INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS
Received: 12-10-2023
Accepted: 22-11-2023
Published: 25-12-2023

Citation: Leonishiya A, Robinson PJ (2023) A Fully Linguistic Intuitionistic Fuzzy Artificial Neural Network Model for Decision Support Systems. Indian Journal of Science and Technology 16(SP4): 29-36. https://doi.org/10.17485/IJST/v16iSP4.ICAMS136

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

A Fully Linguistic Intuitionistic Fuzzy Artificial Neural Network Model for Decision Support Systems

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Abstract

Objectives: To propose a novel Linguistic Intuitionistic Fuzzy Artificial Neural Network (LIF-ANN) model based on Linguistic Intuitionistic Fuzzy Sets (LIFSs), which finds its applications in Decision Support Systems (DSS). **Methods:** In the process of the Neural Network, the Hebbian learning rule is utilized where the Linguistic Intuitionistic Fuzzy nature of the input data is preserved till the end of the process as a pioneering initiative in the field of ANN. **Findings:** The proposed LIF-ANN concentrates on the Multiple Attribute Group Decision Making problem approach in dealing with uncertain data. Also, a novel method of determining attribute weights using the Gaussian method is proposed where the weights are derived in the form of Linguistic Intuitionistic Fuzzy Numbers (LIFNs). The new LinIFOWA operator is also defined and some basic theorems are proved supporting the same. Numerical illustration is presented for the LIF-ANN method and comparisons are made with existing ranking methods. **Novelty:** Unlike most of the research done on Decision support with linguistic Intuitionistic Fuzzy sets, where the weight vector for ANN and the final decision variables become crisp variables, this paper concentrates on maintaining the Linguistic Intuitionistic Fuzzy format throughout and till the end of the problem with the weight vector and the final solution being preserved till the end of the process as Linguistic Intuitionistic Fuzzy Numbers.

Keywords: Linguistic Intuitionistic Fuzzy Sets (LIFSs); OWA Operator; Decision Support System (DSS); MAGDM; Artificial Neural Network

1 Introduction

The Artificial Neural Network (ANN) has its wide range of applications in various fields and a few to mention are image processing, speech recognition, machine learning and in many medical diagnoses. ANN is also one of the DSS which operates with the idea of ranking the best alternative out of the available ones in any decision system whenever attributes with difference of opinion are involved. In [1] Intuitionistic fuzzy time series with hybrid neural network is discussed. In [2] Intuitionistic Fuzzy neural networks with Interval valued Intuitionistic Fuzzy conditions were proposed and the concept of an intuitionistic fuzzy deep neural network is proposed in [3]. An Intuitionistic Fuzzy neural network with Gaussian membership function and Yager-generating function are proposed in [4]. The Multiple Attribute Group Decision Making (MAGDM) problems also operates in the same way as the ANN and the difference between them is the choice of the processing functions and the ranking functions. Decision making methods like TOPSIS method is where the decision-making problem will concentrate its methodology based on the ranking methods done by measuring the closeness to the positive or negative ideal solution in [5]. A rule-based approach predicated on picture fuzzy sets for clinical decision support system is derived in [6]. In [7], practical applications of fuzzy logic in decision-making are proposed. MAGDM method based on the best-worst method with two – dimension linguistic intuitionistic fuzzy variables are proposed in [8]. In recent days, linguistic intuitionistic fuzzy data has gained the attention of researchers to a large extent [9,10]. In [11], prioritized linguistic intuitionistic fuzzy sets and its properties were developed. Competitive advantage assessment using a novel hybrid method with intuitionistic fuzzy linguistic variables is derived in [12]. Authors in [13] and [14] have developed Nonlinear Best-Worst Method for Intuitionistic 2-tuple linguistic sets. There are various aggregation operators for Linguistic Intuitionistic Fuzzy sets which are proposed in [15]. Linguistic intuitionistic fuzzy weighted averaging aggregation operator of linguistic intuitionistic fuzzy numbers can be mentioned [16]. In [17], decision-making problems with fuzzy preference and uncertain linguistic information are developed. The authors in [18] expanded TOPSIS technique for MAGDM problems and it is obvious that the Fuzzy distance and metric measures are extremely important in Fuzzy Decision Making situations which are proposed in [19] and [20] respectively. In this work, we have proposed a new algorithm for Linguistic Intuitionistic Fuzzy ANN (LIF-ANN) based on attribute weight determination and also ranking of the alternatives in the fully Linguistic Intuitionistic Fuzzy environment. The novelty in this paper regarding ANN is that the data set as inputs are themselves fully Linguistic Intuitionistic Fuzzy Numbers, weight vectors for ANN are also Linguistic Intuitionistic Fuzzy Numbers and this nature is maintained till the end of the activation process in the ANN. As a part of the aggregation process, a new operator named the Linguistic Intuitionistic Fuzzy Ordered Weighted Averaging (LinIFOWA) operator is proposed and some theorems are proved for the operator. Different computations are performed with the proposed new algorithm of ANN and comparisons are made with utilizing crisp variables for the final ranking of the best alternatives in the decision algorithms. The study reveals that our new LIF-ANN preserves the Linguistic Intuitionistic Fuzzy nature of the data set throughout the decision process which is completely novel in the field of Fuzzy ANN.

