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RESEARCH ARTICLE

A Novel Radon Transform for Image Registration

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Abstract

Objective: Registration is a critical step in any image analysis. In the Radon Transform, rotation can be expressed as translation, and the Fourier Transform is invariant to translation. In this paper we present an efficient combination of Radon Transform and Fourier Transform for obtaining transformation to correct scale, translation and rotation. This concept is used for image matching. **Method:** This study presents an innovative Radon Transform to register two images. The method involves calculating a Radon Transform first, followed by a Fourier description for each image that needs to be matched. Mono-modal visuals are used in experiments. The Lena dataset is used for correcting scale, translation, and rotation. The same algorithm is used on around 36 images to correct scale, translation, rotation, scale and rotation, rotation and translation, scale and translation etc. **Findings:** This algorithm is tested successfully with different datasets, which differ by translation and/or rotation. Lena image dataset is used that was scaled, rotated and translated with the objective of template matching. Around 36 datasets are used for experimentation. **Novelty :** Innovative method of combination of Fourier and Radon Transform for image registration is attempted with 98 % accuracy.

Keywords: Image registration; image fusion; Radon Transform; Fourier transforms; affine transform

1 Introduction

Image registration is one of the important imaging technologies, which matches two or more images of the same scene. Image registration is used in medical sciences, remote sensing, and computer vision. Identifying a rigid body transformation namely, scale, rotation, and translation of an image is critical step in terms of accuracy, speed and calculation complexity.

Various methods are reported in literature to register images which are in same band. Hjouj, F. Jouini et al. present methods for identifying an image from a given set of Radon projections. They show how the Radon projections of f and g can be used to determine the transformation [1–3]. Cocianu, Cătălina-Lucia et al. reviewed and compared algorithms in terms of evolutionary components, fitness function, image similarity measures and algorithm accuracy indexes used in the alignment process. They observed that new algorithms and algorithm variants are continuously proposed and explored [4]. Radon transforms of ultrasonic data are used in carbon fibre composites to detect fibre direction and stacking sequence, resulting in a fiber-orientation distribution for a particular region [5]. A method is devised that investigates a novel way to invert the Radon transform, which is suitable for dealing with images already transformed using the multiscale approach [6,7]. Jia, X., Bartlett, J., Chen e al proposed the Fourier-Net, replacing the expansive path in a U-Net style network with a parameter-free model-driven decoder, they observed that the Fourier-Net is more efficient than state-of-the art methods in terms of speed while retaining a comparative performance in terms of registration accuracy [8]. In pixel based method it is observed that in natural images like buildings or scenery, method shows match at multiple points. The feature based method shows more accuracy but it is manual [9,10]. Contour based methods do not use the gray values for matching and hence overcomes the limitations of spatial methods. In frequency based method accuracy is more than correlation method but less as compared to other methods. But if we extract image features and then apply Fourier method accuracy increases [11,12]. Image registration is difficult when images are obtained through different sensor types. Mutual Information, Hotelling Transform, Fuzzy logic are some of the approaches that can be used for multimodal image registration.

Feature detection, feature matching, mapping function design, and image transformation and resampling are steps of image registration. The reviewed articles revealed that new algorithms and algorithm variants are continuously proposed and explored. In this study mono modal image registration is done using Radon Transform and Fourier transform.

1.1 Problem Definition

Spatial or geometric variations and intensity or radiometric variations are two categories into which the variations between two or more photographs of the same scene can be divided. Geometric distortion is caused by images of the same scene taken from slightly different angles. This work aims to address geometric variations. Let there be two images as two 2-D arrays of given size denoted by I1(x,y) and I2(x,y), and each map to their respective intensity or other measurable values. Then mapping between images can be expressed as

$$I2\,(x,y) = g(I1\,(f\,(x,y))) \tag{1}$$

Where f is a transformation which maps two spatial coordinates x, y to new spatial coordinates x' and y'.

$$(x^{'}\ y^{'}) = f(x,y) \tag{2}$$

And 'g' is a 1-D intensity or radiometric transformation. The registration problem is to find the optimal spatial and intensity transformations so that the images are matched. Finding the parameters of the optimal spatial or geometric transformation is generally the key to any registration problem. It is frequently expressed parametrically as two single valued functions, *fx* and *fy*.

$$I2\ (x,y) = I1(f_x\,(x,y)\,,\ f_y\,(x,y)) \tag{3}$$

This may be implemented. The fundamental characteristic of any image registration technique is the type of spatial transformations or mapping used to properly overlay two images. The most common transformations are rigid, affine, projective, perspective, and global [1].

