INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 12-10-2023

Accepted: 22-11-2023

Published: 25-12-2023

Citation: Vijayalakshmi R, Thiripurasundari V (2023) On Micro $\widehat{g}\pi$ - Closed Sets in Micro Topological Spaces. Indian Journal of Science and Technology 16(SP4): 18-21. https://doi.org/10.17485/IJST/v16iSP4.ICAMS106

* **Corresponding author**.

vijayalakshmir218@gmail.com

Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

On Micro $\widehat{g}\pi$ - Closed Sets in Micro Topological Spaces

R Vijayalakshmi[1]*, V Thiripurasundari[2]

1 Research Scholar, PG and Research Department of Mathematics, Sri S. Ramasamy Naidu Memorial College (Autonomous), Sattur, Affiliated to Madurai Kamaraj University, Madurai, Tamil Nadu, India
2 Assistant Professor, PG and Research Department of Mathematics, Sri S. Ramasamy Naidu Memorial College (Autonomous), Sattur, Affiliated to Madurai Kamaraj University, Madurai, Tamil Nadu, India

Abstract

Objective: To introduce a new class of sets namely Micro-$\widehat{g}\pi$ -closed (briefly $\mu\widehat{g}\pi$ -closed) sets in Micro topological spaces. **Methods:** The study investigated the concepts of $\mu\widehat{g}\pi$ -closed sets and brief study of $\mu\widehat{g}\pi$ -closed set was made. **Findings:** We derived the inter-relations between $\mu\widehat{g}\pi$ -closed sets with already existing Micro closed sets in Micro topological spaces and found some of its basic properties. **Novelty:** Application of $\mu\widehat{g}\pi$ -sets to introduce a new class of space namely $\mu\widehat{T}_{1/2}$ -space.

Keywords: Micro open set, Micro-gs open set, $\mu\widehat{g}\pi$-closed set, $\mu\widehat{g}\pi$- open set, $\mu\widehat{T}_{1/2}$-space

1 Introduction

Micro topology was introduced by Sakkraiveeranan Chandrasekar[1] and he also introduced the concepts of Micro pre-open and Micro semi-open sets. Recently, we initiated the concept of $\widehat{g}\pi$- closed sets in topological spaces[2] and also studied its properties. In this paper, we have introduced a new class of Micro closed sets called Micro $\widehat{g}\pi$ -closed sets and its properties are studied in Micro topological spaces. Further, we have derived relations between Micro $\widehat{g}\pi$- closed sets with already existing various Micro closed sets. Later, we have defined and analysed $\mu\widehat{T}_{1/2}$- space.

2 Preliminaries

In this paper, (Ω, N, M) denote the micro topological spaces, where $N = \tau_R(X)$, $M = \mu_R(X)$ and MTS denote micro topological space appropriately. For a subset P of a space, $cl_\mu(P)$ and $int_\mu(P)$ denotes the closure of P and the interior of P respectively.

Definition 2.1.[1] Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup \left(N' \cap \mu\right) : N, N' \in \tau_R(X)\}$ and $\mu \neq \tau_R(X)$ and $\mu_R(X)$ satisfies the following axioms:

1. U and φ are in $\mu_R(X)$
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$

Then, $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is Micro topological space and the elements of $\mu_R(X)$ are Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.2. Let (Ω, N, M) be a Micro topological space. A subset of A is

i. Micro-sg-closed, if $scl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro- s- open in U.[2]

ii. Micro-gs-closed, if $scl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro-open in U.[2]

iii. Micro-αg-closed, if $\alpha cl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro- open in U.[3]

iv. Micro-$g\alpha$-closed, if $\alpha cl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro- α- open in U.[3]

v. Micro-g^*-closed, if $cl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro- g-open in U.[4]

vi. Micro-g-closed, if $cl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro- open in U.[5]

vii. Micro-ψ-closed, if $scl_\mu(A) \subseteq L, A \subseteq L$ and L is Micro-sg−open in U.[6]

3 Micro- $\widehat{g}\pi$ -Closed Set

In this section, we have derived the characteristics of Micro- $\widehat{g}\pi$ (shortly $\mu\widehat{g}\pi$) closed set and its inter-relations with existing other Micro closed sets.