2 Methodology

2.1 New Attribute Weights Determination Method and Aggregation Operators for ANN

In the literature [15], some operators for Linguistic Intuitionistic Fuzzy sets were derived and in the following, we propose some novel methods for attribute weight determination and aggregation operations.

2.1.1 Gaussian Method of Attribute Weight Determination

A major issue in MAGDM problems is determining the associated weights using the arithmetic aggregation operators. As far as the previous studies are considered in the literature, the weights of the attributes are mostly crisp in nature. Here we propose a novel method based on the Gaussian method which provides the weights as LIFNs. In the following, we propose the Linguistic Intuitionistic Fuzzy weights for ANN:

Let $w_i = \left[l_{\mu_{i+1}},\ (\alpha(\sigma_{i+1}),\ \gamma(\sigma_{i+1}))\right]$, where μ_n is the mean of the collection 1,2,…,n and σ_n is the standard deviation of the collection 1,2,…,n. $\mu_n$and σ_n are obtained by the following formulae respectively.

$\mu_n = \left(\frac{n+1}{2}\right)$, $\sigma_n = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(i-\mu_i)^2}$.

The membership and non-membership degrees are proposed as:

$\alpha(\sigma_n) = \frac{|\sigma_n|}{\sum_{i=2}^{5}|\sigma_i|}$; $\gamma(\sigma_n) = \frac{|1-\sigma_n|}{\sum_{i=2}^{5}|1-\sigma_i|}$; Where $\sum_{i=1}^{n}\alpha(\sigma_n) = 1$ and $\sum_{i=1}^{n}\gamma(\sigma_n) = 1$.

2.1.2 Linguistic Median Membership (LMM) function

Let $\widetilde{\sigma}_1 =< l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ for j=1,2,…n be a collection of Linguistic Intuitionistic fuzzy numbers. Then the Linguistic Median Membership function is defined as $M_m = \frac{(\theta(x)+(\alpha_A(x)+1-\gamma_A(x))}{2}$. It is used to defuzzify the LIFN.

2.1.3 The LinIFOWA operator

Let $\widetilde{\sigma}_1 =< l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ for j=1,2,…,n be a collection of Linguistic Intuitionistic Fuzzy Numbers. The Linguistic Intuitionistic Fuzzy Ordered Weighted Averaging (LinIFOWA) operator $LinIFOWA : Q^n \to Q$is defined as:

$LinIFOWA(\widetilde{\sigma}_1, \widetilde{\sigma}_2, ..., \widetilde{\sigma}_n) = r_{ij}= \left(S_{\sum_{j=1}^{n}\theta_j}, \prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j{}^{w_j}\right)$

where, the weight vector of $\widetilde{\sigma}_j$, $j = 1,2,\ldots,n$ is $w = (w_1, w_2, \ldots., w_n)^T$ and for $w_j > 0$,$\sum_{j=1}^{n} w_j = 1$. That is $\sum_{i=1}^{n} \alpha(\sigma_n) = 1$and $\sum_{i=1}^{n} \gamma(\sigma_n) = 1$.

