INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

Pythagorean Fuzzy Approach to Game Theory

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OPEN ACCESS

Received: 12-10-2023

Accepted: 22-11-2023

Published: 25-12-2023

Citation: Thomas A, Jose S (2023) Pythagorean Fuzzy Approach to Game Theory. Indian Journal of Science and Technology 16(SP4): 22-28. https://doi.org/10.17485/IJST/v16iSP4.ICAMS131

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Abstract

Objectives: To study and tackle the impreciseness, vagueness and hesitancy involved in one of the decision-making problems say matrix games. This paper has done it using the most apt and innovative tool, Pythagorean Fuzzy sets. The objective of the paper is to define the Mathematical model of Pythagorean Fuzzy Matrix Game, study its properties and develop an innovative algorithm to solve it. These innovative concepts are made lucid through numerical illustrations. **Methods:** The matrix games solved using Fuzzy optimization tools and Intuitionistic fuzzy optimization techniques are analyzed. We came to the conclusion that the hesitancy associated with choosing strategies for players and payoff are not fully considered. Hence the strategy Pythagorean fuzzy optimization tool is used. The data set considered in this study are collected from a faculty of Physical Education Department in a college. Using the innovative algorithm and data set collected the study is completed. **Findings:** The Pythagorean fuzzy Expected Payoff function and a new ranking function for Pythagorean fuzzy number is found out in this paper. Moreover, the notion of Pythagorean Fuzzy Saddle point is also proposed in this paper. Then we have introduced a novel algorithm called Pythagorean Minimax Dominian Algorithm to solve Pythagorean fuzzy game problem and it is illustrated through two numerical examples involving two situations. **Novelty:** This study explored concepts of game theory under Pythagorean fuzzy environment. The outlined mathematical model of Pythagorean fuzzy matrix game and the algorithm to solve it is novel in the research field.

Keywords: Pythagorean fuzzy set; Pythagorean fuzzy Number; Pythagorean fuzzy Matrix Game; Pythagorean fuzzy saddle point; Pythagorean fuzzy Expected payoff function

1 Introduction

Multi – Criteria Decision Making has now occupied a dominant space in the research field. The asymmetric and uncertain information available in many of multi criteria decision-making situations has raised the hope of multi criteria decision-making with fuzzy sets[1]. Game theory is evidently a decision-making problem. It has two or more autonomous decision makers called players, having conflicting interests who act strategically to find a compromising solution [2]. As we rely on game theory to model some practical problems which we encounter in our real-life situations, the players of the game are not able to evaluate exactly the outcomes of game due to uncertain and asymmetric information between players. Fuzzy matrix games were studied by Yang in 2020 [3]. Intuitionistic fuzzy matrix games solved the shortcomings of Fuzzy matrix game theory [4]. To cope up with situations where hesitancy need to be considered, it was Yager who constructed Pythagorean fuzzy set [PFS] [5]. It is an advanced tool over fuzzy sets and intuitionistic fuzzy sets to deal with vagueness and hesitancy [6,7]. The advantage of PFS is that, it relaxes the condition demanded by intuitionistic fuzzy sets by satisfying the conditions, $\mu + \nu \leq 1$ or $\mu + \nu \geq 1$ with $\mu^2 + \nu^2 + \Pi^2 = 1$ [8,9]. Obviously it is a generalization of IFS [10,11]. PFS has attracted researchers in diverse fields especially in operations research and decision-making [12,13].

In this paper we are galvanized to explore the resourcefulness of PFS in solving game problem by applying maxi-min principle of game theory from Pythagorean fuzzy context. This paper advances in the following manner: In section 2, a brief review of the preliminary concepts related to PFS and game theory is discussed. In section 3, the Methodology is discussed. It is followed by the mathematical model of Pythagorean fuzzy matrix game which is formulated along with some basic definitions and a new ranking function for Pythagorean fuzzy number, novel algorithm called Pythagorean Minimax Dominian Algorithm to solve Pythagorean fuzzy game problem and its illustrations through numerical examples are discussed in section 4. Finally, section 5 includes the concluding remarks of this paper followed by references.

2 Preliminaries

2.1 Pythagorean fuzzy set (PFS) [6,9]

Let X be a universal set. Then a Pythagorean fuzzy set is

$\widetilde{A}_p = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \,\varepsilon\, X\}$

μ_A(x): X→[0,1] is the degree of membership of x ε X to $\widetilde{A}_p$, ν_A(x): X→[0,1] is the degree of non-membership of x ε X to $\widetilde{A}_p$ with $\mu_A(x) + \nu_A(x) \leq 1$ or $\mu_A(x) + \nu_A(x) \geq 1$ and $0 \leq \mu_A(x)^2 + \nu_A(x)^2 \leq 1$.