Definition 3.1. Consider (Ω, N, M) as MTS and $P \subseteq \Omega$. Then P is defined as $\mu\widehat{g}\pi$ - Closed set if $\pi cl_\mu(P) \subseteq L$ whenever $P \subseteq L$ and L is Micro-gs -open in Ω.

Theorem 3.2. Every Micro- π - closed set is $\mu\widehat{g}\pi$ -closed set but not conversely.

Proof. Consider a Micro-π -closed set P in Ω such that $P \subseteq L$ where L is a μgs - open. Therefore, $P = \pi cl_\mu(P) \subseteq L$. Thus, P is $\mu\widehat{g}\pi$ -closed set.

Example 3.3. Consider $\Omega = \{u,v,w,x\}$ with $\Omega/R = \{\{u\}, \{w\}, \{v,x\}\}$. Let $X = \{u,v\} \subseteq \Omega$, then $N = \{\Omega, \varphi, \{u\}, \{u,v,x\}, \{v,x\}\}$. If $\mu = \{w\}$, then $M = \{\Omega, \varphi, \{u\}, \{w\}, \{u,w\}, \{v,x\}, \{v,w,x\}, \{u,v,x\}$. Though the set $P = \{u,w,x\}$ is $\mu\widehat{g}\pi$-closed it is not Micro-π -closed.

Theorem 3.4. Every $\mu\widehat{g}\pi$ - closed set is Micro-g -semi closed set but not conversely.

Proof. Consider a $\mu\widehat{g}\pi$ -closed set P in Ω such that $P \subseteq L$ where L is Micro-gs- open. Since $\pi cl_\mu(P) \subseteq L$, $scl_\mu(P) \subseteq \pi cl_\mu(P) \subseteq L$. Hence, P is Micro-gs - closed set.

Example 3.5. Let $\Omega = \{u,v,w,x\}$ with $\Omega/R = \{\{u\}, \{w\}, \{v,x\}\}$. Let $X = \{v,x\} \subseteq \Omega$, then $N = \{\Omega, \varphi, \{v,x\}$. If $\mu = \{v\}$, then $M = \{\Omega, \varphi, \{v\}, \{v,x\}$. Though the set $P = \{u\}$ is Micro-gs – closed, it is not $\mu\widehat{g}\pi$ -closed.

Theorem 3.6. Every $\mu\widehat{g}\pi$ - closed set is Micro- g -closed set but not conversely.

Proof. Consider a $\mu\widehat{g}\pi$ -closed set P in Ω such that $P \subseteq L$ where L is a μg -semi open. We know that every Micro open set is Micro-g-semi open, so $\pi cl_\mu(P) \subseteq L$. Therefore, $cl_\mu(P) \subseteq \pi cl_\mu(P) \subseteq L$. Thus, P is Micro-g-closed set.

Example 3.7. Let $\Omega = \{u,v,w,x\}$, $\Omega/R = \{\{u\}, \{w\}, \{v,x\}\}$. Let $X = \{u,v\} \subseteq \Omega$, then $N = \{U, \varphi, \{u\}, \{u,v,x\}, \{v,x\}\}$. If $\mu = \{w\}$, then the $M = \{\Omega, \varphi, \{u\}, \{w\}, \{u,w\}, \{v,x\}, \{v,w,x\}, \{u,v,x\}$. Though the set $P = \{u,v\}$ is Micro-g –closed, it is not $\mu\widehat{g}\pi$ -closed.

Theorem 3.8. Every $\mu\widehat{g}\pi$ - closed set is Micro-g^* -closed set but not conversely.

Proof. Consider a $\mu\widehat{g}\pi$ -closed set in Ω such that $P \subseteq L$ where L is a μg -semi open. We know that every Micro-g-open set Micro-gs -open, so $\pi cl_\mu(P) \subseteq L$. Therefore, $cl_\mu(P) \subseteq \pi cl_\mu(P) \subseteq L$. Thus, P is μg^* -closed set.