THEOREM 2.1.1:

When LinIFOWA is used to aggregate a set of Linguistic Intuitionistic Fuzzy numbers

$$LinIFOWA_w(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) = \left(S_{\sum_{j=1}^{n}\theta_j}, \prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right),$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is of the LinIFOWA operator with $w_j \in (0,1]$ and$\sum_{j=1}^{n} w_j = 1$, where $\sum_{i=1}^{n} \alpha(\sigma_n) = 1$and $\sum_{i=1}^{n} \gamma(\sigma_n) = 1$.

Proof:

Let us use the induction approach to demonstrate this. Take into account $\widetilde{\sigma_{\sigma(1)}}w_1$ and $\widetilde{\sigma_{\sigma(2)}}w_2$.

$$\widetilde{\sigma_{\sigma(1)}}w_1 = \left\{S_{\theta_{\sigma(1)}w_1}, ([1-\alpha_{\sigma(1)}]^{w_1}, [\gamma_{\sigma(1)}]^{w_1})\right\}, \widetilde{\sigma_{\sigma(2)}}w_2 = \left\{S_{\theta_{\sigma(2)}w_2}, ([1-\alpha_{\sigma(2)}]^{w_2}, [\gamma_{\sigma(2)}]^{w_2})\right\}.$$

Then,

$$LinIFOWA_w\left(\widetilde{\sigma}_{\sigma(1)}, \widetilde{\sigma}_{\sigma(2)}\right) = \widetilde{\sigma}_{\sigma(1)}w_1 \oplus \widetilde{\sigma}_{\sigma(2)}w_2.$$

$$= \left\{S_{\theta_{\sigma(1)}w_1} + \theta_{\sigma(2)}w_2, [[1-\alpha_{\sigma(1)}]^{w_1}[1-\alpha_{\sigma(2)}]^{w_2}, [\gamma_{\sigma(1)}]^{w_1}[\gamma_{\sigma(2)}]^{w_2}]\right\},$$

Continuing the process with $\widetilde{\sigma}_{\sigma(1)}, \widetilde{\sigma}_{\sigma(2)}, \ldots, \widetilde{\sigma}_{\sigma(k)}$ we have:

$LinIFOWA_w\left(\widetilde{\sigma}_{\sigma(1)}, \widetilde{\sigma}_{\sigma(2)}, \ldots, \widetilde{\sigma}_{\sigma(k)}\right) = \left(S_{\sum_{j=1}^{k}\theta_j}, \prod_{j=1}^{k}(1-\alpha_j)^{w_j}, \prod_{j=1}^{k}\gamma_j^{w_j}\right)$.

Then when n=k+1 we have:

$LinIFOWA_w\left(\widetilde{\sigma}_{\sigma(1)}, \widetilde{\sigma}_{\sigma(2)}, \ldots, \widetilde{\sigma}_{\sigma(k)}, \widetilde{\sigma}_{\sigma(k+1)}\right) = \left(S_{\sum_{j=1}^{k+1}\theta_j}, \prod_{j=1}^{k+1}(1-\alpha_j)^{w_j}, \prod_{j=1}^{k+1}\gamma_j^{w_j}\right)$.

Hence, we see that for n=k+1, the operator is valid. The operator is thus true for every n according to the induction principle, concluding the proof. Hence,

$$LinIFOWA_w\left(\widetilde{\sigma}_{\sigma(1)}, \widetilde{\sigma}_{\sigma(2)}, \ldots, \widetilde{\sigma}_{\sigma(k)}, \widetilde{\sigma}_{\sigma(k+1)}, \ldots, \widetilde{\sigma}_{\sigma(n)}\right) = \left(S_{\sum_{j=1}^{n}\theta_j}, \prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right),$$

THEOREM 2.1.2:

Let $\widetilde{\sigma}_j, (j = 1,2,\ldots..,n)$, be a collection of Linguistic Intuitionistic Fuzzy Numbers and $\omega = (\omega_1, \omega_2, \ldots.., \omega_n)^T$ be the weight vector of $\widetilde{\sigma}_j$, with $\omega_j \in (0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$, where $\sum_{i=1}^{n}\alpha(\sigma_n) = 1$ and $\sum_{i=1}^{n}\gamma(\sigma_n) = 1$; Finally, we demonstrate that the LinIFOWA operator is (i) Idempotent, (ii) Bounded,(iii) Monotonic and (iv) Commutative.

