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RESEARCH ARTICLE

Some Properties of Commutative Ternary Right Almost Semigroups

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Abstract

Objective/Background: In this paper, the concept of commutative ternary right almost semigroups is introduced. The properties of ternary right almost semigroups and commutative ternary right almost semigroups are also discussed. Finally, regular only and the regularity are also explored in ternary right almost semigroups. **Methods:** Properties of ternary right almost semigroup have been employed to carry out this research work to obtain all the characterizations of commutative ternary right almost semigroups, regular and normal corresponding to that ternary semigroup. **Findings:** We call an algebraic structure $(S,+,.)$ is a ternary semigroup if $(S,\ .)$ is a Semigroup, S is a ternary semigroup under ternary multiplication. Let S be a groupoid. Then it is a right almost semigroup (RA-semigroup), if we have $a_1\,(a_2a_3) = a_3\,(a_2a_1) = a_2\,(a_1a_3)$, for all $a_i \in S, 1 \le i \le 3$. (i) RA-semigroup - R-cyclic if $(a_1a_2)a_3 = (a_3a_1)a_2 = (a_2a_3)a_1$, for all $a_i \in S, 1 \le i \le 3$. (ii) RA-semigroup - L-cyclic if $a_1(a_2a_3) = a_3(a_1a_2) = a_2(a_3a_1)$, for all $a_i \in S, 1 \le i \le 3$. In this ternary structure we try to study commutative ternary semigroups concept and obtain their properties. **Novelty:** In this study, we define the notion of some properties of commutative ternary right almost semigroups, regular and normal. We also find some of their interesting results.

AMS Subject Classification code: 20M12, 20N10

Keywords: Ternary semigroups; Ternary right almost semigroup; Commutative ternary right almost semigroups; Quasi- commutative ternary right almost semigroups; Regular ternary right semigroups and Normal ternary right almost semigroups

1 Introduction

A. Anjaneyulu[1] extended the ideal theory of commutative semigroup to duo semigroup. D.D. Arlderson and E.W. Johnson[2] used the term semigroup to mean a commutative multiplicative semigroup with 0 and 1. The multiplicative theory of ideals in a commutative is a highly developed area of research in ternary semigroup. Ronnason

Chinram, Wichayaporn Jantanan, Natee Raikham, Pattarawan Singavananda [3] introduced covered left ideals and covered right ideals of a ternary semigroup. We here study some results of a ternary semigroup containing covered left ideals and give the conditions for every proper left ideal of a ternary semigroup to be a covered left ideal. The theory of ternary algebraic system was introduced by D.H. Lehmer [4] in 1932, but earlier such structures were studied by Kasner who gave the idea of $n-ary$ algebras. [5] F.M. Sioson introduced the notion of regular ternary semigroup. Y. Sarala, A. Anjaneyulu and D. Madhusudhana Rao initiated the study of quasi commutative, pseudo commutative and normal ternary semigroups. We define the notion of ternary semigroup to some properties of commutative ternary right almost semigroups, regular and normal. We also find some of their interesting results.

2 Methodology

In this article, some properties of commutative ternary right almost semigroups, some characterizations of the quasi commutative ternary right almost semigroups, regular ternary right almost semigroups and normal ternary right almost semigroups are discussed.

3 Results and Discussion

This section the deals preliminary concepts and some basic results of ternary right almost semigroups [6–8].

Definition 3 .1. A class S with an operation between triplets of elements is called a triplex if the following postulates hold.

Postulate I. $(a.b.c)d.e = d.(a.b.c).e = d.e(a.b.c)$

$$= (a.b.d).c.e = (a.b.e).c.d = (a.c.d)b.e$$

$$= (a.c.e).b.d = (a.d.e)b.c = (b.c.d).a.e$$

$$= (b.c,e).a.d = (b,d.e).a.c = (c.d.e).a.b$$

provided a,b,c,d,e and all the expressions belong to S.

Postulate II. If $a,b,c \in S$, then there is an element x of S such that $a.b.x = c$.