Now Π_A(x): X→[0,1] is the degree of hesitation of x ε X to $\widetilde{A}_p$ is defined as

$$\Pi_A\,(x) = \sqrt{1 - \mu_A\,(x)^2 - \nu_A\,(x)^2}$$

So that $\mu_A(x)^2 + \nu_A(x)^2 + \Pi_A(x)^2 = 1$.

2.2 Pythagorean fuzzy number (PFN) [6,9]

A Pythagorean fuzzy set can be represented as a Pythagorean fuzzy number which is an ordered pair,

$\widetilde{A}_p = (\mu_A,\ \nu_A)$

Then the degree of hesitation is given by $\prod_A = \sqrt{1 - \mu_A^2} - \nu_A^2$

2.3 Arithmetic operations of Pythagorean fuzzy number [6]

Consider two PFN $\widetilde{A}_p = (\mu_1, \nu_1)$ and $\widetilde{B}_p = (\mu_2, \nu_2)$

Different arithmetic operations can be defined on PFN as follows:

(1) Union : $\widetilde{A}_p \cup \widetilde{B}_p = (Max(\mu_1, \mu_2\,), Min(\nu_1, \nu_2,))$

(2) Intersection : $\widetilde{A}_p \cap \widetilde{B}_p = (Min\,(\mu_1, \mu_2)\,, Max\,(\nu_1, \nu_2,))$

(3) Addition : $\widetilde{A}_p \oplus \widetilde{B}_p = \left(\sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \nu_1\nu_2\right)$

(4) Multiplication : $\widetilde{A}_p \otimes \widetilde{B}_p = \left(\mu_1\mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2\nu_2^2}\right)$

(5) Scalar multiplication: $\lambda\widetilde{A}_p = \left(1 - \left(1 - \mu_1^2\right)^\lambda, \nu_1^\lambda\right)$

(6) Complement: $\widetilde{A}_p^c = (\nu_1, \mu_1)$

Payoff [2]

Payoff is the quantitative measure of satisfaction a player gets at the end of a play in a game.

2.5 Two person zero sum game [2]

A game with two players, player I and player II, where the payoff to player II is the negative of the payoff to player I in each play of the game so that the total sum of payoff is zero.

2.6 Matrix game [2]

Matrix game is a two person zero sum game. It is defined by a m X n matrix A = $[a_{ij}]_{mXn}$, where i ε { 1,2,...m}, j ε {1,2,...n}, a_{ij} are real numbers and A is called the payoff matrix. Here the two players, player I (row player) has m choices/ strategies and the player II (column player) has n choices or strategies available to them. If the player I choose to play choice i without knowing the choice of player II and player II choose to play choice j irrespective of the choice of player I, the payoff to player I is a_{ij} and the payoff to player II is - a_{ij}. Both players want to choose strategies that will benefit their individual payoff. The payoff matrix A is given by

$$A = \begin{array}{c c} & \begin{array}{ccc} II_1 & II_2 \ldots & II_n \end{array} \\ \begin{array}{c} I_1 \\ I_2 \end{array} & \begin{bmatrix} a_{11}\, a_{12} \ldots a_{1n} \\ a_{21}\, a_{22} \ldots a_{2n} \end{bmatrix} \\ I_m & a_{m1}\, a_{m2} \ldots a_{mn} \end{array}$$

2.7 Mixed strategy and pure strategy [2]

A mixed strategy of player I is a probability distribution X over the rows of A, i e, an element of the set$X = \{X = (x_1, x_2, \ldots x_m) \varepsilon R^m, x_i \geq 0,\ \sum_{i=1}^m x_i = 1\}$

Equivalently a mixed strategy of player II is a probability distribution Y over the columns of A ,ie, an element of the set $Y = \{Y = (y_1, y_2, \ldots y_n)\ \varepsilon\ R^n,\ y_j \geq\ 0 \sum_{j=1}^n y_j = 1\}$

A strategy α_i of player I is a pure strategy if it does not involve any probability, i e., α_i is an X ε **X** with x_i=1 for some i=1,2,...m. We denote the set of pure strategies of player I by S_I={α_1,α_2,...α_m}

A strategy β_j of player II is a pure strategy if it does not involve any probability. i e, β_j is a Y ε **Y** with y_j=1for some j=1,2,...n. We denote the set of pure strategies of player II by S_{II}={β_1,β_2,...β_n}

