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RESEARCH ARTICLE

Solving Fuzzy Fractional Biological Population Model using Shehu Adomian Decomposition Method

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Abstract

Objectives: In applied sciences and engineering, fuzzy fractional differential equations (FFDEs) are a crucial topic. The main objective of this study is to find the exact solution of the nonlinear Fuzzy Fractional Biological Population Model (FFBPM). In the Caputo concept, fractional derivatives are regarded. **Methods:** For nonlinear problems, the Shehu transform is difficult to exist. So, the Shehu transform is combined with the Adomian decomposition method is called the Shehu Adomian Decomposition Method (SHADM) and has been proposed to solve the FFBPM. **Findings:** The main favor of this method is rapidly converging to the exact solution for nonlinear FFDEs. The theoretical proof of convergence for the SHADM and the uniqueness of the solution is given. **Novelty:** Adomian polynomials are used for nonlinear terms. Figures and numerical examples demonstrate the expertise of the suggested approach. This method is applied for both linear and nonlinear ordinary and partial FFDEs. The proposed approach is rapid, exact, and simple to apply and produce excellent outcomes.

Keywords: Caputo fractional derivative; Shehu transform; triangular fuzzy number; Fuzzy Fractional Biological Population Model; Shehu Adomian decomposition method

1 Introduction

Fractional calculus is expanded upon in ordinary calculus. This involves computing a function's derivative in any order. The memory and heredity characteristics of numerous substances and processes in porous media, electrical circuits, control, biology, electromagnetic processes, biomechanics, and electro chemistry have been documented using fractional differential operators. Over the past few decades, fractional calculus and its applications have gained popularity, mostly because it has proven to be a helpful tool for modelling a number of complex processes in a wide range of seemingly unrelated fields of science and engineering. Measurement uncertainty is represented by a fuzzy number. Since its invention by Lotfi Zadeh in 1965, fuzzy sets have found utility in a variety of contexts.

The solution for fractional differential equations in uncertain settings was first introduced by Agarval in 2010. There have been many implementations in the area of Fuzzy Fractional Differential Equations (FFDEs) and Fuzzy Fractional Partial Differential Equations (FFPDEs) to find exact and numerical solutions. Consider the general FFPDE with fuzzy initial conditions of the form:

$$\frac{\partial^{\delta}\widetilde{w}(\zeta,t)}{\partial t^{\delta}}+p\widetilde{w}(\zeta,t)+Q\widetilde{w}(\zeta,t)=h(\zeta,t), \qquad m-1<\delta<m,\ m\in N \tag{1}$$

with the fuzzy initial condition $\frac{\partial^{k}\widetilde{w}(\zeta,0)}{\partial t^{k}}=\widetilde{q}^{k}(\zeta), \quad k=0,1.2,\ldots,m-1.$

where $\frac{\partial^{\delta}\widetilde{w}(\zeta,t)}{\partial t^{\delta}}$ represents the Caputo fractional derivative of $\widetilde{w}(\zeta,t)$and $\widetilde{w}(\zeta,t)$ is the fuzzy function. The general nonlinear and linear differential operators are denoted by Q and P. The source term is $h(\zeta,t)$.

The focus of this work is to calculate the analytical solution for the nonlinear FFBPM in the form

$$\frac{\partial^{\delta}\widetilde{w}}{\partial t^{\delta}}=\frac{\partial^{2}}{\partial\zeta_{1}^{2}}\left(\widetilde{w}^{2}\right)+\frac{\partial^{2}}{\partial\zeta_{2}^{2}}\left(\widetilde{w}^{2}\right)+f(\widetilde{w})$$

with fuzzy initial condition w$_{\sim}(\zeta_1, \zeta_2, 0)$. These factors of position $\zeta=(\zeta_1,\zeta_2)$ in

B and time t can be used to model the spread of a biological species in a region

$B=\widetilde{w}(\zeta,t)$ is the population density, and $\widetilde{f}(\zeta,t)$is the population supply due to birth and death rate.

Moez Benhamed et al.[1] applied the ZZ transformation and the new iterative transform technique to obtain a fascinating explicit pattern for outcomes of the biological population model. Muhammad Shakeel et al.[2] applied a modified exp-function method to calculate closed-form solutions of a degenerate parabolic equation arising in the spatial diffusion of biological populations. Maysaa Al-Qurashi et al.[3] proposed the Shehu decomposition method for the fractional Zakharov-Kuznetsov model in plasma fluid. Safaa Hamid Mahdi et al.[4] introduced the Elzaki Adomian decomposition method for solving the nonlinear fractional partial differential equations. Rashid et al.[5] constructed an approximate analytical solution for multi-dimensional fractional Zakharov–Kuznetsov equation via the Aboodh Adomian decomposition method. Mostafa M.A. Khater et al.[6] proposed the nonlinear biological population model.

