

RESEARCH ARTICLE



Dynamic Behaviour Modelling of Magneto-Rheological Fluid Damper Using Machine Learning

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Abstract

Objectives: Despite significant advancements in the field of automotive suspension systems, a notable research gap exists in accurately predicting the intricate nonlinear damping behavior of Magnetorheological Dampers (MRDs), which hinders the comprehensive enhancement of automotive comfort and safety. This study aims to address this gap by developing a novel machine learning-based black box model capable of precisely forecasting the complex damping characteristics exhibited by MRDs, thereby paving the way for substantial improvements in both ride comfort and vehicle safety. **Methods:** A methodology integrating machine learning and real-time feedback control is employed, utilizing Linear, Nonlinear Autoregressive with Exogenous Variables (ARX), and Hammerstein-Wiener models for input selection and parameter estimation. Experimental data from hydraulic testing is used to develop a nonlinear black box model using a NARX structure. Inputs of MRD force and velocity predict the corresponding damping force, improving stability and generality compared to physical modeling methods. **Findings:** The successful implementation of the proposed methodology enables the identification of a model that closely matches experimental data obtained from an MR damper. The developed non-linear black box model, combined with constructive parameter estimation models, improves the understanding and control of MR damping behaviour. **Novelty:** This advancement contributes to the field's progress by offering a novel approach for predicting the damping behaviour of MRDs, facilitating their effective utilization in various applications across the automotive and other industries.

Keywords: Magnetorheological fluids; Nonlinear dynamics; Feedback control; Machine learning; MR damper; Dynamic modeling

1 Introduction

Magnetorheological dampers, leveraging Magnetorheological Fluids (MR Fluids), have garnered substantial research interest due to their controllable properties with wide applications spanning industries⁽¹⁾. Particularly appealing to the automotive sector for enhanced driving comfort and safety, these dampers' intricate damping behavior remains elusive to conventional methods due to nonlinear dynamics and hysteresis. This study pioneers an innovative approach, fusing machine learning and real-time feedback control to craft a non-linear black box model for precise prediction of Magnetorheological Damper (MRD) damping. Vital to this approach is problem decomposition, untangling input resolution and parameter estimation via diverse models like Linear, Nonlinear Autoregressive with Exogenous Variables (ARX), and Hammerstein-Wiener. This paradigm shift from conventional methods holistically addresses the complexities tied to MR damper behavior, offering heightened insights into their intricate dynamics.

Recent strides highlight the proposed model's potential in elucidating MR damper dynamics⁽²⁾. Incorporating excitation amplitude and input voltage influence on damping coefficient⁽³⁾, an adaptive neuro-fuzzy inference system, driven by experimental data, adeptly forecasts MR fluid damper behavior, capturing nonlinear hysteretic traits^(4,5). This model computes damping force effectively, aligned with experimental outcomes within a preset frequency and coil current span⁽⁶⁾. To validate further, a proof-of-concept design underwent rigorous testing, unraveling real-world MR damper performance⁽⁷⁾. Given MR dampers' role in vibration control systems⁽⁸⁾, the pursuit of accurate models is imperative. GMDH-based solution's comparative identification accuracy to computational counterparts underscores its potential⁽⁹⁾. This study journeys into comprehensive modeling of magnetorheological dampers, interweaving the proposed model, adaptive neuro-fuzzy inference systems, and validation. The developed non-linear black box model discussed in the next, combined with constructive parameter estimation models, improves the understanding and control of MR damping behaviour. The findings promise insight into intricate damping behavior, fostering MR dampers' optimal cross-industry utilization, notably in vibration control systems.