The number of elements in S is called the order of triplex and is specified, when necessary, by adding one of the postulates:

Postulate III$_1$. S contains $'n'$ elements.

Postulate III$_2$ .S contains infinitely many elements.

According as III$_1$, or III$_2$ holds, the triplex is called finite or infinite.

Definition 3.2. A ternary semigroup is a nonempty set S together with a ternary operation $(a_1,a_2,a_3) \to (a_1a_2a_3)$, satisfying the associative law of the first kind

$$((a_1a_2a_3)(a_4a_5)) = (a_1(a_2a_3a_4)a_5) = (a_1a_2(a_3a_4a_5))$$

for all $a_i \in S, 1 \leq i \leq 5$.

Definition 3.3. Let S be a groupoid. Then it is a right almost semigroup (RA-semigroup), if we have

$$a_1(a_2a_3) = a_3(a_2a_1) = a_2(a_1a_3),$$

for all $a_i \in S, 1 \leq i \leq 3$.

(i) RA-semigroup - R-cyclic if $(a_1a_2)a_3 = (a_3a_1)a_2 = (a_2a_3)a_1$, for all $a_i \in S, 1 \leq i \leq 3$.

(ii) RA-semigroup - L-cyclic if $a_1(a_2a_3) = a_3(a_1a_2) = a_2(a_3a_1)$, for all $a_i \in S, 1 \leq i \leq 3$.

Remark 3.4. A groupoid S is medial if for all $a_i \in S, 1 \leq i \leq 4$, S satisfies medial (or) bi-symmetry law, (i.e)$(a_1a_2)(a_3a_4) = (a_1a_3)(a_2a_4)$, for all $a_i \in S, 1 \leq i \leq 4$.

Example 3.5 . Let $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

Then S is a ternary semigroup under usual multiplication.

Example 3.6 . Let $S = \{0,1,2,3,4,5\}$ and $abc = (a*b)*c$ for all $a,b,c \in S$, where $'*'$ is defined in the following table:

$*$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	1	1	1	1
2	0	1	2	2	1	1
3	0	1	1	1	2	2
4	0	1	4	5	1	1
5	0	1	1	1	4	5

Then $(S, *)$ is a ternary semigroup.

Definition 3.7. A ternary right almost semigroup S is said to be commutative if $abc = bca = cab = bac = cba = acb$ for all $a, b, c \in S$.

A ternary right almost semigroup S is said to be quasi commutative if for any $a, b, c \in S$, there exists a natural number $'n'$ such that

$$abc = b^n ac = bca = c^n ba = cab = a^n cb.$$

Definition 3.8. A ternary right almost semigroup S is said to be normal if $abS = Sab$ for all $a, b \in S$.

Definition 3.9. A ternary right almost semigroup S is said to be right pseudo commutative if $abcde = abdec = abecd — abdce = abedc = abced$ for all $a, b, c, d, e \in S$.

Example 3.10. Let $S = \{a, b, c, d, e\}$ be a set. Define a ternary operation '.' on S. where $'.'$ is defined by the following table:

$.$	a	b	c	d	e
a	a	a	a	a	a
b	b	a	a	a	a
c	a	a	a	a	a
d	a	a	a	a	a
e	a	b	c	d	e

Then $(S, .)$ is a right pseudo commutative ternary right almost semigroup.

Definition 3.11. An element 'a' of a ternary right almost semigroup S is said to be right identity if $saa = s$ for all $s \in S$.

Definition 3.12. An element $'a'$ of ternary right almost semigroup S is said to be identity or unital if $saa = s$ for all $s \in S$.

Example 3.13. Let Z_0^- be the set of all non-positive integers. Then with the usual ternary operation '.', Z_0^- forms a ternary right almost semigroup with the identity element -1.

Theorem 3.14. Any ternary right almost semigroup S has almost one identity.

Note 2. The identity of ternary right almost semigroup is usually denoted by $'1'$ (or) $'e'$.

Definition 3.15. [9] An element $'a'$ of a ternary right almost semigroup S is said to be right zero of S if $bca = a$ for all $b, c \in S$.