To solve FFPDEs, various techniques have been devised. Shehu transform method was suggested by Gethsi Sharmila et al.[7] to solve linear FFDEs. One-dimensional FFPDEs were suggested to be evaluated by Kamal Shah et al.[8]. Sahar Askari et al.[9] solved the fuzzy fractional differential equations by the Adomian decomposition method used in optimal control theory. It is difficult to obtain the exact solution for the nonlinear problem because of its complexity. So here the Shehu transform is combined with Adomian decomposition method is called the Shehu Adomian Decomposition Method (SHADM) has been proposed. SHADM does not need perturbation, linearization, or discretization because it is an enhanced version of ADM. The considered technique is unique in that it uses a simple method to assess the result and is based on Adomian polynomials, which allows quick convergence of the found solution for nonlinear terms of the problem. When compared to traditional methods, the proposed method can reduce the volume of computing effort while retaining high numerical accuracy; the size reduction equates to an improvement in the approach's performance.

This article is organized as follows: The fundamental definitions of the fractional derivative, Shehu transform, and fuzzy numbers are provided in Section 1. The construction of SHADM for FFBPM is described in Section 2. And also the uniqueness and convergence of the suggested approach for FFPDEs are demonstrated in Section 2. Some numerical examples and demonstrations of the given figures are discussed in section 3. The conclusion is provided in Section 4.

1.1 Basic Tools

The essential definitions of fractional calculus and the Shehu transform of fractional derivatives are provided[10] and used throughout the rest of the work.

Definition 1.1. The fractional derivative of Caputsense, is defined as follows:

${}^{c}D_{t}^{\delta}f(t)=\frac{1}{\Gamma(m-\delta)}\int_{0}^{t}(t-\Phi)^{m-\delta-1}f^{(m)}(\Phi)\,d\Phi, \quad m-1<\delta\leq m,\ t>0,\ m\in N$

Definition 1.2. Over the set of functions, the Shehu transform of is defined as[11],

$B=\left\{f(t)\setminus\ni M,\Upsilon_1,\Upsilon_2>0,|f(t)|<Mexp\left(\frac{|t|}{\Upsilon_j}\right), \ if\ t\in(-1)^{j}\times(0,\infty)\right\},$

Through the integral

$S(f(t)]=F(s,u)=\int_{0}^{\infty}exp\left(-\frac{st}{u}\right)f(t)dt, \quad t>0,$

Next, the inverse of the Shehu transform is denoted by $S^{-1}\left[F\left(s,u\right)\right]=f\left(t\right),\ t\geq 0$.
For δ is a fractional number,

$$S\left[t^{\delta}\right]=\left(\frac{u}{s}\right)^{\delta+1}\Gamma\left(\delta+1\right)$$

Definition 1.3. The Shehu transform of the Caputo derivative is defined as
$S\left[{}^{c}D_{t}^{\delta}f\left(t\right)\right]=\frac{s^{\delta}}{u^{\delta}}F\left(s,u\right)-\sum_{q=0}^{m-1}\left(\frac{s}{u}\right)^{\delta-(q+1)}f^{(q)}\left(0\right),\quad m-1<\delta\leq m$
Definition 1.4 (Triangular fuzzy number)
It's a three-pointed fuzzy number represented by $N=\left(\theta_1,\theta_2,\theta_3\right)$. N's membership function is as follows:

$$\mu_N(x)\ =\ \begin{cases} 0, & \chi<\theta_1 \\ \frac{\chi<\theta_1}{\theta_2-\theta_1} & \theta_1\leq\chi<\theta_2 \\ \frac{\theta_3-\chi}{\theta_3-\theta_2} & \theta_2\leq\chi<\theta_3 \\ 0, & \chi>\theta_3 \end{cases}$$

2 Methodology

This section explains the methodology of Shehu Adomian decomposition method for FFBPM.

