The convex polytope D_n admits d-lucky face labeling and the d-lucky face number is 2.

Proof:

The graph D_n consists of three, five and n-sided faces. We prove that the graph D_n has a d-lucky face labeling. So it should satisfy the condition $f(f_i) \neq f(f_j)$ for every pair of adjacent faces. The faces are grouped in the following way. The centre face is f_1, there are $2n$ 3-sided faces between the concentric circles, n 3-sided faces adjacent to the concentric circle and n 5-sided faces lie outermost.

We begin the labeling from f_1 to the outermost 5-sided faces. The labels are as follows:

$$f_i = 1 \begin{cases} for\; i = 1 \\ for\; 2 \leq i \leq 2n, 3n+2 \leq i \leq 4n, i\; even \\ for\; 2n+2 \leq i \leq 3n+1 \end{cases}$$

$f_i = 2, \; 3n+3 \leq i \leq 4n+1, \; i\; odd$

We claim $f(f_i) \neq f(f_j)$. For illustration, we consider D_8. See Figure 1. We take 2 adjacent faces f_1 and f_2. For $f(f_1)$: $d(f_1) = 8$, $l(f_1) = 1$, $\sum_{f_j \in N(f_i)} l(f_j) = 8$ and $|v_i| = 8$. Therefore, $f(f_1) = 8 + 8 + 1 + 8 = 25$. For $f(f_2)$: $d(f_2) = 3$, $l(f_2) = 1$, $\sum_{f_j \in N(f_i)} l(f_j) = 5$ and $|v_i| = 3$. Therefore, $f(f_2) = 3 + 1 + 5 + 3 = 12$. This shows that $f(f_1) \neq f(f_2)$. Therefore, D_n satisfies d-lucky face labeling for each pair of faces adjacent in D_n and the $\eta_{df}(G) = 2$. Since 2 belongs to the Euler lucky number set, we say that the graph D_n is an Euler lucky graph.

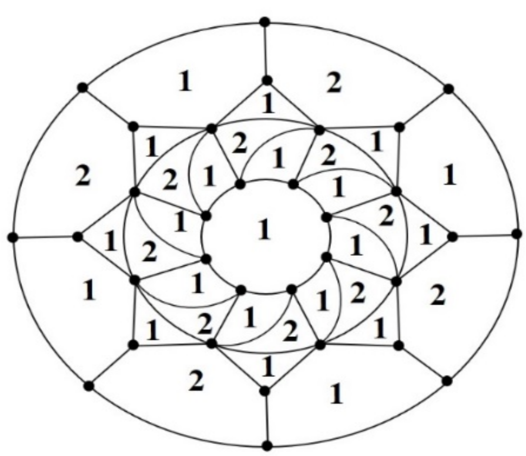

Fig 1. ***d*** **-lucky face labeling of** D_8

Theorem 2. 2: The convex polytope S_n admits d-lucky face labeling and the d-lucky face number is 2.

Proof:

The graph S_n consists of three, four, five and n-sided faces. To prove that the graph S_n has a d-lucky face labeling, it should satisfy the condition $f(f_i) \neq f(f_j)$ for each pair of adjacent faces. The faces are grouped as follows: The centre face is f_1, there are n 3-sided faces, n 6-sided faces adjacent to 3-sided faces and n 5-sided faces lie outermost.

We begin the labeling from f_1 to the outermost 5-sided faces. By definition of $f(f_i)$, we label the faces satisfying the condition $f(f_i) \neq f(f_j)$ for each pair of adjacent faces as follows.

$$f_i = 1 \begin{cases} for\ i = 1 \\ for\ 2 \leq i \leq n+1 \\ for\ n+2 \leq i \leq 3n, \quad i\ even \end{cases}$$

$f_i = 2 \quad for\ n+3 \leq i \leq 3n+1, \quad i\ odd$

We claim $f(f_i) \neq f(f_j)$. We take 2 adjacent faces f_2 and f_{10}. For $f(f_2)$: $d(f_2) = 3,\ l(f_2) = 1,\ \sum_{f_j \in N(f_i)} l(f_j) = 4$ and $(v_i| = 3$. Therefore, $f(f_2) = 11$. For $f(f_{10})$: $d(f_{10}) = 6,\ l(f_{10}) = 1,\ \sum_{f_j \in N(f_i)} l(f_j) = 9$ and $(v_i| = 6$. Therefore, $f(f_3) = 22$. This show that $f(f_2) \neq f(f_{10})$. Therefore, S_n satisfies d-lucky face labeling for each pair of faces adjacent in S_n. The $\eta_{df}(G) = 2$. Since 2 is an Euler lucky number, the graph S_n is an Euler lucky graph. See Figure 2 for illustration.