1.1 Black box modelling

In the realm of machine learning and system identification, nonlinear modeling has risen as a crucial approach for understanding complex dynamic behaviors. A recent study focused on modeling Magnetorheological Dampers (MRDs) using a nonlinear black box framework. This involved various models such as linear, nonlinear ARX, and Hammerstein-Wiener models. MATLAB was utilized to capture key input parameters, including MRD velocity and damping force. The study highlighted that the damping force of MR dampers increases with the applied current⁽¹⁰⁾. The initial phase of the study validated a linear model's precision using validation data. Researchers then progressed to enhance damping force prediction by exploring nonlinear ARX and Hammerstein-Wiener models of differing complexities. This intricate process aimed to identify the best model fit among various orders. A significant achievement was the demonstration of intelligent control for dampers through prediction-control methods, using state data from the damped system. Experimental efforts successfully captured Desired Damping Force and Desired Current for the Magnetorheological Damping System. In a computational investigation⁽¹¹⁾, facets like Pressure Drop Across the Annular Orifice, Fluid Velocity Within the Magnetized Annular Orifice, and Damping Characteristics of MR Fluid Dampers were examined. This led to the discovery of a proportional decline in fluid velocity within the magnetized annular orifice with increasing magnetic field strength, reaching a saturation point. Another experimental exploration⁽¹²⁾ introduced a k-nearest neighbor prediction model to estimate current in a Magnetorheological Damping System and predict resulting Damping Force Output. This model outperformed conventional BP neural networks in accuracy. Regarding dynamic behavior of MR Dampers⁽¹³⁾, particularly in semi-active damping control for lateral secondary dampers, the Acceleration Driven Damper Linear algorithm demonstrated superior efficiency. A comprehensive literature review⁽¹⁴⁾ emphasized MR fluid dampers with variable stiffness and damping as effective semi-active controllers in vibration control. Analyzing Dynamic Behavior⁽¹⁵⁾ encompassing an MR Fluid Damper's Hysteresis Loop and Force-Displacement Relationship established the Extreme Learning Machine (ELM) model with sigmoid activation functions as highly predictive. For measuring damping force, an inventive approach using the ELM method⁽¹⁶⁾ proved remarkably efficient, achieving rapid training times for substantial data points.

Nonlinear black box system identification involves creating data-driven mathematical abstractions of dynamic systems with limited or no knowledge of their intrinsic properties. The construction of the model is experimental and involves subjective decision-making. After defining the experiment protocol and designing the excitation signal, the model structure and its related parameters are determined, followed by evaluating the model quality and validating its adherence to metrics. If the model fails to meet the validation criteria, a new model is constructed based on the outputs in terms of model accuracy and residual properties. This process is subjective and may involve tedious and error-prone activities. In this study, a general nonlinear black box structure was proposed to model the MR damping behavior on the force-velocity plane, with a dynamic sample system

modeling shown in Figure 1.

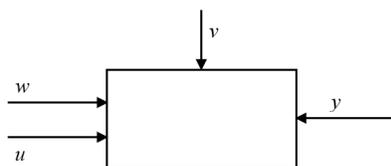


Fig 1. Dynamic system

Here the data assigned are

$$u(t) = [u(1), u(2), \dots, u(t)] \tag{1}$$

$$y(t) = [y(1), y(2), \dots, y(t)] \tag{2}$$

Using the above a relationship between past observations for $u(t-1)$, $y(t-1)$ and future outputs $y(t)$ is obtained as:

$$y(t) = g[u(t-1), y(t-1) + v(t)] \tag{3}$$

The additive term $v(t)$ accounts for the fact that the next output, $y(t)$, will not be an exact function of past data. However, a goal must be that $v(t)$ is small so that we may think of $[u(t-1), y(t-1)]$ as a good prediction of $y(t)$ given past data⁽¹⁷⁾.

The equation model is designed for general discrete-time dynamic systems and the choice of the nonlinear mapping in $g(t)$ are separated into two partial problems. One involves selecting the regression vector t from past inputs and outputs, and the other involves selecting the nonlinear mapping g from the regressor space to the output space.

Black box modeling typically involves a trial-and-error process of estimating parameters for various model structures and comparing the results. Some of these structures can be estimated using non-iterative estimation algorithms, which can reduce complexity. Model structures are usually defined by the model order, which can vary depending on the selected model type. More complex models can be created by specifying a higher model order for the same linear model structure, increasing model flexibility to capture complex phenomena. However, an unnecessarily high order can make the model less reliable. Explicitly modeling noise by including the $He(t)$ term, as shown in the equation, can improve model accuracy.

$$y(t) = Gu(t) + He(t) \tag{4}$$

In this context, the variable H represents the additive disturbance and assumes that it is generated by a linear system driven by a white noise source $e(t)$. Explicitly incorporating the additive disturbance in the model structure can enhance the precision of the measured component G . Moreover, such a model structure is beneficial when the primary goal is to forecast future response values. This advancement contributes to the field's progress by offering a novel approach for predicting the damping behaviour of MRDs, facilitating their effective utilization in various applications across the automotive and other industries.