Example 3.9. Let $\Omega = \{u,v,w,x\}$ with $\Omega/R = \{\{w\}, \{x\}, \{u,v\}\}$. Let $X = \{w\} \subseteq \Omega$, then $N = \{\Omega, \varphi, \{w\}\}$. If $P = \{v\}$, then $\mathscr{M} = \{\Omega, \varphi, \{\mathrm{v}\}, \{\mathrm{w}\}, \{\mathrm{v, w}\}\}$. Then $A = \{u,v,x\}$ is Micro-g^* -closed but it is not $\mu\widehat{g}\pi$ –closed.

Theorem 3.10. Every $\mu\widehat{g}\pi$ - closed set is Micro-sg -closed set but not conversely.

Proof. Consider a $\mu\widehat{g}\pi$ -closed set P in Ω such that $P \subseteq L$ where L is a μg -semi open. We know that every Micro-s-open set is Micro-gs-open, so $\pi cl_\mu(P) \subseteq L$. Then, $scl_\mu(P) \subseteq \pi cl_\mu(P) \subseteq L$. Thus, P is Micro-sg -closed set.

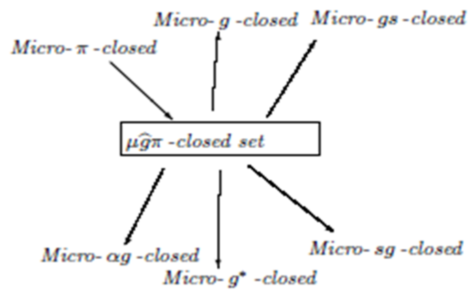
Example 3.11. Let $\Omega = \{u,v,w,x\}$ with $\Omega/R = \{\{w\}, \{x\}, \{u,v\}\}$. Let $X = \{w\} \subseteq \Omega$, then $N = \{\Omega, \varphi, \{w\}\}$. If $\mu = \{w\}$, then $M = \{\Omega, \varphi, (v), (w), \{v,w\}\}$. Though the set $P = \{l\}$ is Micro-sg –closed, it is not $\mu\widehat{g}\pi$ -closed.

Theorem 3.12. Every $\mu\widehat{g}\pi$ - closed set is Micro-αg -closed set but not conversely.

Proof. Consider $\mu\widehat{g}\pi$ -closed set in Ω such that $P \subseteq L$ where L is a μg - semi open. We know that every Micro-open set is Micro-gs -open, so $\pi cl_\mu(P) \subseteq L$. Then, $scl_\mu(P) \subseteq \pi cl_\mu(P) \subseteq L$. Thus, P is Micro-αg -closed set.

Example 3.13. Let $\Omega = (u,v,w,x\}$ with $\Omega/R = \{\{u\}, \{v\}, \{w,x\}\}$. Let $= \{u,w\} \subseteq \Omega$, then $\tau_R(X) = \{\Omega, \varphi, \{u\}, \{u,w,x\}, \{w,x\}\}$. If $= \{w\}$, then $M = \{\Omega, \varphi, \{u\}, \{w\}, \{u,w\}, \{v,x\}, \{v,w,x\}, \{u,v,x\}$. Though the set $P = \{u,v,x\}$ is Micro-αg –closed, it is not Micro-$\widehat{g}\pi$ -closed.

Remark 3.14. The following Implication diagram shows that the inter-relations with some other existing sets.



4 Characteristics of $\mu\widehat{g}\pi$- Closed Set

Theorem 3.15. The union of two $\mu\widehat{g}\pi$ -closed subset of (Ω,N,M) is also a $\mu\widehat{g}\pi$ -closed.

Proof. Consider two $\mu\widehat{g}\pi$ -closed sets P and Q in (Ω,N,M). L is Micro g -semi open sets in Ω containing $P\cup Q$. Then, $\pi cl_{\mu}(P\cup Q)\subseteq \pi cl_{\mu}(P)\cup \pi cl_{\mu}(P)\subseteq L$. Thus, $P\cup Q$ is $\mu\widehat{g}\pi$ closed.