A ternary right almost semigroup S is said to be right zero ternary right almost semigroup if every element of S is right zero element.

Definition 3.16. An element $'a'$ of a ternary right almost semigroup S is said to be zero of S if $bca = a$ for all $b, c \in S$.

A ternary right almost semigroup S is said to be zero ternary right almost semigroup if every element of S is zero element.

Example 3.17. Let $0 \in S$ and $\|S\| > 2$. Then S with the ternary operation $'.'$ defined by $x.y.z =—$ is ternary right almost semigroup with 0 (zero).

Result 3.18. Any ternary right almost semigroup S has at most one nonzero element.

Definition 3.19 . An element $'a'$ of a ternary right almost semigroup S is said to be an idempotent if $a^3 = a$.

Note 3. The set of all idempotent elements in a ternary right almost semigroup S is denoted by $I(S)$.

Definition 3.20. An element $'a'$ of a ternary right almost semigroup S is said to be a proper idempotent element provided $'a'$ is an idempotent and which is not an identity of S when identity exists.

Definition 3.21. A ternary right almost semigroup S is said to be an idempotent ternary right almost semigroup or a ternary band if every element of S is an idempotent.

Definition 3.22 . A ternary right almost semigroup S is said to be a right cancellative if $xab = yab => x = y$.

Definition 3.23 . An element $'a'$ of a ternary right almost semigroup S is said to be a regular if there exists $x, y \in S$ such that $axaya = a$.

A ternary right almost semigroup S is said to be a regular ternary right almost semigroup if every element of S is regular.

Result 3.24. Every idempotent element of a ternary right almost semigroup S is regular.

Definition 3.25. An element $'a'$ of a ternary right almost semigroup S is said to be a right regular if there exists $x, y \in S$ such that $a = xya^2$.

An element 'a' of a ternary right almost semigroup S is said to be intra regular if there exists $x, y \in S$ such that $a = xa^5y$.

Definition 3.26 . An element $'a'$ of a ternary right almost semigroup S is said to be completely regular if there exists $x, y \in S$ such that $axaya = a$ and $axa = aax = xaa = aya = aay = yaa = axy = yxa = xay = yax$.

A ternary right almost semigroup S is said to be completely regular ternary right almost semigroup if every element of S is completely regular.

Result 3.27. Let S be a ternary right almost semigroup and $a \in S$. If $'a'$ is a completely regular element in S, then ' a ' is right regular in S.

Result 3.28. If S is a commutative ternary right almost semigroup, then S is a quasi-commutative ternary right almost semigroup.

Result 3.29. If S is a quasi-commutative ternary right almost semigroup, then S is a normal ternary right almost semigroup.

Result 3.30. Every commutative ternary right almost semigroup S is a normal ternary right almost semigroup.

Result 3.31. If S is a commutative ternary right almost semigroup, then S is a pseudo commutative ternary right almost semigroup.

4 Ternary right almost Semigroups Satisfying the Identity abc=ba

In this section we prove some properties of ternary right almost semigroups satisfying the identity $abc = ba$ [10–12].

Theorem 4.1. A quasi-commutative ternary right almost semigroup S is a commutative ternary right almost semigroup if all elements of S are idempotent.

Proof. Let S be a quasi-commutative ternary right almost semigroup.

Then

$$abc = b^n ac - bca = c^n ba = cab = a^n cb \tag{1}$$

for all $a, b, c \in S$,

where $'n'$ is a natural number.

Since $a \in S \Rightarrow a^3 \in S$

$\Rightarrow aa^3 = aa \Rightarrow a^4 = a^2 \Rightarrow a^5 = a^3 = a \Rightarrow a^5 = a, \ a^7 = a, \ \ldots.$

In generally we write this $a^{2n+l} = a$ for $n = 1, 2, 3, \ldots.$

From result 3.21, every idempotent element of S is regular.

Here $'a'$ is regular. Then there exists $x, y \in S$ such that $a = axaya$.

Now, we have to prove that S is commutative,

i.e., $abc = bca = cab = bac = cba = acb$ for all $a, b, c \in S$.