2 Methodology

This section presents innovative solutions to several transformation difficulties. Images of the same scene are captured from several perspectives (view points). Lena image dataset is used that was scaled, rotated and translated objective was template matching. Around 36 datasets are used for experimentation. An innovative Radon Fourier approach is used to find rigid body transformation. Detailed procedure is explained in following sections.

2.1 Radon Transform for Image Registration

To detect linear features, the Radon Transform is utilised. The Radon Transform may convert two-dimensional images with lines into a domain of line parameters, with each line in the image producing a peak at the corresponding line parameters. Figure 1 depicts the geometry of the Radon Transform.

Radon Transform of two dimensional function f (x,y) in (s, α) plane is defined as

$$R\,(s,\,\propto) = R[f\,(x,y)] \tag{4}$$

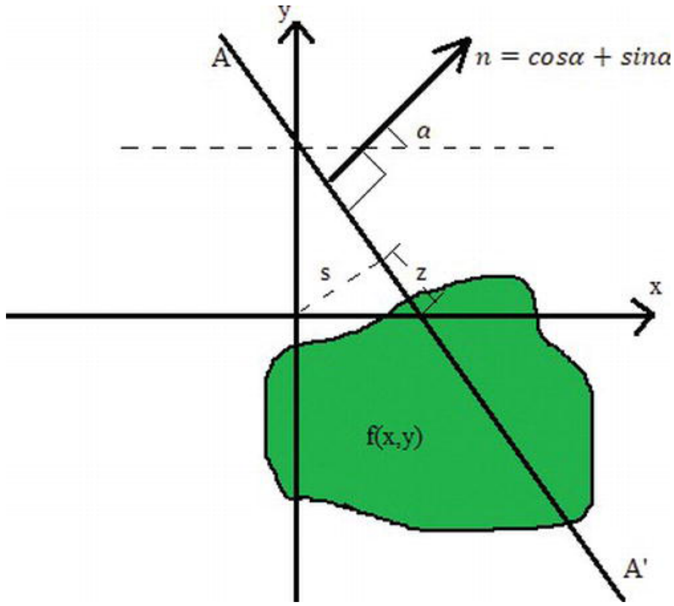

Fig 1. Geometry of Radon Transform

$$R(s,\propto) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\,\delta\ ((s) - (xcos\alpha) - (ysin\alpha)]\,dxdy \tag{5}$$

2.2 RTFT for Image Matching

The Radon Transform allows for the derivation of a large number of features from an image.

Let *f(X)* be the original image and *g(X)* be the deformed image of *f(X)*, in two-dimensional space R^2. Let B is a linear transform.

$$B = \rho\, A \tag{6}$$

Where A is rotation matrix and ρ is a positive number, assume that g(X) = f (BX +X_0) where $X_0$is vector in R^2. Our objective is to determine ρ, R and X_0. Let S^{n-1} denote the unit sphere of R^n, each element $\overline{n}$of S^{n-1} is a unit direction of Rn. Let H $(\overline{n}, s)$ is a hyper plane where $\overline{n}$is normal direction and s is distance from origin. Then Radon Transform of image *f (X)* is

$$Rf\left(\bar{n},s\right) f(x)\,d\mu \tag{7}$$

Where $d\mu$ is standard measure on hyperplane. If T is translation then g(X) = f(X +X_0). Radon Transform of *g(X)* is