Proof:

(i) Idempotent: If all $\widetilde{\sigma}_j, (j = 1,2,\ldots..,n)$, are equal, i e. $\widetilde{\sigma}_j = \widetilde{\sigma}$ for all j, we must then demonstrate $LinIFOWA_\omega(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) = \widetilde{\sigma}$. Since $\widetilde{\sigma}_j = \widetilde{\sigma}$, for all j, then we should have,

$LinIFOWA_\omega(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) = \sum_{j=1}^{n} a\,\widetilde{\omega}_j = \left\langle \sum_{j=1}^{n}\theta_j\omega_j, \left[\prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right]\right\rangle$

$= \left\langle S_{\theta_j}, [\alpha_{\widetilde{\sigma}_j}], \gamma_{\widetilde{\sigma}_j}]\right\rangle = \widetilde{\sigma}$.

(ii) Boundedness: Let, $\sigma - \leq LinIFOWA_\omega\ (\sigma\ 1, \sigma\ 2, \ldots, \sigma\ n) \leq \sigma +$.

The following illustrates this: $\widetilde{\sigma}^- = \left\langle \min_i S_{\theta_i}; [\min_i \alpha_{\widetilde{\sigma}_i}], [\max_i \gamma_{\widetilde{\sigma}_i}]\right\rangle$.

(iii) Monotonicity: Let $\widetilde{\sigma}_j^*, (j = 1,2,\ldots..,n)$, be a collection of LIFNs. If $\widetilde{\sigma}_j \leq \widetilde{\sigma}_j^*$ for all j, then we, show that, $LinIFOWA_\omega(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) \leq LinIFOWA_\omega(\widetilde{\sigma}_1^*, \widetilde{\sigma}_2^*, \ldots., \widetilde{\sigma}_n^*)$ for all ω.

Let, $LinIFOWA_\omega(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) = \sum_{j=1}^{n}\widetilde{\sigma}_i\omega_i \& LinIFOWA_\omega(\widetilde{\sigma}_1^*, \widetilde{\sigma}_2^*, \ldots., \widetilde{\sigma}_n^*) = \sum_{j=1}^{n}\widetilde{\sigma}_1^*\omega_i$.

Since $\widetilde{\sigma}_j \leq \widetilde{\sigma}_j^*$ for all j, then we have

$LinIFOWA_\omega(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) \leq LinIFOWA_\omega(\widetilde{\sigma}_1^*, \widetilde{\sigma}_2^*, \ldots., \widetilde{\sigma}_n^*)$.

When you consider the fact that,$\widetilde{\sigma}_j = \left\langle S_{\theta_j}, [\alpha_{\widetilde{\sigma}_j}], [\gamma_{\widetilde{\sigma}_j}]\right\rangle$ *and* $\widetilde{\sigma}_j* = \left\langle S_{\theta_j*}, [\alpha_{\widetilde{\sigma}_j}*], [\gamma_{\widetilde{\sigma}_j}*]\right\rangle$, we should have, $S_{\theta_j} \leq S_{\theta_j^*}$ *and* $\alpha_{\widetilde{\sigma}_j} \leq \alpha_{\widetilde{\sigma}_j^*}; \gamma_{\widetilde{\sigma}_j} \leq \gamma_{\widetilde{\sigma}_j^*}, \forall j$.

(iv) Commutative: Let $\widetilde{\sigma}_j, (j = 1, 2, \ldots\ldots., n)$, be a collection of LIFNs. Then we have to prove, $LinIFOWA_\omega\,(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots\ldots, \widetilde{\sigma}_n) = LinIFOWA_\omega\,(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots\ldots.\overline{\widetilde{\sigma}}_n),\ j = 1, 2, \ldots, n$

and for all ω where $\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots\ldots.\overline{\widetilde{\sigma}}_n$is any permutation of $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots\ldots, \widetilde{\sigma}_n$. Let

$$LinIFOWA_\omega\,(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots\ldots, \widetilde{\sigma}_n) = \sum_{j=1}^n \widetilde{a}\omega_j = \left\langle \sum_{j=1}^n \theta_j \omega_j,\ [\prod_{j=1}^n (1-\alpha_j)^{w_j}, \prod_{j=1}^n (\gamma_j^{w_j}] \right\rangle$$
$$= \left\langle S_{\theta_j}, [\alpha_{\widetilde{\sigma_j}]}, [\gamma_{\widetilde{\sigma_j}]} \right\rangle,$$