Remark 3.16. The intersection of two $\mu\widehat{g}\pi$ -closed sets in (Ω,N,M) need not be $\mu\widehat{g}\pi$ -closed in (Ω,N,M)

Example 3.17. Consider $\Omega = \{w,x,y,z\}$ with $\Omega/R = ((w\},(y\},(x,z\}\}$. Let $X = \{w,x\} \subseteq \Omega$, then $N = \{\Omega,\ \varphi,\ \{w\},\ \{w,x,z\},\ \{x,z\}\}$. If $\mu = \{y\}$ then $M = \{\Omega,\ \varphi,\ \{w\},\ \{y\},\ \{w,y\},\ \{x,z\},\ \{x.y,z\},\ \{w,x,z\}\}$. Then the sets $\{w,y,z\}$ and $\{w,x,z\}$ are $\mu\widehat{g}\pi$ -Closed sets but their intersection {z} is not Micro- $\widehat{g}\pi$ -Closed.

Theorem 3.18. If P is $\mu\widehat{g}\pi$ - closed in Ω and $P\subseteq Q\subseteq \pi cl_{\mu}(P)$, then Q is also $\mu\widehat{g}\pi$ closed in Ω.

Proof. Consider a $\mu\widehat{g}\pi$- closed set P in X here with $P\subseteq Q\subseteq \pi cl_{\mu}(P)\subseteq L$. Suppose that L is μgs-open of X with $Q\subseteq L$, then $P\subseteq L\Rightarrow \pi cl_{\mu}(P)\subseteq L$. So $\pi cl_{\mu}(Q)\subseteq L$ and Q is $\mu\widehat{g}\pi$ - closed in Ω.

Theorem 3.19. If P is a $\mu\widehat{g}\pi$ - closed set of Ω if and only if $\pi cl_{\mu}(P)-P$ does not consists of any non-empty Micro-gs - closed.

Proof. Suppose there exist a non-empty Micro-gs- closed set V of Ω such that $V\subseteq \pi cl_{\mu}(P)-P$, then $V\subseteq \pi cl_{\mu}(P)$. Since P is $\mu\widehat{g}\pi$ - closed and $\Omega - V$ Micro-gs-open, $\pi cl_{\mu}(P)\subseteq\Omega - V$. This implies $V\subseteq\Omega-\pi cl_{\mu}(P)$. So $V\subseteq(\pi cl_{\mu}(P)-P)\cap(\Omega-\pi cl_{\mu}(P))\subseteq\pi cl_{\mu}(P)\cap(\Omega-\pi cl_{\mu}(P)) = \varphi\Rightarrow V=\varphi$.

Conversely, assume that $\pi cl_{\mu}(P)-P$ consists of no non-empty Micro-gs- closed such that $P\subseteq L$ where L is Micro-gs - open set. Assume that $\pi cl_{\mu}$is not in L. Then $\pi cl_{\mu}(P)\cap L^{c}$ is a non-empty Micro-gs-closed set in $\pi cl_{\mu}(P)-P$ which is a contradiction. Then $\pi cl_{\mu}(P)\subseteq L$ and hence P is $\mu\widehat{g}\pi$ - closed set.

Theorem 3.20. The intersection of Micro-gs - closed and $\mu\widehat{g}\pi$ –closed is always $\mu\widehat{g}\pi$ -Closed.

Proof. Consider $\mu\widehat{g}\pi$ –closed set P and μgs – closed set V. This implies U is μgs – open set with $P\cap V\subseteq U$. Then, $P\subseteq U\cup V^{c}$ is μgs -open. Since P is $\mu\widehat{g}\pi$ –closed, $\pi cl_{\mu}(P)\subseteq U\cup V^{c}\Rightarrow \pi cl_{\mu}(P)\subseteq V\subseteq U$. Thus, $\pi cl_{\mu}(P\cap V)\subseteq\pi cl_{\mu}(P)\cap\pi cl_{\mu}(V)\subseteq \pi cl_{\mu}(P)\cap V\subseteq U$. Hence, $P\cap V$ is $\mu\widehat{g}\pi$ -closed.

Definition 3.21. Let $P\subseteq Y\subseteq\Omega$. Then P is $\mu\widehat{g}\pi$ – closed with relative to Y if $\pi cl_{\mu Y}(A)\subseteq U$ where $A\subseteq U$ and U is Micro- g – semi open in Y.