2 Methodology

The process of system identification follows a sequential pattern: gathering data, selecting a set of models, and determining the best model within the set. However, the initial model generated may fail to pass validation tests, necessitating revisions to the different steps of the procedure⁽¹⁸⁾. Figure 2 illustrates the procedure. OE models, which are a type of polynomial models, have a distinct structure that involves two active polynomials, B and F . These models are designed to represent traditional transfer functions, which relate measured inputs to outputs while accounting for the effects of white noise as an additive output disturbance.

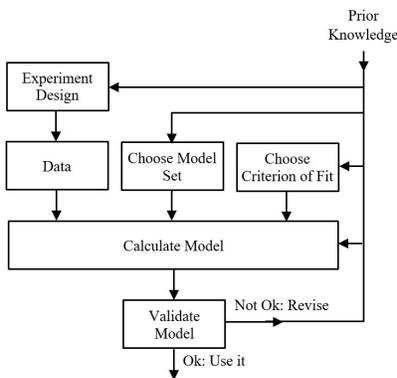


Fig 2. System identification loop

2.1 AIC (Akaike’s information criteria)

In the process of model selection, the AIC is commonly used to compare the performance of different models by evaluating their AIC scores. The AIC takes into account both the goodness of fit of the model and the number of parameters used in the model. It penalizes models with a higher number of parameters, as more parameters can potentially lead to overfitting, where the model becomes too complex and may not generalize well to new data. Therefore, models with fewer parameters are favored, as they are deemed to be more parsimonious and can provide a more concise representation of the data. To utilize the AIC in model selection, researchers typically develop a set of candidate models by considering various combinations of predictors or different model structures. These models are then estimated using maximum likelihood estimation, and their AIC scores are calculated. The model with the lowest AIC score is considered the best-fit model, as it strikes a balance between goodness of fit and model complexity. In other words, it provides the best trade-off between model accuracy and simplicity. One of the key advantages of using the AIC is its ability to handle models with different numbers of parameters, allowing for a fair comparison among models with varying levels of complexity. By considering both the goodness of fit and model complexity, the AIC helps researchers to identify the most appropriate model that adequately captures the underlying data patterns while avoiding overfitting. The Akaike Information Criterion (AIC) ability to penalize models with more parameters and its focus on finding the best trade-off between model accuracy and simplicity make it a widely used criterion for assessing model fit and selecting the most appropriate model in various statistical applications.

2.2 Non-linear model

In the System Identification Toolbox software, dynamic models refer to mathematical relationships that connect a system’s inputs, $u(t)$, to its outputs, $y(t)$, which can be utilized to calculate the current output based on the previous inputs and outputs. Discrete-time models have a standard form in the software:

$$y(t) = [f(u(t - 1), y(t - 1), u(t - 2), y(t - 2)...)] \tag{5}$$

A model can be considered nonlinear if its function f is nonlinear. This function can take the form of various nonlinearities, including saturations and switches. Figure 3 block diagram displays the configuration of a nonlinear ARX model in a simulation scenario. The model processes the output y in two stages.