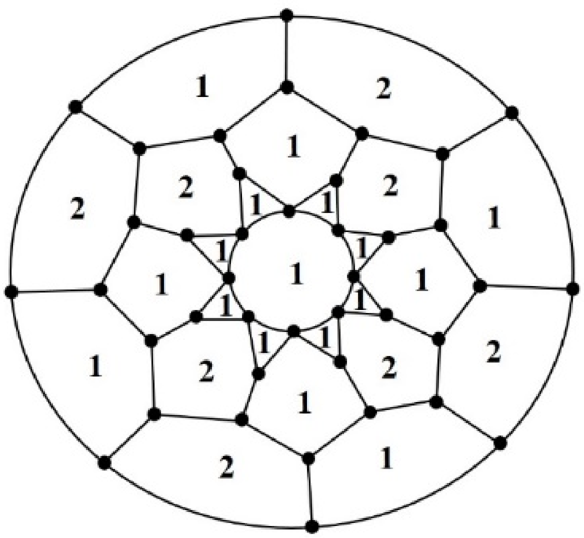

Fig 2. *d* -lucky face labeling of S_8

Theorem 2. 3: The convex polytope T_n admits d-lucky face labeling and the d-lucky face number is 2.

Proof:

The graph T_n consists of three, five and n-sided faces. To prove that the graph S_n has a d-lucky face labeling, it should satisfy the condition $f(f_i) \neq f(f_j)$ for every pair of adjacent faces. The faces are grouped as follows: The centre face is f_1, there are n 3-sided faces and n 5-sided faces adjacent to 3-sided faces.

We begin the labeling from f_1. By definition of $f(f_i)$, we label the faces as follows.

$$f_i = 1 \begin{cases} for\ i = 1 \\ for\ 2 \leq i \leq n+1 \\ for\ n+2 \leq i \leq 2n, \quad i\ even \end{cases}$$

$f_i = 2 \quad for\ n+3 \leq i \leq 2n+1, \quad i\ odd$

Here we consider the graph T_8. Figure 3 shows the d-lucky face labeling of T_8. Therefore, T_n admits d-lucky face labeling and the d-lucky face number is 2. Since $\eta_{df}(G) = 2$ is an Euler lucky number, the graph T_n is an Euler lucky graph.

Theorem 2. 4: The convex polytope U_n admits d-lucky face labeling and the d-lucky face number is 2.

Proof:

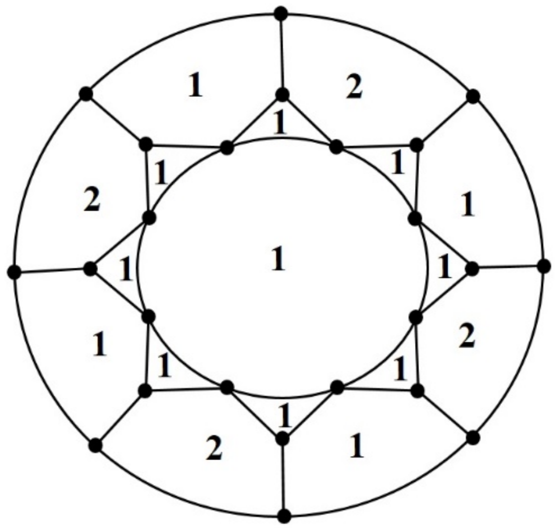

Fig 3. ***d*** **-lucky face labeling of** T_8

The graph U_n consists of n 4-sided faces, $2n$ five sided faces and a pair of n-sided faces. To prove that the graph S_n has a d-lucky face labeling, it should satisfy the condition $f(f_i) \neq f(f_j)$ for every pair of adjacent faces. The faces are grouped as follows: The centre face is f_1, there are n 4-sided faces and $2n$ 5-sided faces.

We begin the labeling from f_1. By definition of $f(f_i)$, we label the faces as follows.

$$f_i = 1 \begin{cases} for\ i = 1 \\ for\ n+2 \leq i \leq 3n \end{cases}$$

$$f_i = 2 \begin{cases} for\ n+2 \leq i \leq 2n, & i\ even \\ for\ n+3 \leq i \leq 2n+1, & i\ odd \end{cases}$$

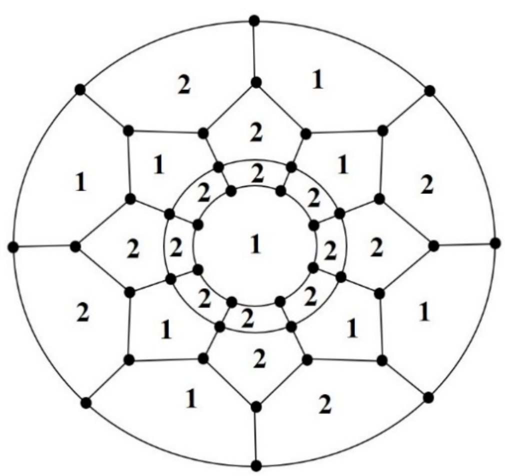

Fig 4. ***d*** **-lucky face labeling of** U_8

This satisfies the condition for d-lucky face labeling and $\eta_{df}(G) = 2$. Since 2 is an Euler lucky number, the graph U_n is an Euler lucky graph. Figure 4 shows the d-lucky face labeling of U_8. Hence, the theorem holds for any even n.

4 Conclusion

In this study, we have proved that convex polytopes admit d-lucky face labeling, with d-lucky face number being 2. Based on the above results the graphs are found to be Euler lucky graphs. We would like to continue working on some of the networks in the future. We also try to apply the notion of d-lucky face labeling in the field of image segmentation for the purpose of advanced research.

5 Declaration

Presented in "International Conference On Recent Trends In Applied Mathematics (ICRTAM 2023)" during 24th -25th February 2023, organized by Department of Mathematics, Loyola College, Chennai, Tamil Nadu, India. The Organizers claim the peer review responsibility.

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