Theorem 3.22. Let $P\subseteq Y\subseteq\Omega$ and suppose that P is $\mu\widehat{g}\pi$ closed in X. Then P is $\mu\widehat{g}\pi$ - closed with relative to Y.

Proof. Let us assume that $P\subseteq Y\cap Z$ where Z is μgs-open in X. $P\subseteq Z\Rightarrow\pi cl_{\mu}(P)\subseteq Z$. This implies that $\pi cl_{\mu}(P)\cap Y\subseteq Z\cap Y$. Thus, P is $\mu\widehat{g}\pi$ – closed relative to Y.

Definition 3.23. A subset P in Ω is defined as $\mu\widehat{g}\pi$ -open in Ω if $\Omega-P$ is $\mu\widehat{g}\pi$ closed in Ω.

Theorem 3.24. If $\pi int_{\mu}(P)\subseteq Q\subseteq P$ and P is $\mu\widehat{g}\pi$ -open in Ω, then Q is $\mu\widehat{g}\pi$ -open in Ω.

Proof. Suppose that $\pi int_{\mu}(P)\subseteq Q\subseteq P$ and P is $\mu\widehat{g}\pi$ -open in Ω. Then $\Omega\setminus P\subseteq\Omega\setminus Q\subseteq\pi cl_{\mu}(\Omega\setminus P)$. Since $\Omega\setminus P$ is $\mu\widehat{g}\pi$ closed in Ω implies that $\Omega\setminus Q$ is $\mu\widehat{g}\pi$ closed in Ω. Hence, Q is $\mu\widehat{g}\pi$ in Ω.

Theorem 3.25. Consider a MTS Ω and $S,T\subseteq\Omega$. If S is $\mu\widehat{g}\pi$ -open and $\pi int_{\mu}(T)\subseteq S$, then $S\cap T$ is $\mu\widehat{g}\pi$ -open.

Proof. Given T is $\mu\widehat{g}\pi$-open and $\pi int_{\mu}(T)\subseteq S$, $\pi int_{\mu}(T)\subseteq S\cap T\subseteq T$. Hence, $S\cap T$ is $\mu\widehat{g}\pi$ -open.

Theorem 3.26. A set P is $\mu\widehat{g}\pi$ -open in Ω if and only if $V\subseteq\pi int_{\mu}(P)$ whenever V is Micro-gs - closed in Ω and $V\subseteq P$.

Proof. Suppose $V\subseteq\pi int_{\mu}(P)$, V is μgs -closed in Ω and $V\subseteq P$. Let $\Omega - P\subseteq G$ where G is μgs- open in Ω. So that $G\subseteq\Omega-P$ and $\Omega-G\subseteq\pi int_{\mu}(P)$. Thus, $\Omega-P$ is $\mu\widehat{g}\pi$ -closed in Ω. Hence, P is $\mu\widehat{g}\pi$ -open in Ω.

Conversely, suppose that P is $\mu\widehat{g}\pi$ -open, $V\subseteq P$ and V is μgs - closed in Ω. Then $\Omega - V$ is Micro-gs-open and $\Omega - P\subseteq\Omega - V$. But $\pi int_{\mu}(\Omega - P) = \Omega-\pi int(P)$. Hence $V\subseteq\pi int_{\mu}(P)$.

Theorem 3.27. If P is $\mu\widehat{g}\pi$ - open in Ω, then $U=\Omega$ when U is Micro-gs -open and $\pi int_{\mu}(P)\subseteq P^{c}\subseteq U$.

Proof. Given P is a $\mu\widehat{g}\pi$ open and U is a Micro-gs - open, $\pi int_{\mu}(P) \cup P^c \subseteq U$. This gives $U^c \subseteq (X - \pi int_{\mu}(P)) \cap P = \pi int_{\mu}(P^c) - P^c$. Since U^c is μgs -closed and P is $\mu\widehat{g}\pi$ - open. We have $U^c = \varphi$. Thus, $U = \Omega$.

Definition 3.28. Let (Ω, N, M) be a MTS. Then Ω is said to be $\mu\widehat{T}_{1/2}$ -space if every $\mu\widehat{g}\pi$ -closed set in Ω is Micro-π -closed in Ω.