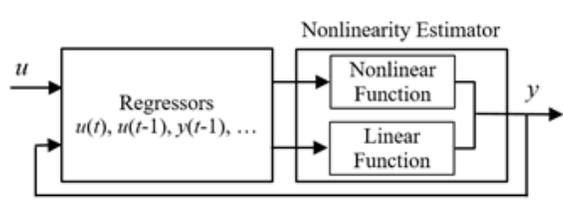


Fig 3. Configuration of a nonlinear ARX model

2.3 Hammerstein-Wiener Models

The Hammerstein-Wiener models, as illustrated in Figure 4, are a popular class of models used to represent dynamic systems. These models consist of one or two static nonlinear blocks connected in series with a linear block, where the linear block incorporates a discrete transfer function that captures the dynamic component of the model. The nonlinear blocks introduce nonlinearity into the model, allowing for the representation of complex system behaviors. In this study, Hammerstein-Wiener models are employed as black box models, which do not provide explicit physical insight into the underlying system processes. However, they have been widely used in various applications, including electro-mechanical systems, radio frequency components, audio and speech processing, and predictive control of chemical processes, due to their versatility and effectiveness in capturing the nonlinear dynamics of complex systems.

One of the key advantages of Hammerstein-Wiener models is their simple block representation, which makes them easily interpretable and implementable in practice. Moreover, these models have a clear relationship to linear systems, making them suitable for systems with both linear and nonlinear behaviors. This makes Hammerstein-Wiener models a preferred choice over more complex nonlinear models, such as neural networks and Volterra models, in certain applications⁽¹⁷⁾. The simplicity and ease of implementation of Hammerstein-Wiener models make them attractive for practical use in real-world scenarios. The utilization of Hammerstein-Wiener models in this study provides a valuable approach for representing dynamic systems. Despite being black box models without explicit physical insight, Hammerstein-Wiener models are widely used in diverse applications due to their simple block representation, clear relationship to linear systems, and ease of implementation, making them a preferred choice in various fields of research and practical applications⁽¹⁷⁾.



Fig 4. Hammerstein-Wiener Models

Both ARX and Hammerstein-Wiener models provide different levels of complexity and accuracy in capturing the behavior of MR dampers. These models help capture the dynamic behavior of the dampers and provide a mathematical representation that can be used for analysis, control design, and simulation purposes. In the context of MR dampers, the non-linear static block can be designed to capture the force-displacement relationship, while the linear dynamic block can capture the response dynamics.

2.4 Experimental Design

The study’s data was obtained from Jiandong et al.⁽¹⁹⁾, in which an RD-1097-01 MR damper was connected to a shake table and ground. The experimental setup is depicted in Figure 5. The voltage of the MR damper was fixed at 1.25V, while the shake table generated vibrations that caused the piston rod of the MR damper to move along its chamber. Since the shake table weighed about 60lbs and had large inertia, it was controlled under closed-loop operation. The proportional derivative (PD) controller determined the displacement by counting the turns of a circulating shaft and sent out currents to drive the shake table at a sampling period of 0.001s. It also obtained readings of the damping force $y(t)$ and displacement via a strain meter and an infrared sensor, respectively, at a sampling period of 0.005s. After down sampling the measurements by a factor of 5 and synchronizing all measurements by comparing them with the two displacement measurements, the displacement measurements with relatively more noise were discarded. Velocity $u(t)$ was obtained by numerically differentiating the displacement measurements using a simple differentiation filter. Therefore, the velocity was roughly piece wise constant for every 40 samples (the sampling period h is 0.005s).

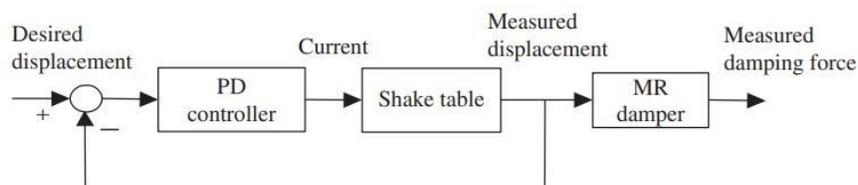


Fig 5. A diagram of the experimental setup Jiandong et. al⁽²⁰⁾ .

Using the above, authors collected two groups of experimental data, each with 3700 samples. The data presented in Figure 6(a) and (b) are used for parameter estimation and for model validation, respectively.