$$Rg(X) = Rg\left(\left(\bar{n},s\right)\right) = f(X + X_0)\,d\mu \tag{8}$$

$$Rg(\bar{n},s)\ = f(x)\,d\mu \tag{9}$$

$$Rg\left(\bar{n},s\right) = Rf\ (\bar{n},s+\bar{n}\,X_0) \tag{10}$$

If T is rotation around a unit direction $\overline{n}$and θ is a rotation angle and when $\overline{n}$ = 6 $\overline{r}$then

$$Rg\left(\bar{n},s\right) = Rf\ (\bar{n},s) \tag{11}$$

Where $\overline{n}\,.\,\overline{r} \;=\; \overline{\overline{n}}\,\overline{1}\,.\,\overline{r}$ and angle between two vectors is equal to θ and

$$Rg\left(\bar{r},s\right) = R\,f\,\left(\bar{r},s\right) \tag{12}$$

If T is only scaling transform, then g(X) = f (ρX) then

$$Rg\;(X) = Rg\left(\bar{n},s\right) = \;\rho^{n-1} f\,(X)\,d\mu \tag{13}$$

$$Rg\left(\bar{n},s\right) = \rho^{n-1}\,Rf\left(\bar{n},\;\rho^{n-1},s\;\right) \tag{14}$$

If g(X) = f (ax + X_0) then

$$Rg\left(\bar{n},s\right) = \rho^{n-1}\,Rf\left(\bar{n},\;\rho^{-1}\;(s+\;\bar{n}X_0)\;\right) \tag{15}$$

Uniform scaling can be easily determined by following equation

$$\rho^n = \frac{g\,(X)\,dx}{f\,(X)\,dy} \tag{16}$$

Therefore, we will always assume that uniform scaling is already registered, and considered transformation is only rigid motion that is translation and rotation.

2.3 Fourier Transform

Let F1(ω) and F2(ω) be their Fourier Transform of two functions f1(X) and f2(X) respectively. If f_2(X) = f_1(X + X_0), then

$$F_2\,(\omega) = e^{(-j\,2\pi\omega X_0)}\;F_1\,(\omega) \tag{17}$$

Here determining X_0 is equivalent to finding Dirac function δ(X − X_0). Cross power spectrum of two functions is defined as

$$\frac{F_1\,(\omega)\,F_2*(\omega)}{|F_1\,(\omega)\,F_2*(\omega)|} \tag{18}$$

Where * denotes complex conjugate.

The phase of the cross-power spectrum is equivalent to the phase difference between two functions, as guaranteed by the Shift theorem. By taking inverse Fourier Transform we can determine X_0. Assume that f (X) is an image and g(X) = f (ρAX + X_0) then Radon Transform of f (X) and g(X) is

$$R\,[f\,(X)] = Rf(\bar{n},s) \tag{19}$$

$$R\,[g\,(X)] = Rg(\bar{n},s) \tag{20}$$

Power spectra of Rf $(\bar{n},s)$ is Rf$(\bar{n},\omega)$ and of Rg$(\bar{n},s)$ is Rgf$(\bar{n},\omega)$ with respect to variable s, we have

$$Rg\left(\bar{n},\;\omega\right) = \rho^{n-1}\;Rf\;(\bar{n_1}\;.\;\rho^{-1}\omega). \tag{21}$$

Where

$$\rho^{n-1} = \frac{Rg\left(\bar{n},\;\;s\right)ds}{Rf\left(\bar{n},\;\;s\right)ds} \tag{22}$$

Assume that

$$\rho^{n-1}\,\bar{R}f\left(\bar{n}.\rho^{-1}\omega\right) = \widehat{R}f(\bar{n},\omega) \tag{23}$$

Hence,

$$\rho^{n-1}\,\bar{R}f\left(\bar{n_1}.\rho^{-1}\omega\right) = \widehat{R}f(\bar{n_1},\omega) \tag{24}$$