From Equation (1) it is enough to prove that

$b^n ac = bac, \ \ c^n ba = cba, \ a^n cb = acb$ for $n = 1, 2, 3, \ldots.$

Consider $b^n ac = bb^{n-1}ac$,

$= b^{2n+1} b^{n-1} ac$, (since $b = b^{2n+1}$)

$= bb^{3n-1}ac$,

$= bb^{3n-2}bac$,

$= bb^{3n-2}b^{2n+1}ac$,

$= bb^{3n-2}bb^{2n}ac$,

$= bb^{3n-2}bb^{2n-1}bac$,

$= bxbybac$, $\quad (x = b^{3n-2} \ and \ y = b^{2n-1})$

$= bac$ (Since 'b' is regular)

$$b^n ac = bac,$$

Similarly, $c^n ba = cba, \ a^n cb = acb$.

Therefore, $abc = bca = cab = bac = cba = acb$ for all $a,b,c \in S$.

Hence, S is a commutative ternary right almost semigroup.

Theorem 4.2. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a,b,c \in S$, then S is a commutative ternary right almost semigroup.

Proof. Let S be a regular ternary right almost semigroup. Then for every $a \in S$, there exists $x,y \in S$ such that $a = axaya$. Given that S satisfies the identity $abc = ba$ for all $a,b,c \in S$, we have to prove that S is a commutative ternary right almost semigroup,

i.e., $abc = bca = cab = bac = cba = acb$ for all $a,b,c \in S$.

Consider $abc = a(bcx)cyc \quad (c = cxcyc)$

$$= ac(bcy)c \quad (bcx = cb)$$

$$= (ac)cbc \quad (bcy = cb)$$

$$= c(abc)bc \quad (ac = cab)$$

$$= cb(abc) \quad (abc = ba)$$

$$= (cb)ba \quad (abc = ba)$$

$$= b(cab)a \quad (cb = bca)$$

$$= (ba)ca \quad (cab = ac)$$

$$= ab(xca) \quad (for\ all\ x \in S,\ ba = abx)$$

$$= a(bcx) \quad (xca = cx)$$

$$= acb \quad (bcx = cb)$$

$$abc = acb \tag{1}$$

Consider $acb = (acb)ubvb = c(aub)vb(b = bubvb$ and $acb = ca)$

$$= c(uav)b \quad (aub = ua)$$

$$= ca(ub) \quad (uav = au)$$

$$= c(abu)v \quad (ub = buv)$$

$$= c(bav) \quad (abu = ba)$$

$$= cab$$

$$acb = cab \tag{2}$$

Consider $cab = (cab)ubvb = a(cub)vb \qquad (b = bubvb \; and \; cab = ac)$

$$= a(ucv)b \quad (cub = uc)$$

$$= ac(ub) \quad (ucv = cu)$$

$$= a(cbu)c \quad (ub = buc)$$

$$= (abc)c \quad (cbu = bc)$$

$$= bac \quad (abc = ba)$$

$$cab = bac \tag{3}$$

Consider $bac = b(acx)cyc = bca(cy)c \quad (c = cxcyc \; and \; acx = ca)$

$$= bc(ayc)xc \quad (cy = ycx)$$

$$= (bcy)axc \quad (ayc = ya)$$

$$= c(bax)c \quad (bcy = cb)$$

$$= c(abc) \quad (bax = ab)$$

$$= cba \quad (abc = ba)$$

$$bac = cba \tag{4}$$

Consider $cba = (cba)xaya = b(cxa)ya \; (axaya = a \; and \; cba = bc)$

$$= b(xcy)a \quad (cxa = xc)$$

$$= bc(xa) \quad (xcy = cx)$$

$$= b(cax)y \quad (xa = axy)$$

$$= b(acy) \quad (cax = ac)$$

$$= bca \quad (acy = ca)$$

$$cba = bca \tag{5}$$

From Equations (1), (2), (3), (4) and (5) we get $abc = acb = cab = bac = cba = bca$ implies that $abc = bca = cab = bac = cba = acb$ for all $a,b,c \in S$.