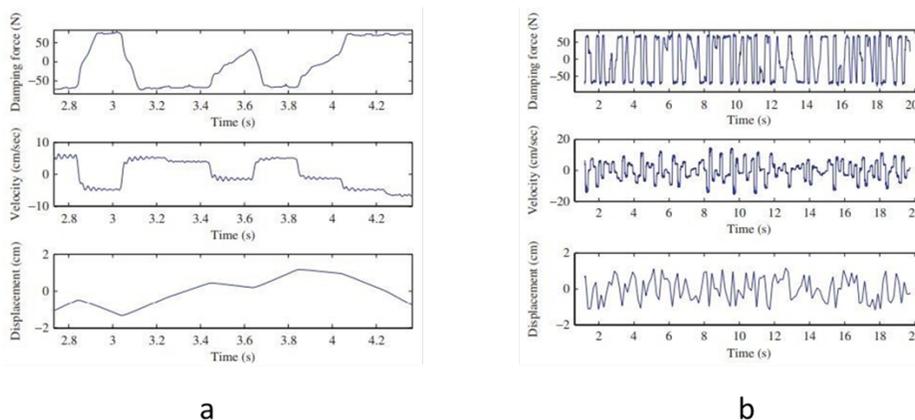


Fig 6. (a). Experimental Data for Estimation (b). Experimental Data for Validation

Figure 6(a), displays parametric estimation data, which serves as the basis for predicting the future behavior of a specific parameter. The anticipated output is presented in Figure 6 (b). It's important to understand that this parametric data is derived from experimental results outlined in the reference⁽¹⁹⁾.

One of the major shortcomings in this method is that in order to remove the error propagation⁽¹⁴⁾, omits the step of estimating the numerator, and only identifies the denominator of the linear dynamics and uses a subspace direct equalization method to estimate the unmeasurable inner signal

2.5 Input-Output Data

For all estimation and validation tasks performed in this project, a dataset consisting of 3499 measurements was used, corresponding to a sampling rate of 0.005s. The input, denoted as $v(t)$, represents the velocity [cm/s] of the damper, and the output, denoted as $f(t)$, represents the damping force [N]. The dataset was divided into two subsets: the first 2000 samples were used for estimation (ze), and the remaining samples were used to validate the results (zv), as shown in Figure 7. This figure provides an overview of how the dataset has been utilized. A 20-second dataset is divided into two equal segments, each lasting 10 seconds and containing 2000 data samples. The first segment, referred to as "ze" (0 - 10 seconds), is utilized to forecast future estimations, denoted as "zv". In this study, a variety of models are applied to predict the estimated data, and these predictions are compared with "zv" to determine the closest match.

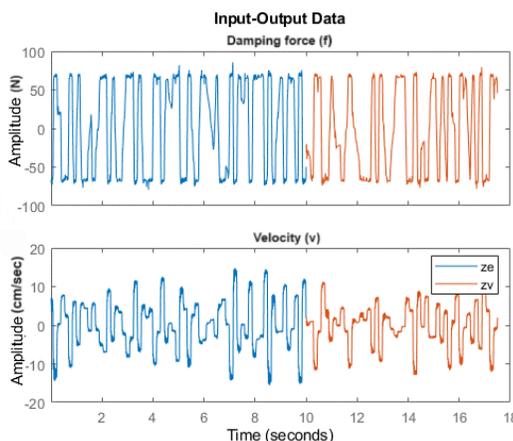


Fig 7. Splitted experimental data for estimation validation

2.6 Model Order Selection

In system identification, the orders of linear and nonlinear models are determined by several parameters^(21–26). For linear Autoregressive with exogenous inputs (ARX) models, these parameters include n_a , n_b , and n_k , which determine the number of past output and input values, as well as input delays, used for predicting future outputs. On the other hand, for Hammerstein-Wiener models, which represent a class of linear models with static input/output nonlinearities, the orders are represented by parameters n_b (number of zeros plus one), n_f (number of poles), and n_k (input delay in the number of lags). Typically, selecting the model orders is done through a trial and error process, although MATLAB functions like `arxstruc` and `selstruc` can be used to automatically compute the orders of linear ARX models. These orders provide guidance on the potential orders to use for nonlinear models. In a specific study, the optimal order for a linear ARX model was determined using the Akaike Information Criterion (AIC) criterion. The chosen order was $[2\ 4\ 1]$, which indicates that the ARX model structure used six regressors - $f(t-1)$, $f(t-2)$, $v(t-1)$, $v(t-2)$, $v(t-3)$, and $v(t-4)$ - to predict the damper force $f(t)$.