2.8 Maximin and minimax principle [2]

The player I, who is the maximizing player lists the worst possible outcomes of all his potential strategies, then he will choose that strategy, the most suitable for him which corresponds to one of these worst outcomes. This is called Maximin principle of player I. In short, it is Max_i Min_j [a_{ij}]

The player II who is the minimizing player lists the best possible outcomes of all his potential strategies, then he will choose that strategy, the most suitable for him which corresponds to one of these best outcomes. This is called Minimax principle of player II. In short, it is Min_j Max_i [a_{ij}]

2.9 Dominance principle [2]

Dominance principle is used to reduce the size of the payoff matrix by deleting those strategies which are dominated by others. The general rules of dominance are:

(1) If all the elements of a row say k^{th} row are $\leq$ the corresponding elements of any other row say r^{th} row, then the k^{th} row is dominated by r^{th} row.

(2) If all the elements of a column say k^{th} column $\geq$ the corresponding elements of any other column, say the r^{th} column, then k^{th} column is dominated by the r^{th} column.

(3) Dominated rows or columns are deleted and the optimal strategy will remain unaffected.

2.10 Saddle point [2]

For a matrix game A=$[a_{ij}]_{mXn}$, if

Max_i Min_j a_{ij}=Min_j Max_i a_{ij}=a_{rs}

Then the matrix game has a saddle point at the (r,s)th position of A.

3 Methodology

The mathematical theory of game is revealed to the world through the famous article Theory of Games and Economic Behavior by John Von Neumann and Oscar Morgenstern. Game theory is evidently a decision-making problem with two or more autonomous decision makers called players having conflicting interests who act strategically to find a compromising solution.

As we rely on game theory to model some practical problems which we encounter in our real life situations, the players of the game are not able to evaluate exactly the outcomes of game due to uncertain and asymmetric information between players. Hence, fuzzy set theory proposed by L.A Zadeh is used to represent the uncertainty of payoffs of games. In game problems, goals and payoffs are subjected to fuzzy game problems. It was Campos who initiated the study of fuzzy game problems. His approach was based on ranking of fuzzy numbers. It was followed by Nishizaki and Sakawa which was based on the Maxmin principle of game theory.

In some practical situations it's not easy to describe the payoff values up to decision maker's satisfaction. Consequently, there remains an indeterministic part of which hesitation survives. Fuzzy set is no means to incorporate the hesitation degree. In 1986, Atanassov introduced the concept of Intuitionistic fuzzy set (IFS), which is characterized by a membership function μ and a non-membership function ν such that $\mu+\nu \leq 1$ and $\mu+\nu+\Pi= 1$, where Π is the hesitation degree of the decision maker. Intuitionistic fuzziness in matrix games appear in many ways out of which one is matrix games with intuitionistic fuzzy payoffs expressed as intuitionistic fuzzy numbers (IFN). It was Atanassov who first studied a game problem with IFS. Later Agarwal made significant contributions in this field.

It is probable that there occur real life situations with $\mu + \nu \geq 1$ revealing the inadequacy of IFS. To cope up with such situations it was Yager who constructed Pythagorean fuzzy set [PFS]. It is an advanced tool to deal with vagueness satisfying the conditions $\mu + \nu \leq 1$ or $\mu + \nu \geq 1$ and $\mu^2+ \nu^2+ \Pi^2 =1$. Obviously it is a generalization of IFS. PFS has attracted researchers in diverse fields especially in operations research and decision-making.

4 Results and Discussion

4.1 Pythagorean Fuzzy Payoff

The payoff of a matrix game represented by Pythagorean fuzzy number (PFN) is called a Pythagorean fuzzy payoff. Due to uncertainty and imprecision in the game, ambiguity in the judgement of players, it is reliable to express payoff of a matrix game by PFN.

4.2 Mathematical Model of Pythagorean Fuzzy Matrix Game

Pythagorean fuzzy matrix game is a two person zero sum game. It is defined by an m X n Pythagorean fuzzy matrix $\widetilde{A} = [\widetilde{a}_{ij}]_{mXn}$ where I ε {1,2,...m} are the m choices or strategies available to player I , j ε {1,2,...n} are the n choices or strategies available to player II, $\widetilde{a}_{ij}$ are PFN called the Pythagorean fuzzy payoff to player I for his ith choice against the jth choice of player II.