2.1 Fuzzy Fractional Shehu Adomian Decomposition Method

Consider the FFBPM given as

$$D_t^{\delta}\,\widetilde{w}\left(\zeta_1,\zeta_2,t\right)=D_{\zeta_1}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+D_{\zeta_2}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+\widetilde{k}\left(\sigma\right),\ 0<\delta\leq 1$$

with fuzzy initial conditions

$$\widetilde{w}\left(\zeta_1,\zeta_2,0\right)=\widetilde{g}(\zeta)$$

where $D_t^{\delta}=\frac{\partial^{\delta}}{\partial t^{\delta}},\ D_{\zeta_1}^{2}=\frac{\partial^{2}}{\partial\zeta_1^{2}},\ D_{\zeta_2}^{2}=\frac{\partial^{2}}{\partial\zeta_2^{2}}$
Apply the Shehu transformation to as follows:

$$S[D_t^{\delta}\,\widetilde{w}\left(\zeta_1,\zeta_2,t\right)]=S[D_{\zeta_1}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+D_{\zeta_2}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+\widetilde{k}\left(\sigma\right)]$$

Using the initial condition

$$\frac{s^{\delta}}{u^{\delta}}S[\,\widetilde{w}\left(\zeta_1,\zeta_2,t\right)]=\frac{s^{\delta-1}}{u^{\delta-1}}\widetilde{g}(\zeta_1)+S[D_{\zeta_1}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+D_{\zeta_2}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+\widetilde{k}\left(\sigma\right)]$$

$$S[\,\widetilde{w}\left(\zeta_1,\zeta_2,t\right)]=\frac{u}{s}\widetilde{g}(\zeta_1)+\frac{u^{\delta}}{s^{\delta}}S[D_{\zeta_1}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+D_{\zeta_2}^{2}\,\widetilde{w}^{2}\left(\zeta_1,\zeta_2,t\right)+\widetilde{k}\left(\sigma\right)]$$

The ADM assumes that $\widetilde{w}(t,r)$can be broken into an infinite series[12]

$$\widetilde{w}\left(t,r\right)=\sum_{n=0}^{\infty}\widetilde{w}_n(t,r) \tag{2}$$

where $\widetilde{w}_n(t,r)$ is specified recursively. Here the nonlinear term $\widetilde{w}^2$ is decomposed as follows:

$$\widetilde{w}^2=\sum_{n=0}^{\infty}A_n \tag{3}$$

where $A_n=A_n\left(\widetilde{w}_0,\widetilde{w}_1,\ldots,\widetilde{w}_n\right)$ is the Adomian polynomial,

$$A_n\left(\widetilde{w}_0,\widetilde{w}_1,\ldots,\widetilde{w}_n\right)=\frac{1}{n!}\frac{d^n}{d\mu^n}\left[N\left(\sum_{k=0}^{n}\mu^k\widetilde{w}_k\right)\right]\ ,\ n=0,1,2,\ldots$$

Here μ is a parameter. Then, the Adomian polynomial A_n is described as follows. Thus,

$$A_0 = \widetilde{w}_0^2$$

$$A_1 = 2\widetilde{w}_0\widetilde{w}_1,$$

$$A_2 = 2\widetilde{w}_0\widetilde{w}_2 + \widetilde{w}_1^2,$$

$$A_3 = 2\widetilde{w}_0\widetilde{w}_3 + 2\widetilde{w}_1\widetilde{w}_2,$$

$$\vdots$$

$$S\left[\sum_{n=0}^{\infty} \widetilde{w}_n(\zeta_1, \zeta_2, t)\right] = \frac{u}{s}\widetilde{g}(\zeta) + \frac{u^\delta}{s^\delta} S\left[D_{\zeta_1}^2 \sum_{n=0}^{\infty} A_n + D_{\zeta_2}^2 \sum_{n=0}^{\infty} A_n + \widetilde{k}(\sigma)\right]$$

Comparisons on both sides

$$S\left[\widetilde{w}_0(\zeta_1, \zeta_2, t)\right] = \frac{u}{s}\widetilde{g}(\zeta) + \frac{u^\delta}{s^\delta} S\left[\widetilde{k}\right],$$

$$S\left[\widetilde{w}_1(\zeta_1, \zeta_2, t)\right] = \frac{u^\delta}{s^\delta} S\left[D_{\zeta_1}^2 A_0 + D_{\zeta_2}^2 A_0\right]$$

$$S\left[\widetilde{w}_2(\zeta_1, \zeta_2, t)\right] = \frac{u^\delta}{s^\delta} S\left[D_{\zeta_1}^2 A_1 + D_{\zeta_2}^2 A_1\right]$$