3 Results and Discussion

3.1 Preliminary Data Analysis: Creating Linear Models

In order to simplify the analysis, linear models are initially considered. If more than the results obtained from these models is needed, nonlinear models can be investigated. A linear state-space model will be used with an order that will be determined automatically. The output data from the first subset of measurements, z_e , will be used to evaluate and compare the responses of the models: The Figure 8 shows the comparison of linear model to estimation data (Figure 8(a)) and against data set z_v (Figure 8(b)). LinMod1-3 are three models used to data estimation, the difference/adaptability of the model depends upon the best fit to the actual output which is varying from 48.52% to 55.3%.

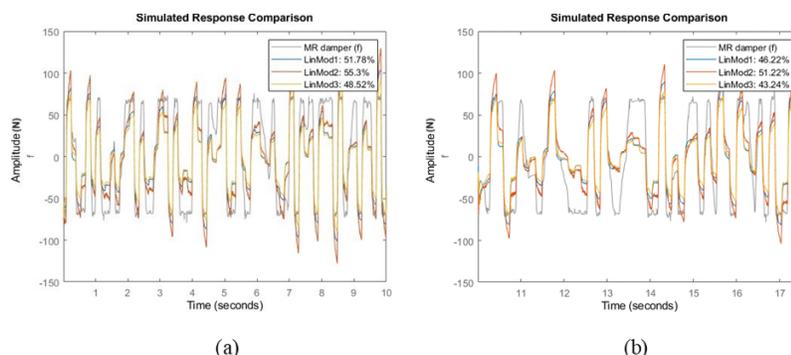


Fig 8. Comparison of linear model to (a) estimation data and against (b) data set z_v

3.2 Creating Nonlinear ARX Models

After finding that the ARX model provided a poor fit to the validation data, Nonlinear ARX (IDNLARX) models were employed to improve the results. The decision to use nonlinear models was also supported by the advice utility, which helps determine if a nonlinear model would be more effective than a linear one. Nonlinear ARX models are similar to ARX models but allow for more flexible modeling by using nonlinear functions to combine regressors or by making the regressors arbitrary functions of I/O variables in addition to time-delayed variable values. While ARX models use the order matrix $[n_a n_b n_k]$ to create regressors when they have contiguous lags, the linear Regressor object provides more flexibility for regressors with arbitrary lags or when based on the absolute values of variables. Polynomial regressor objects can be used for regressors based on time-delayed variables that are polynomials, while custom Regressor objects can be used for regressors that utilize arbitrary user-specified formulas.

Detailed analysis was done with following model orders and results obtained are plotted in Figure 9.

Narx2 1 = `nlarx(ze, [3 4 1])` Narx2 2 = `nlarx(ze, [2 5 1])`

Narx2 3 = `nlarx(ze, [3 5 1])` Narx2 4 = `nlarx(ze, [1 4 1])`

Narx2 5 = `nlarx(ze, [2 3 1])` Narx2 6 = `nlarx(ze, [1 3 1])`

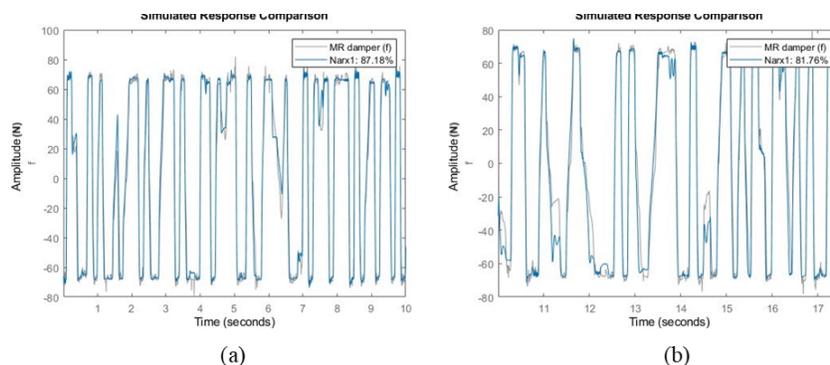


Fig 9. Comparison of non-linear ARX model to (a) estimation and (b) validation data