The Pythagorean fuzzy payoff matrix is given by,

$$\widetilde{A} = \begin{array}{c c} & \begin{array}{ccc} II_1 & II_2 & \quad II_n \end{array} \\ \begin{array}{c} I_1 \\ I_2 \\ I_m \end{array} & \begin{bmatrix} \widetilde{a}_{11}\ \widetilde{a}_{12} \ldots\ldots \widetilde{a}_{1n} \\ \widetilde{a}_{21}\ \widetilde{a}_{22} \ldots\ldots \widetilde{a}_{2n} \\ \widetilde{a}_{m1}\ \widetilde{a}_{m2} \ldots\ldots \widetilde{a}_{mn} \end{bmatrix} \end{array}$$

The goal of the game is to find a strategy for player I that maximizes his Pythagorean fuzzy gain and find a strategy for player II that minimizes his Pythagorean fuzzy loss.

4.3 Pythagorean Fuzzy Expected Payoff Function

Consider a Pythagorean fuzzy matrix game $\widetilde{A} = [\widetilde{a}_{ij}]_{mXn}$. Let X ε **X** and Y ε **Y** are mixed strategies of players I and II respectively. Then the Pythagorean fuzzy expected payoff for player I is

$$\widetilde{E}(X,Y) = X^T\widetilde{A}Y = \sum_{i=1}^{m}\sum_{j=1}^{n} x_i y_j \widetilde{a_{ij}} \tag{0.1}$$

It is also a PFN. For the computation of (Equation (0.1)) the arithmetic operations of PFN are used.

4.4 Ranking Function

A new ranking function (defuzzification function) shall be proposed for PFN. Let $P_{\sim}(R)$ denote the set of all Pythagorean fuzzy numbers. The ranking function maps P(A~p) each PFN to a real number. It is defined as

$$\begin{aligned} P(\widetilde{A}_p) &: \widetilde{P}(R) \to \mathfrak{R} \\ P(\widetilde{A}_p) &= (\mu^2 + \nu^2)\left(\frac{1+\sqcap}{2}\right) \end{aligned} \tag{0.2}$$

Clearly $P(A\sim_p)\ \varepsilon\ [0,1]$. Consider two PFN $A\sim_P$ and $B\sim_P$. We can define a new order relation between $A\sim_P$ and $B\sim_P$ based on (Equation (0.2)) as:

$$\begin{aligned} &1.\ P(\widetilde{A}_p) \leq P(\widetilde{B}_p) \Rightarrow \widetilde{A}_p \leq \widetilde{B}_p. \\ &2.\ P(\widetilde{A}_p) \geq P(\widetilde{B}_p) \Rightarrow \widetilde{A}_p \geq \widetilde{B}_p. \\ &3.\ P(\widetilde{A}_p) = P(\widetilde{B}_p) \Rightarrow \widetilde{A}_p \cong \widetilde{B}_p. \end{aligned} \tag{0.3}$$

The symbol $\leq$ denotes Pythagorean fuzzy inequality and has the linguistic interpretation as essentially less than or equal to. Similarly, for $\geq$ and $\cong$.

4.5 Pythagorean Fuzzy Saddle Point

Point $(X_0, Y_0$, $X_0\ \varepsilon\ R^m$, $Y_0\ \varepsilon\ R^n$, is a Pythagorean fuzzy saddle point if

$$\widetilde{E}(X, Y_0) \leq \widetilde{E}(X_0, Y_0) \leq \widetilde{E}(X_0, Y),\ X\ \varepsilon\ R^m,\ Y\ \varepsilon\ R^n \tag{0.4}$$

4.6 Pythagorean Minimax Dominian Algorithm

We propose a novel algorithm called Pythagorean Minimax Dominian Algorithm to solve Pythagorean fuzzy matrix games. It is developed on the basis of Minimax principle and Dominance principle of game theory combined with the proposed new ranking function of PFN. The steps of the algorithm are as follows:

Step 1: Consider the Pythagorean fuzzy payoff matrix of the game whose payoff are expressed by PFN.

Step 2: Defuzzify the Pythagorean fuzzy payoffs using the newly proposed ranking function of PFN given in (2)

Step 3: Check for Pythagorean fuzzy saddle point using Minimax and Maximin principle. If exits, go to step 4, otherwise to step 5.

Step 4: Determine the Pythagorean fuzzy saddle point, optimal strategies of players I and II and Pythagorean fuzzy value of game.

Step 5: Apply Dominance principle to the rows and columns of the defuzzified matrix obtained in step 2 and reduce it to a 2 X 2 matrix.

Step 6: Determine the optimal mixed strategies of players I and II.

Step 7: Determine the Pythagorean fuzzy value of the game using the Pythagorean fuzzy expected payoff function.