We know that every unit direction can be expressed as vector (cos α, sin α) where α is rotation angle. In the Radon Transform rotation can be expressed as translation. Hence, we have

$$\bar{R}g\,(\alpha,\omega) = \widehat{R}f(\alpha+\alpha_0.\,\omega) \tag{25}$$

For every α, compute inverse Fourier Transform of $\bar{R}f_{(\alpha,\omega)}$ *and* $\bar{R}g_{(0,\omega)}$

$$\bar{R}f\,(\alpha,\omega) \xrightarrow{IFT} \widetilde{R}f(\alpha,s) \tag{26}$$

$$\bar{R}g\,(0,\omega) \xrightarrow{IFT} \widetilde{R}g(0,s) \tag{27}$$

Hence, cross power spectrum of $\widetilde{R}f(\propto,s)$ *and* $\widetilde{R}g(o,s)$ is

$$\frac{\widetilde{R}f\,(\alpha,s)\,\widetilde{R}*g(0,s)}{\left|\widetilde{R}f\,(\alpha,s)\,\widetilde{R}*g(0,s)\right|} \tag{28}$$

Let C_α (ω) is Fourier Transform of above phase function then C_α (ω) = $\delta(\omega)$if α = α0

2.4 Algorithm

Calculate Radon Transform of *g(X)* and *f(X)*.

$$f(X) \xrightarrow{RT} Rf(\alpha,s) \tag{29}$$

$$g(X) \xrightarrow{RT} Rg(\alpha,s) \tag{30}$$

Take Fourier Transform of R f (α,s) and R g (α,s) we get

$$Rf(\alpha,s) \xrightarrow{FT} \widetilde{R}f(\alpha,\varepsilon) \tag{31}$$

$$Rg(\alpha,s) \xrightarrow{FT} \widetilde{R}g(\alpha,\varepsilon) \tag{32}$$

Where $0 < \alpha < \pi$

Calculate cross power spectrum of R f(α,s) and (α,s), then determine Fourier Transform of phase function, so we get C_α (ω).

$$\theta_1,\ let\ \theta_2 = \theta_1 + \frac{\pi}{2}$$

For angle, then phase function is given by

$$\frac{\widetilde{R}f\,(\theta_1+\alpha_0,\varepsilon)\,\widetilde{R}g*(\theta_1,\ \varepsilon)}{\left|\widetilde{R}f\,(\theta_1+\alpha_0,\varepsilon)\,\widetilde{R}g*(\theta_1,\ \varepsilon)\right|} \tag{33}$$

Calculating inverse Fourier Transform of above phase function we get C(θ_1,s).

C(θ_2,s)is given by

$$\frac{\widetilde{R}f\left(\theta_2+\alpha_0,\varepsilon\right)\widetilde{R}g*\left(\theta_2,\ \varepsilon\right)}{\left|\widetilde{R}f\left(\theta_2+\alpha_0,\varepsilon\right)\widetilde{R}g*\left(\theta_2,\ \varepsilon\right)\right|} \overset{IFT}{\Rightarrow} C\left(\theta_2,\ s\right) \tag{34}$$

C(θ_1,s) = δ(s − x^0) and C(θ_2,s) = δ(s − y^0) where x^0 and y^0 are constants dependent on translation x_0 and y_0. We know that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta_1 & sin\theta_1 \\ -sin\theta_1 & cos\theta_1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{35}$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} cos\theta_1 & sin\theta_1 \\ -sin\theta_1 & cos\theta_1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \tag{36}$$

3 Result and discussion

We present our discussion through the following examples.

3.1 Implementation on Lena dataset to match image with translated, rotated and scaled template image

The performance of the proposed algorithm is evaluated on images with translation, rotation and scaling and result of registration has been studied. Figure 2 shows the results of experiments performed on Lena image. Actual Lena template was scaled of 0.9, rotated by 25, translated along x–axis =35, translated along y-axis=35. Registration results are, scale=0.8, rotation=25, translation along x-axis=35, translation along y-axis=35. Result shows that the method improved the results of all methods proposed in [4,12].