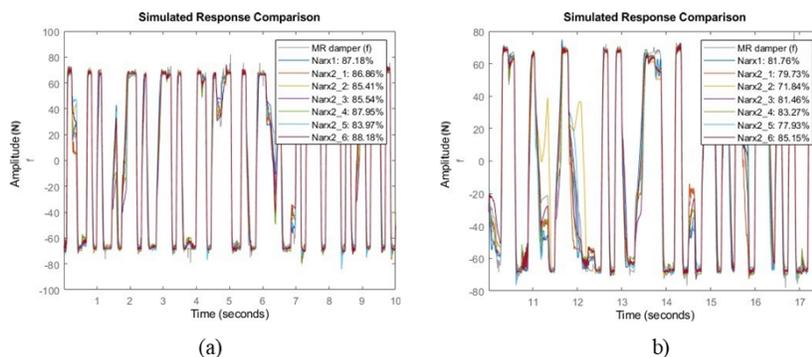


Fig 10. Comparison of various non-linear ARX model with different model order to (a) estimation data and (b) validation data

Comparison data obtained from the non-linear ARX model to estimation and validation, a comparison of different model order to estimation data and validation data has been carried out using non-linear ARX model and plotted in and Figure 10. Hammerstein-Wiener models can be created by connecting static nonlinear elements to a linear model in series. This type of model extends the linear output-error (OE) model by subjecting the input and output signals of the linear model to static nonlinearities like saturation or dead zones. The following nonlinear functions and model orders were used for creating the Hammerstein-Wiener models. The performance of the created models was evaluated by comparing them to both estimation and validation data, as illustrated in Figure 11.

- Nhw1 = nlhw(ze, [3 4 1])
- Nhw2 = nlhw(ze, [4 2 1])
- Nhw3 = nlhw(ze, [4 2 1])
- Nhw4 = nlhw(ze, [3 1 1])
- Nhw5 = nlhw(ze, [3 1 1])

Figure 12 illustrates various nonlinear models used to represent the relationship between voltage input and damping force output. The results indicate that Narx2, Narx6, and Narx5 models outperformed other Nonlinear ARX models, while the model Nhw1 outperformed other Hammerstein-Wiener models. The study found that the Nonlinear ARX models provided the most optimal solution for describing the MR damper’s dynamics.

Various literature has already reported on the ARX and Hammerstein-Wiener models offering valuable insights into the behavior of MR dampers, their implementation are challenging due to the non-linear nature of the dampers, the complexity of parameter identification, model generalization limitations, sensitivity to disturbances, and the need for real-time performance. This study has done careful consideration and validation when applying these modeling techniques to MR dampers and from the comparison of various generated models it is established that by the use of nonlinear autoregressive with exogenous input structure the nonlinear behavior of MR damper can be predicted to higher accuracy.

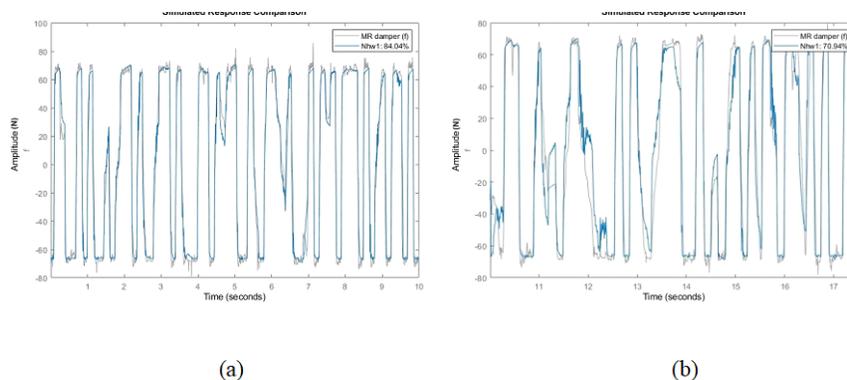


Fig 11. Comparison of Hammerstein Wiener model to estimation data and validation data