$$\vdots$$

$$S\left[\widetilde{w}_{n+1}(\zeta_1, \zeta_2, t)\right] = \frac{u^\delta}{s^\delta} S\left[D_{\zeta_1}^2 A_n + D_{\zeta_2}^2 A_n\right]$$

$$\vdots$$

Applying inverse Shehu transform,

$$\widetilde{w}_0(\zeta_1, \zeta_2, t) = \frac{u}{s}\widetilde{g}(\zeta) + S^{-1}\left[\frac{u^\delta}{s^\delta} S\left[\widetilde{k}\right]\right]$$

$$\widetilde{w}_1(\zeta_1, \zeta_2, t) = S^{-1}\left[\frac{u^\delta}{s^\delta} S\left[D_{\zeta_1}^2 A_0 + D_{\zeta_2}^2 A_0\right]\right]$$

$$\widetilde{w}_2(\zeta_1,\zeta_2,t) = S^{-1}\left[\frac{u^\delta}{s^\delta}S\left[D^2_{\zeta_1}A_1 + D^2_{\zeta_2}A_1\right]\right]$$

$$\vdots$$

$$\widetilde{w}_{n+1}(\zeta_1,\zeta_2,t) = S^{-1}\left[\frac{u^\delta}{s^\delta}S\left[D^2_{\zeta_1}A_n + D^2_{\zeta_2}A_n\right]\right]$$

$$\vdots$$

The series type solution is obtained as

$$\widetilde{w}(\zeta_1,\zeta_2,t) = \widetilde{w}_0(\zeta_1,\zeta_2,t) + \widetilde{w}_1(\zeta_1,\zeta_2,t) + \widetilde{w}_2(\zeta_1,\zeta_2,t) + \ldots \tag{4}$$

2.2 Convergence Analysis

The essential requirement that guarantees the existence of a unique solution and convergence of the SHADM for general FFPDE is laid out here.

Theorem 2.1 Uniqueness theorem of SHADM for FFPDE

The FFPDE Equation (1) has a unique solution when $0 < \gamma = (\gamma_1 + \gamma_2)\ \frac{t^\delta}{r(\delta+1)} < 1$.

Proof:

Let χ be the Banach space of all continuous functions on the interval $J = [0,T]$ with the norm $||\widetilde{w}(\zeta,t)|| = max|\widetilde{w}(\zeta,t)|$. Define a mapping $F:\chi \to \chi$ where

$$\widetilde{w}(\zeta,t) = \theta(t) + S^{-1}\left[\left(\frac{u}{s}\right)^\delta S[P\widetilde{w}(\zeta,t) + Q\widetilde{w}(\zeta,t) + h(\zeta,t)]\right]$$

Let $\widetilde{w},\widetilde{w}^* \in \chi$. Assume $|P\widetilde{w} - P\widetilde{w}^*| < \delta_1|\widetilde{w} - \widetilde{w}^*|$ and $|Q\widetilde{w} - pQ| < \delta_2|\widetilde{w} - \widetilde{w}^*|$ And the function values $\widetilde{w}$ and $\widetilde{w}^*$ are distinct.

$$||G\widetilde{w} - G\widetilde{w}^*|| = \max_{t\in J}\left|S^{-1}\left(\left(\frac{u}{s}\right)^\delta (S[P\widetilde{w}(\zeta,t) - P\widetilde{w}^*(\zeta,t) + Q\widetilde{w}(\zeta,t) - Q\widetilde{w}^*(\zeta,t)]]\right|\right|$$

$$\leq \max_{t\in J}(\gamma_1+\gamma_2)\left[S^{-1}\left[\left(\frac{u}{s}\right)^\delta |S\widetilde{w}(\zeta,t) - \widetilde{w}^*(\zeta,t)|\right]\right]$$

$$= (\gamma_1+\gamma_2)\frac{t^\delta}{\Gamma(\delta+1)}||\widetilde{w}(\zeta,t) - \widetilde{w}^*(\zeta,t)||$$

G is contraction as $0 < (\gamma_1 + \gamma_2)\ \frac{t^\delta}{r(\delta+1)} < 1$. Hence, Equation (1) has a unique solution.