Therefore, S is a commutative ternary right almost semigroup.

Theorem 4.3. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a,b,c \in S$, then S is right regular.

Proof. Let S be a regular ternary right almost semigroup. Then for any $a \in S$, there exists $x,y \in S$ such that $a = axaya$.

Given that S satisfies the identity $abc = ba$ for all $a,b,c \in S$,

i.e., S is commutative, We have to prove that S is right regular. i.e., for any $a \in S$, there exists $x,y \in S$ such that $a = xya^2$.

Consider $a = axaya = a(xya)a \quad (aya = yaa)$

$$= (ayx)a \quad (xya = yx)$$

$$= xyaa \quad (ayx = xay)$$

$$= xya^2$$

$a = xya^2$ for all $a \in S$. Therefore, S is right regular.

Theorem 4.4. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a,b,c \in S$, then S is completely regular.

Proof. Let S be a regular ternary right almost semigroup. Then for any $a \in S$, there exist $x,y \in S$ such that $a = axaya$. Given that S satisfies the identity $abc = ba$ for all $a,b,c \in S$,

from theorem 4.2, S is commutative, and we have to prove that S is completely regular,

i.e., if $a \in S$, then there exist $x,y \in S$ such that $a = axaya$ and

$$axa = aax = xaa = aya = aay = yaa = axy = yxa = xay = yax,$$

By the regularity of S we have $axaya = a$ for all $a \in S$. To prove that

$$axa = aax = xaa = aya = aay = yaa = axy = yxa = xay = yax,$$

Consider $axa = axa(yax)a \quad (a = axaya)$

$$= ax(aay)a \quad (yax = ay)$$

$$= ax(aya)a \quad (aay = aya)$$

$$= a(xya)aa \quad (aya = yaa)$$

$$= a(yxa)a \quad (xya = yx)$$

$$= a(xya) \quad (yxa = xy)$$

$$= ayx \quad (xya = yx)$$

$$axa = ayx \tag{1}$$

Consider $ayx = axayayx \quad (a = axaya)$

$$= aax(ayx) \quad (xay = ax)$$

$$= aa(xya) \quad (axy = ya)$$

$$= a(ayx) \quad (xya = yx)$$

$$= aya \quad (ayx = ya)$$

$$ayx = aya \tag{2}$$

From Equations (1) and (2) we get $axa = ayx = aya$

$$=> axa = aya = ayx \tag{3}$$

Since S is commutative, from Equation (3) we get,

$$axa = aax = xaa = aya = aay = yaa = axy = yxa = xay = yax.$$

Therefore, S is completely regular.

Theorem 4.5. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a, b, c \in S$, then S is right cancellative.

Proof. Let S be a regular ternary right almost semigroup satisfying the identity $abc = ba$ for all $a, b, c \in S$.

We have to prove that S is right cancellative,
i.e., if $xab = yab => x = y$ for all $a, b, c,\ x,\ y \in S$.
Consider $xab = yab$

$$xux(vxa)b = ypy(qya)b \quad (x = xuxvx \text{ and } y = ypyqy\ )$$

$$xwc(xvb) = ypyiyqb) \quad for\ all\ u, v, p, q \in S.$$

$$xuxvx = ypyqy$$

$$x = y.$$

Therefore, S is right cancellative.

Corollary 4.6. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a, b, c \in S$, then S is quasi commutative.

Proof. Let S be a regular ternary right almost semigroup satisfying the identity $abc = ba$ for all $a, b, c \in S$. Then from theorem 4.2, S is commutative. From result 3.31, every commutative ternary right almost semigroup is a quasi-commutative ternary right almost semigroup. Hence, S is quasi commutative.

Corollary 4.7. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a, b, c \in S$, then S is normal.

Proof. Let S be a regular ternary right almost semigroup satisfying the identity $abc = ba$ for all $a, b, c \in S$. Then from corollary 4.6, S is quasi commutative. From result 3.29, every quasi-commutative ternary right almost semigroup is normal. Hence, S is normal.

Corollary 4.8. If a regular ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a, b, c \in S$, then S is pseudo commutative.