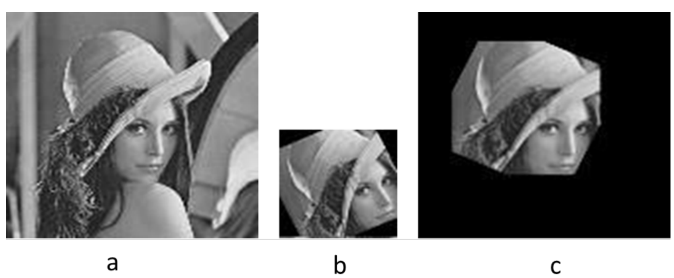

Fig 2. (a) Original Lena Image (256 256). (b) Deformed Lena image. (c) Recovered Lena image with scale=0.8, rotation=25, translation along x-axis=35, translation along y axis=35.

3.2 Radon Transform to correct rotation, scale and translation on image

Due to inherent properties of Radon Transform, it is a useful tool to capture the directional features of the images. It enables the implementation of very effective detection algorithms

The sinogram (all projections taken along the different θ direction). Figure 3 shows us rotation, translation and scaling property of Radon Transform. We recognized the transform parameters k, φ, Δx, Δy. From equation, the uniform scaling is easily derived. For a given spatial position, the rotation angle θ can be uniquely determined by local minima of two profiles of two images from Figure 3(e).

Regardless of the image registration method, or application area, it is extremely desired to provide the user with an estimate how accurate the registration actually is. A lot of work has been done on validation of rigid-body registration, but accuracy was always a problem. Errors were in the form of alignment, localization and matching. It has been found that the proposed approach achieves 98% accuracy over the methods proposed in [4,12].

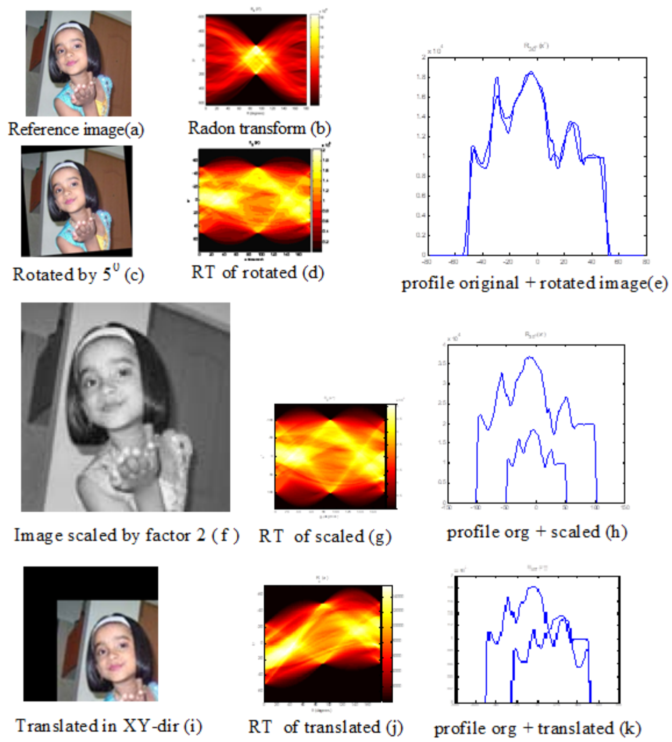

Fig 3. Rotated, translated and scaled sample images, their Radon Transform and profiles indicating rotation translation and scale and rotation corrected image

4 Conclusion

We have seen that this work improved the results of all methods proposed by various researchers. The main challenge of image registration approaches is their computational complexity. Although computer speeds have increased, the necessity to reduce the computational time of methods persists. The complexity of methods as well as the size of data ill keeps increasing day by day. The accuracy (98 %) and speed of the RTFT approach is good. As the Radon Transform converts rotation into translation and magnitude of DFT is invariant to translation, the features extracted using their combinations are invariant to rotation. The suggested RTFT technique can be utilized successfully for multimodal picture registration since it is invariant to changes in illumination. Method is also robust to white Gaussian noise. It can be concluded that RTFT method is the best method for image registration.

Even after a great deal of effort, automatic image registration still remains an open problem. Image registration is hot areas and also very promising research directions.

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