Theorem 2.2 Convergence theorem of SHADM for FFPDE

The solution $\widetilde{w}(\zeta,t) = \sum_{i=0}^{\infty}\widetilde{w}_i(\zeta,t)$ of Equation (1) using SHADM converges if $0 < \gamma < 1$ and $((\widetilde{w}_n|| < \infty$ where $\gamma = (\gamma_1+\gamma_2)\frac{t^\delta}{\Gamma(\delta+1)}$.

Proof:

Let $\widetilde{Z}_m = \sum_{r=0}^{m}\widetilde{w}_r(\zeta,r)$ be the partial sum of series. To show that in Banach space χ, $\widetilde{Z}_m$ is a Cauchy sequence.

$\left|\left|\widetilde{Z}_m-\widetilde{Z}_n\right|\right| = \max_{t\in J}|\widetilde{Z}_m-\widetilde{Z}_n| = \max_{t\in J}\left|\sum_{r=n+1}^{m}\widetilde{w}_r(\zeta,r)\right|, \quad n=1,2,3,\ldots$
$=\max_{t\in J}\left(S^{-1}\left(\left(\frac{u}{s}\right)^{\delta}S\left(\left(P(\widetilde{Z}_{m-1})\right)-P(\widetilde{Z}_{n-1})+Q(\widetilde{Z}_{m-1})-Q(\widetilde{Z}_{n-1})\right]\right]\right|$
$\leq \max_{t\in J}\left(S^{-1}\left(\left(\frac{u}{s}\right)^{\delta}S\left(\left(P(\widetilde{Z}_{m-1})\right)-P(\widetilde{Z}_{n-1})\right]\right]\right| + \max_{t\in J}\left(S^{-1}\left(\left(\frac{u}{s}\right)^{\delta}S\left(Q(\widetilde{Z}_{m-1})-Q(\widetilde{Z}_{n-1})\right]\right]\right|$
$\leq \gamma_1\max_{t\in J}\left|S^{-1}\left[\left(\frac{u}{s}\right)^{\delta}S\left[\left(P(\widetilde{Z}_{m-1})\right)-P(\widetilde{Z}_{n-1})\right]\right]\right| + \gamma_2\max_{t\in J}\left|S^{-1}\left[\left(\frac{u}{s}\right)^{\delta}S\left[Q(\widetilde{Z}_{m-1})-Q(\widetilde{Z}_{n-1})\right]\right]\right|$
$=(\gamma_1+\gamma_2)\frac{t^{\delta}}{\Gamma(\delta+1)}||\widetilde{Z}_{m-1}-\widetilde{Z}_{n-1}||$
If $m=n+1$,

$$\left|\left|\widetilde{Z}_{n+1}-\widetilde{Z}_n\right|\right| \leq \gamma\left|\left|\widetilde{Z}_n-\widetilde{Z}_{n-1}\right|\right| \leq \gamma^2\left|\left|\widetilde{Z}_{n-1}-\widetilde{Z}_{n-2}\right|\right| \leq \cdots \leq \gamma^n|\left(\widetilde{Z}_1-\widetilde{Z}_0|\right|$$

Where $\gamma=(\gamma_1+\gamma_2)\frac{t^{\delta}}{\Gamma(\delta+1)}$. In a similar way,

$$\left|\left|\widetilde{Z}_m-\widetilde{Z}_n\right|\right| \leq \left|\left|\widetilde{Z}_{n+1}-\widetilde{Z}_n\right|\right| + ||\widetilde{Z}_{n+2}-\widetilde{Z}_{n+1}|| + \cdots + ||\widetilde{Z}_m-\widetilde{Z}_{m-1}||$$

$$\leq (\gamma^n+\gamma^{n-1}+\cdots+\gamma^{m-1})||\widetilde{Z}_1-\widetilde{Z}_0||$$

$\leq \gamma^n\left(\frac{1-\gamma^{m-n}}{1-\gamma}\right)||\widetilde{w}_1||$
Since $\gamma\in(0,1)$, $1-\gamma^{m-n}<1$. As a result

$$\left|\left|\widetilde{Z}_m-\widetilde{Z}_n\right|\right| \leq \left(\frac{\gamma^n}{1-\gamma}\right)||\widetilde{w}_1||$$

Since $||\widetilde{w}_1||<\infty$. (Since $\widetilde{w}(\zeta)$ is bounded). $\left|\left|\widetilde{Z}_m-\widetilde{Z}_n\right|\right| \to 0$ when $n\to\infty$. Hence, $\widetilde{Z}_m$ is the Cauchy sequence. As a result, the series $\widetilde{Z}_m$ is convergent.