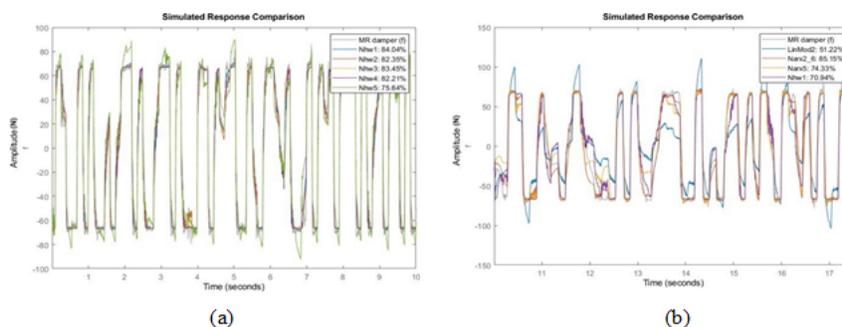


Fig 12. Comparison of various Hammerstein Wiener model with different model order to (a) estimation data and (b) validation data

3.2.1 Standard Error Analysis

To evaluate the performance of the proposed non-linear models, we have conducted standard error analysis. Table 1 shows the error occurred during training and testing phase for ARX models. The results indicate that Narx2, Narx6, and Narx5 have less error compared to other ARX models proposed. While in Table 2 tabulated the error analysis of Hammerstein Wiener models. Here the model Nhw1 have less prediction error compared to other models. Both these errors analysis validates the performance of the proposed models.

Table 1. Standard error analysis for ARX Models (Narx2)

Non-linear ARX Models (Narx2)	1	2	3	4	5	6
Training error	1.362	1.157	1.432	1.782	1.123	1.321
Test error	1.232	0.961	1.379	1.485	0.954	1.104

Table 2. Standard error analysis for Hammerstein Wiener model (Nhw1)

Hammerstein Wiener model (Nhw1)	1	2	3	4	5
Training error	1.453	1.496	1.634	1.588	1.556
Test error	0.965	1.210	1.392	1.359	1.482

4 Conclusion

This study stands as a groundbreaking effort to bridge the research gap in understanding and predicting the complex nonlinear damping behavior of Magnetorheological Dampers (MRDs). Through the development of an innovative machine learning-

based black box model, underpinned by real-time feedback control, the intricate dynamics of MRDs have been illuminated. The integration of Linear, Nonlinear Autoregressive with Exogenous Variables (ARX), and Hammerstein-Wiener models has effectively addressed the limitations of conventional methods, enabling precise prediction of damping characteristics. The successful implementation of this methodology has yielded a non-linear black box model that closely aligns with experimental data, significantly enhancing the control and comprehension of MR damping behavior.

The novel approach presented in this study has far-reaching implications. By capturing the nuances of MRDs' damping behavior, this research opens avenues for transformative advancements in ride comfort and vehicle safety across the automotive industry. The demonstrated potential of the model to accurately represent the dynamic properties of MR dampers paves the way for enhanced vibration control systems and broader applications. The utilization of adaptive neuro-fuzzy inference system techniques further fortifies the model's accuracy and reliability. As the automotive sector and related industries continue to seek refined solutions for optimal system design and performance, the contributions of this study are poised to revolutionize the integration of MRDs.

Looking ahead, this study suggests promising avenues for further research. Exploring the adaptability of the proposed model to various excitation scenarios, extending its application to broader frequency ranges, and investigating the implications of varying environmental conditions could offer comprehensive insights into the behavior of MR dampers. Additionally, delving into the integration of this predictive model within real-time control systems, and evaluating its performance across a range of vehicles and scenarios, could enhance the practicality of its implementation. The potential synergy between the model and emerging technologies such as autonomous vehicles and smart suspension systems offers a captivating realm for exploration. In sum, this study not only advances the knowledge base concerning MR dampers but also stimulates ongoing innovation in automotive comfort and safety technologies, setting a dynamic trajectory for future research and practical implementation.

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