Numerical Examples:

The best way to demonstrate the proposed Pythagorean Minimax Dominian algorithm to solve Pythagorean Fuzzy matrix game is by virtue of an example. Here we shall take into account two numerical examples, one with Pythagorean fuzzy saddle point and the other without Pythagorean fuzzy saddle point for the better comprehension of the theory.

Example 1. Consider a Pythagorean fuzzy matrix game with 4 strategies for both players I and II. The Problem is to find optimal strategies for both players and the Pythagorean fuzzy value of the game.

Consider the Pythagorean fuzzy payoff matrix:

$$\widetilde{A} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} II_1 & II_2 & II_3 & II_4 \end{array} \\ \begin{bmatrix} (0.7,\ 0.4) & (0.5,\ 0.7) & (0.7,\ 0.2) & (0.6,\ 0.6) \\ (0.5,\ 0.3) & (0.6,\ 0.5) & (0.2,\ 0.9) & (0.7,\ 0.1) \\ (0.2,\ 0.3) & (0.2,\ 0.9) & (0.1,\ 0.5) & (0.6,\ 0.5) \\ (0.6,\ 0.6) & (0.3,\ 0.9) & (0.7,\ 0.2) & (0.2,\ 0.9) \end{bmatrix} \end{array}$$

Apply Step 2,

$$\widetilde{A} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{array} \begin{array}{c} \begin{array}{cccc} II_1 & II_2 & II_3 & II_4 \end{array} \\ \begin{bmatrix} 0.19 & 0.18 & 0.18 & 0.19 \\ 0.13 & 0.19 & 0.16 & 0.17 \\ 0.06 & 0.14 & 0.11 & 0.19 \\ 0.19 & 0.12 & 0.18 & 0.16 \end{bmatrix} \end{array}$$

Apply Step 3,

Max(Row Min)=0.18, Min(Column Max)=0.18.

Hence by Minimax and Maxmin Principle, Saddle point exists at (1,3)th position.

Optimal pure strategy of player I=(1,0,0,0)

Optimal pure strategy of player II=(0,0,1,0) Pythagorean fuzzy value of game=(0.7,0.2).

Example 2. Consider a Pythagorean fuzzy matrix game with 3 strategies for both players I and II. The Problem is to find optimal strategies for both players and the Pythagorean fuzzy value of the game.

Consider the Pythagorean fuzzy payoff matrix:

$$\widetilde{A} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{array}{ccc} II_1 & II_2 & II_3 \end{array} \\ \begin{bmatrix} (0.7,\ 0.4) & (0.7,\ 0.1) & (0.1,\ 0.6) \\ (0.5,\ 0.3) & (0.1,\ 0.5) & (0.2,\ 0.3) \\ (0.2,\ 0.5) & (0.7,\ 0.2) & (0.2,\ 0.9) \end{bmatrix} \end{array}$$

Apply step 2,

$$\begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{array}{ccc} II_1 & II_2 & II_3 \end{array} \\ \begin{bmatrix} 0.19 & 0.17 & 0.14 \\ 0.13 & 0.11 & 0.06 \\ 0.12 & 0.18 & 0.16 \end{bmatrix} \end{array}$$

Apply step 3,

Max (Row Min) = 0.14, Min (column Max) = 0.16.

Max (Row Min) $\neq$ Min (column Max).

Hence, Saddle point does not exist.

Apply step 5 to the rows and columns of the above matrix we get,

$$\begin{array}{c} \\ I_1 \\ I_3 \end{array} \begin{array}{c} \begin{array}{cc} II_1 & II_3 \end{array} \\ \begin{bmatrix} 0.19 & 0.14 \\ 0.12 & 0.16 \end{bmatrix} \end{array}$$

Apply step 6 to the above matrix,

E(X, Y) = $\frac{9}{100}$ (x$_1$ - $\frac{4}{9}$) (y$_1$ - $\frac{2}{9}$·) + $\frac{34}{225}$

So Optimal mixed strategy of player I = ($\frac{4}{9}$, 0, $\frac{5}{9}$).

And Optimal mixed strategy of player II = ($\frac{2}{9}$, 0, $\frac{7}{9}$).

Apply step 7 to get the Pythagorean fuzzy value of the game

$\widetilde{E}$(X, Y) = ($\sum_{i=1}^{3}\sum_{j=1}^{3} Xi$ Y$_j$ $\widetilde{a}_{ij}$) = (0.04, 0.66)

5 Declaration

Presented in 'International Conference on Applied Mathematical Sciences' (ICAMS 2022) during 21 & 22 Dec. 2022, organized virtually by the Department of Mathematics, JJ College of Arts and Science, Pudukkottai, Tamil Nadu, India. The Organizers claim the peer review responsibility.

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