Theorem 3.29. For a MTS (Ω, N, M) the following conditions are equivalent.

(i) (Ω, N, M) is a $\mu\widehat{T}_{1/2}$ -space

(ii) Every singleton set $\{p\}$ is either Micro-gs - closed or Micro-π -open.

Proof. (i) =) (ii) Take $p \in \Omega$. If $\{p\}$ is not a Micro-gs- closed set of (Ω, N, M). Then $\Omega - \{p\}$ is not a Micro-gs- open set. Thus, $\Omega - \{p\}$ is an $\mu\widehat{g}\pi$ -Closed set of (Ω, N, M). Since (Ω, N, M) is $\mu\widehat{T}_{1/2}$ -space, $\Omega - \{p\}$ is Micro-π -closed set of (Ω, N, M) , That is $\{p\}$ is Micro-π -open set of (Ω, N, M).

(ii) =) (i) Let P be an $\mu\widehat{g}\pi$ -Closed set of (Ω, N, M). Let $p \in \pi cl_{\mu}(P)$. By (ii), $\{p\}$ is either Micro-gs -closed or Micro-π -open.

Case(i): If $\{p\}$ is Micro-gs -closed and $p \notin P$. Then $\pi cl_{\mu}(P) - P$ contains a non-empty Micro-gs- closed set. This contradicts Theorem 3.19 as P is a Micro-gs - closed set. Therefore, $p \in P$.

Case(ii): Consider a Micro-π-open set $(p\}$. Then $\Omega - (p\}$ is Micro-π -closed. If $p \notin P$, then $P \subseteq \Omega - (p\}$. Since $p \, \varepsilon \, \pi cl_{\mu}(P)$, we have $p \in \Omega - \{p\}$, which is a contradiction. Hence, $p \in P$.

So in both cases we have $\pi cl_{\mu}(P) \subseteq P$. Trivially $P \subseteq \pi cl_{\mu}(P)$. Therefore, $P = \pi cl_{\mu}(P)$ or equivalently P is Micro-π -closed. Hence, (Ω, N, M) is a $\mu\widehat{T}_{1/2}$ -space.

5 Conclusion

A new class of sets called $\mu\widehat{g}\pi$-closed sets have been introduced and some of their properties have been studied. Also, $\mu\widehat{T}_{1/2}$-spaces is presented and its properties are analyzed. Furthermore, $\mu\widehat{g}\pi$-sets can be used to derive a new class of continuity, closed maps, homeomorphism.

6 Declaration

Presented in ‘International Conference on Applied Mathematical Sciences’ (ICAMS 2022) during 21 & 22 Dec. 2022, organized virtually by the Department of Mathematics, JJ College of Arts and Science, Pudukkottai, Tamil Nadu, India. The Organizers claim the peer review responsibility.

References

1) Chandrasekar S. On Micro topological spaces. *Journal of New Theory*. 2019;(26):23–31. Available from: https://dergipark.org.tr/en/pub/jnt/issue/42082/506329.
2) Thiripurasundari V, Vijayalakshmi R. A New type of Generalized Closed Set in Topological Spaces. *Journal of Information and Computational Science*. 2019;9(10). Available from: http://www.joics.org/gallery/ics-1552.pdf.
3) Ibrahim HZ. Micro β-open sets in Micro topology. *General Letters in Mathematics*. 2020;8(1):8–15. Available from: https://www.refaad.com/Files/GLM/GLM-8-1-2.pdf.
4) Ibrahim HZ. On Micro T1/2 -space. *International Journal of Applied Mathematics*. 2020;33(3):369–384. Available from: https://doi.org/10.12732/ijam.v33i3.1.
5) Sandhiya S, Balamani N. Micro g* -closed sets in Micro Topological Spaces. *Journal of Research in Applied Mathematics*. 2022;8(4):16–20. Available from: https://www.questjournals.org/jram/papers/v8-i4/B08041115.pdf.
6) Sowmiya T, Balamani N. Micro ψ-Closed Sets in Micro Topological Spaces. *Quest Journals Journal of Research in Applied Mathematics*. 2022;8(4):11–15. Available from: https://www.questjournals.org/jram/papers/v8-i4/B08041115.pdf.