3 Results and discussion

This section discusses some examples for FFBPM using the methodology of SHADM.

Example 3.1 Consider the FFBPM

$$\frac{\partial^{\delta}\widetilde{w}}{\partial t^{\delta}} = \frac{\partial^2}{\partial\zeta_1^2}\left(\widetilde{w}^2\right) + \frac{\partial^2}{\partial\zeta_2^2}\left(\widetilde{w}^2\right) + h\widetilde{w}$$

with the fuzzy initial condition

$$\widetilde{w}(\zeta_1,\zeta_2,0) = \widetilde{q}(\sigma)\sqrt{\zeta_1\zeta_2}$$

Where $\widetilde{q}(\sigma) = \left[\underline{q}(\sigma),\overline{q}(\sigma)\right] = [\sigma-1, 1-\sigma],\ 0\leq\sigma\leq1,\ h=1.$

Applying the Shehu Adomian decomposition method

$$\underline{w}_0(\zeta_1,\zeta_2,t) = \underline{q}\sqrt{\zeta_1\zeta_2}$$

$$\overline{w}_0(\zeta_1,\zeta_2,t) = \overline{q}\sqrt{\zeta_1\zeta_2}$$

$$\underline{w}_1(\zeta_1,\zeta_2,t) = \underline{q}\sqrt{\zeta_1\zeta_2}\frac{t^{\delta}}{\Gamma(\delta+1)}$$

$$\overline{w}_1(\zeta_1,\zeta_2,t)=\overline{q}\sqrt{\zeta_1\zeta_2}\frac{t^{\delta}}{\Gamma(\delta+1)}$$

$$\underline{w}_2(\zeta_1,\zeta_2,t)=\underline{q}\sqrt{\zeta_1\zeta_2}\frac{t^{2\delta}}{\Gamma(2\delta+1)}$$

$$\overline{w}_2(\zeta_1,\zeta_2,t)=\overline{q}\sqrt{\zeta_1\zeta_2}\frac{t^{2\delta}}{\Gamma(2\delta+1)}$$

$\vdots$

$$\widetilde{w}(\zeta_1,\zeta_2,t)=\widetilde{w}_0(\zeta_1,\zeta_2,t)+\widetilde{w}_1(\zeta_1,\zeta_2,t)+\widetilde{w}_2(\zeta_1,\zeta_2,t)+\ldots$$

The exact solution is

$$\underline{w}(\zeta_1,\zeta_2,t)=\underline{q}\sqrt{\zeta_1\zeta_2}E_{\delta,1}(t^{\delta})$$

$$\overline{w}(\zeta_1,\zeta_2,t)=\overline{q}\sqrt{\zeta_1\zeta_2}E_{\delta,1}(t^{\delta})$$