Proof. Let S be a regular ternary right almost semigroup satisfying the identity $abc = ba$ for all $a, b, c \in S$. Then from theorem 4.2, S is commutative. Again, from result 3.31, every commutative ternary right almost semigroup is a pseudo commutative ternary right almost semigroup. Hence, S is pseudo commutative.

Theorem 4.9. If a right pseudo commutative ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a, b, c \in S$, then S is commutative.

Proof. Let S be a right pseudo commutative ternary right almost semigroup. Then for all $a, b, c \in S$ such that

$$abcde = abdec = abecd = abdce = abedc = abced \tag{1}$$

we have to prove that S is commutative. Since S satisfies the identity $abc = ba$ for all
$a, b, c \in S$,
we Consider $a(bcd)e = a(cbe) \quad [\,bed = cb]$

$$= abc \quad (cbe = bc)$$

$$abcde = abc \tag{2}$$

Consider $(ab)\,dec = b\,(acd)\,ec \quad [\,ab = bac\,]$

$$= bca\,(ec) \quad [\,acd = ca\,]$$

$$= (bca)\,cef \quad [ec = cef]$$

$$= cb(cef) \quad [bca = cb]$$

$$= cbe(caf) \quad [ce = eca]$$

$$= cb(eac) \quad [caf = ac]$$

$$= c(bae) \quad [eac = ae]$$

$$= cab \quad [bae = ab]$$

$$abdec = cab \tag{3}$$

Consider $ab(ecd) = a(bce) \quad [ecd = ce]$

$$= acb$$

$$abecd = acb \tag{4}$$

Consider $(abd)ce = b(ace) \quad [abd = ba]$

$$= bca \quad [ace = ca]$$

$$abdce = bca \tag{5}$$

Consider $(ab)edc = b(ace)dc \quad [ab = bac]$

$$= (bc)adc \quad [ace = ca]$$

$$= cb(ead)c \quad [bc = cbe]$$

$$= c(bae)c \quad [ead = ae]$$

$$= c(abc) \quad [bae = ab]$$

$$= cba \quad [abc = ba]$$

$$abedc = cba \tag{6}$$

Consider $ab(ced) = (abe)c\,[\,ced = ec]$

$$= bac$$

$$abced = bac \tag{7}$$

Substituting Equations (2), (3), (4), (5), (6) and (7) in Equation (1)

we have that $abc = cab = acb = bca = cba = bac$ implies that $abc = bca = cab = bac = cba = acb$ for all $a,b,c \in S$.

Therefore, S is commutative.

Theorem 4.10. If a pseudo commutative ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a,b,c \in S$, then S is commutative.

Proof. The theorem follows from the above three theorems.

Corollary 4.11. If a pseudo commutative ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a,b,c \in S$, then S is quasi commutative.

Proof. Let S be a pseudo commutative ternary right almost semigroup and S satisfies the identity $abc = ba$ for all $a,b,c \in S$. Then from theorem 4.10, S is commutative. From result 3.31, every commutative ternary right almost semigroup is quasi commutative. Hence S is quasi commutative.

Corollary 4.12. If a pseudo commutative ternary right almost semigroup S satisfies the identity $abc = ba$ for all $a,b,c \in S$, then S is normal.

Proof. Let S be a pseudo commutative ternary right almost semigroup satisfying the identity $abc = ba$ for all $a,b,c \in S$. Then from corollary 4.11, S is a quasi-commutative ternary right almost semigroup. From result 3.29, every quasi-commutative ternary right almost semigroup is normal. Hence, S is normal.

5 Conclusion

The properties of ternary semigroups, ternary right almost semigroups, commutative ternary right almost semigroups, regular and normal ternary right almost semigroups were discussed. We also proved that a regular ternary right almost semigroup satisfying the identity $abc = ba$ for all $a,b,c \in S$, is a commutative ternary right almost semigroup, completely regular, right cancellative, quasi-commutative and pseudo commutative ternary right almost semigroup satisfying the identity $abc = ba$ for all $a,b,c \in S$, is commutative.

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