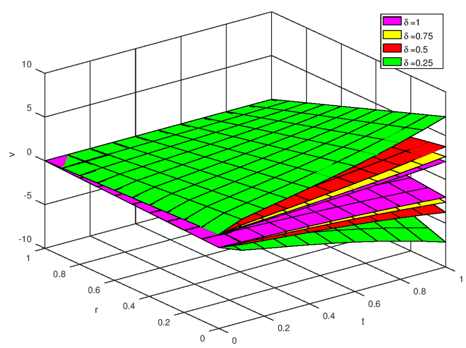

Fig 1. 3D Exact Solution for Example 3.1 at $\zeta_1,\zeta_2=$ 0.75

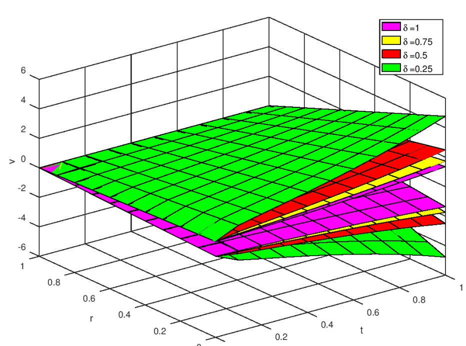

Fig 2. 3D Exact Solution for Example 3.1 at $\zeta_1,\zeta_2=$ 0.5

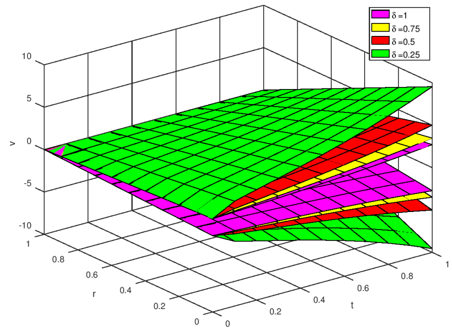

Fig 3. 3D Exact Solution for Example 3.1 at ζ_1, ζ_2= 1

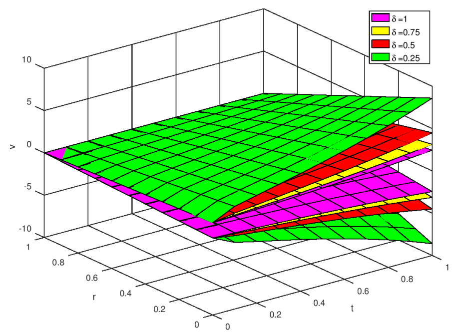

Fig 4. 3D Exact Solution for Example 3.1 at ζ_1, ζ_2= 0.9

From Figures 1, 2, 3 and 4, the surface plots are given for the exact solution of Example 3.1 corresponding to different fractional order (δ= 0.5, 0.75, 0.9 and 1) for various values of both t and r (varies from 0 to 1) at different values for ζ_1 and ζ_2.

Example 3.2 Consider the FFBPM

$$\frac{\partial^\delta \widetilde{w}}{\partial t^\delta} = \frac{\partial^2}{\partial \zeta_1^2}\left(\widetilde{w}^2\right) + \frac{\partial^2}{\partial \zeta_2^2}\left(\widetilde{w}^2\right) + \widetilde{w}$$

with the fuzzy initial condition

$$\widetilde{w}(\zeta_1, \zeta_2, 0) = \widetilde{q}(\sigma)\sqrt{sin\zeta_1 sinh\zeta_2}$$

where $\widetilde{q}(\sigma) = \left[\underline{q}(\sigma), \overline{q}(\sigma)\right] = [\sigma - 1, 1 - \sigma],\ 0 \le \sigma \le 1.$

Applying the Shehu Adomian decomposition method

$$\underline{w}_0(\zeta_1, \zeta_2, t) = \underline{q}\sqrt{sin\zeta_1 sinh\zeta_2}$$

$$\overline{w}_0(\zeta_1, \zeta_2, t) = \overline{q}\sqrt{sin\zeta_1 sinh\zeta_2}$$

$$\underline{w}_1(\zeta_1, \zeta_2, t) = \underline{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^\delta}{\Gamma(\delta+1)}$$

$$\overline{w}_1(\zeta_1, \zeta_2, t) = \overline{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^\delta}{\Gamma(\delta+1)}$$

$$\underset{-}{w}_2(\zeta_1,\zeta_2,t) = \underset{-}{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^{2\delta}}{\Gamma(2\delta+1)}$$

$$\overline{w}_2(\zeta_1,\zeta_2,t) = \overline{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^{2\delta}}{\Gamma(2\delta+1)}$$

$$\vdots$$

$$\widetilde{w}(\zeta_1,\zeta_2,t) = \widetilde{w}_0(\zeta_1,\zeta_2,t) + \widetilde{w}_1(\zeta_1,\zeta_2,t) + \widetilde{w}_2(\zeta_1,\zeta_2,t) + \ldots$$

In general,

$$\underset{-}{w}(\zeta_1,\zeta_2,t) = \underset{-}{q}\sqrt{sin\zeta_1 sinh\zeta_2} + \underset{-}{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^{\delta}}{\Gamma(\delta+1)} + \underset{-}{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^{2\delta}}{\Gamma(2\delta+1)} + \ldots$$

$$\overline{w}(\zeta_1,\zeta_2,t) = \overline{q}\sqrt{sin\zeta_1 sinh\zeta_2} + \overline{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^{\delta}}{\Gamma(\delta+1)} + \overline{q}\sqrt{sin\zeta_1 sinh\zeta_2}\frac{t^{2\delta}}{\Gamma(2\delta+1)} + \ldots$$

The exact solution is

$$\underset{-}{w}(\zeta_1,\zeta_2,t) = \underset{-}{q}(\sigma)\sqrt{sin\zeta_1 sinh\zeta_2}E_{\delta,1}(t^{\delta})$$

$$\overline{w}(\zeta_1,\zeta_2,t) = \overline{q}(\sigma)\sqrt{sin\zeta_1 sinh\zeta_2}E_{\delta,1}(t^{\delta})$$

From Figures 5, 6, 7 and 8, the surface plots are given for the exact solution of Example 3.2 corresponding to different fractional order (δ= 0.5, 0.75, 0.9 and 1) for various values of both t and r (varies from 0 to 1) at different values for ζ_1 and ζ_2.

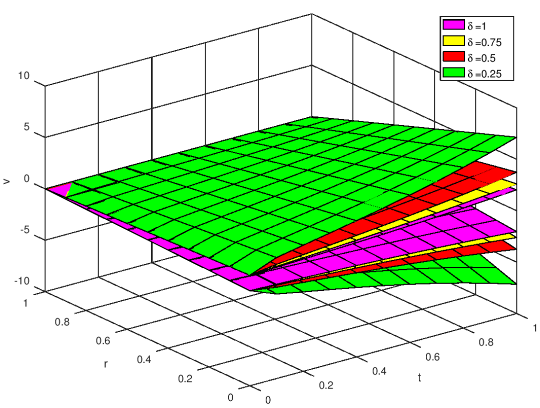

Fig 5. 3D Exact Solution for Example 3.2 at ζ_1, ζ_2= 0.75

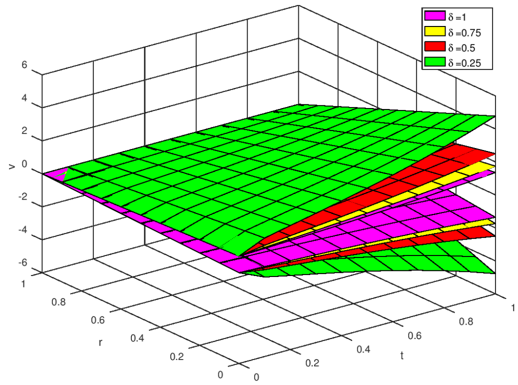

Fig 6. 3D Exact Solution for Example 3.2 at ζ_1, ζ_2= 0.5

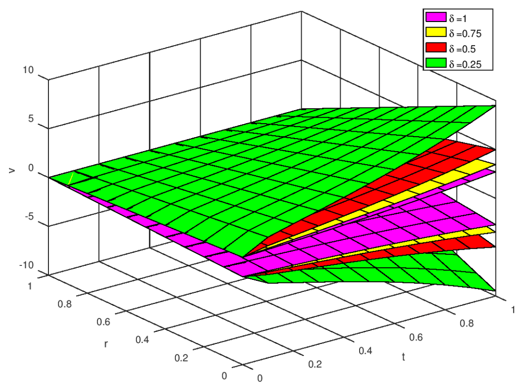

Fig 7. 3D Exact Solution for Example 3.2 at ζ_1, ζ_2= 1.0

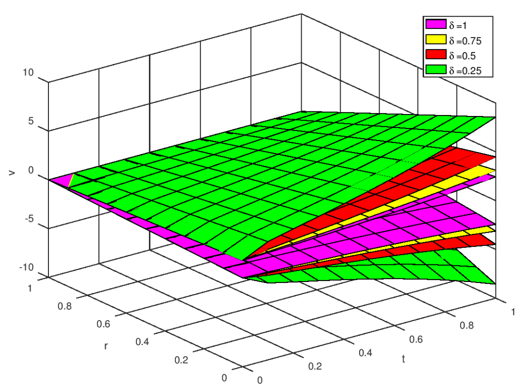

Fig 8. 3D Exact Solution for Example 3.2 at ζ_1, ζ_2= 0.9

4 Conclusion

In this study, the SHADM is successfully used for nonlinear FFBPM. The benefits of using this technique is: The problem does not need any small or big parametric assumptions. This approach can be used to solve fuzzy fractional ordinary and partial differential equations that are both linear and nonlinear. Both weak and highly nonlinear problems can be solved using this technique. The method under consideration is distinctive in that it is based on Adomian polynomials, which enables speedy convergence of the discovered solution for the nonlinear term of the problem, and it employs a straightforward method to evaluate the solution for FFBPM. The proposed method's uniqueness and convergence theorem are investigated.

The given figures demonstrate that it is possible to calculate the exact solution for different values of the fractional orders δ and t as well as r. It may be inferred that the suggested approach is rapid, exact, and simple to apply and produce excellent outcomes. As fractional order differential systems in uncertain environments, this method has been used in the future for many significant applications in engineering and science.

5 Declaration

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