Now,

$$LinIFOWA_\omega\,((\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots\ldots, \overline{\widetilde{\sigma}}_n)) = \sum_{j=1}^n \overline{\widetilde{a}}\omega_j = \left\langle \sum_{j=1}^n \overline{\theta}_j \omega_j,\ [\prod_{j=1}^n (1-\overline{\alpha}_j)^{w_j}, \prod_{j=1}^n (\overline{\gamma}_j^{w_j}] \right\rangle$$
$$= \left\langle S_{\overline{\theta}_j}, [\overline{\alpha}_{\widetilde{\sigma_j}]}, [\overline{\gamma}_{\widetilde{\sigma_j}]} \right\rangle.$$

Since $\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)$ is any permutation of $(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n)$ we can have $\widetilde{\sigma}$

$_{\sigma(j)} = \widetilde{\sigma}_{\sigma(j)}$, for $j = 1, 2, \ldots, n$ and hence $LinIFOWA_\omega\,(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) = LinIFOWA_\omega\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)$.

THEOREM 2.1.3:

Let $\widetilde{\sigma}_j, (j = 1, 2, \ldots.., n)$, be a collection of LIFNs and $\omega = (\omega_1, \omega_2, \ldots.., \omega_n)^T$ be the weight vector of LinIFOWA operator, with $\omega_j \in (0, 1], \sum_{j=1}^n \omega_j = 1$and, where $\sum_{i=1}^n \alpha(\sigma_n) = 1and\sum_{i=1}^n \gamma(\sigma_n) = 1$. Then we have:

1. If $\omega = (1, 0, 0, \ldots., 0)^T$, *then* $LinIFOWA_\omega\,(\sigma\ 1, \sigma\ 2, \ldots, \sigma\ n) = \max_j\,(\sigma\ j)$.

2. If $\omega = (0, 0, 0, \ldots., 1)^T$, *then* $LinIFOWA_\omega\,(\sigma\ 1, \sigma\ 2, \ldots, \sigma\ n) = \min_j\,(\sigma\ j)$.

3. If $\omega_j = 1, \omega_i = 0$ and $i \neq j$, then $LinIFOWA_\omega(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) = \left(\widetilde{\sigma}_{\sigma(j)}\right)$, where j$^{\text{th}}$ greatest value of $\widetilde{\sigma}_{\sigma(j)}$ is $\widetilde{\sigma}_i, (i = 1, 2, \ldots, n)$.

2.2 The Hebbian Learning Rule based on Linguistic Intuitionistic Fuzzy Set for ANN

The Hebbian learning rule describes how to change the weights of the relevant network nodes. The weight linking two neighboring neurons should grow if they are both activated and terminated simultaneously. The weight between neurons should decrease for those that are active in the opposing phase. The weight should not alter if there is no signal correlation. A strong positive weight exists between the nodes when both of their inputs are either positive or negative. A significant negative weight exists between two nodes if one input is positive while another is negative. All weight values are initially set to zero. Both soft and hard activation functions are compatible with this learning rule. This is the unsupervised learning rule since desirable neuronal responses are not utilised in the learning process. The absolute values of weights are typically inversely correlated with learning time, which is undesirable. If w_i are the fully linguistic intuitionistic fuzzy weights and X_i are the input linguistic intuitionistic fuzzy decision matrices, then the learning signal is given by the function $r = f(w_i X_i)$. The increment of the weight vector is given as $\triangle w_{ij} = cf(w_i X_i)X_j = co_i X_j$, where $o_i$is the actual output and $o_i = f(w_i X_i)$.

2.2.1 The Pseudo code for Linguistic Intuitionistic Fuzzy ANN based on Hebbian Learning Rule

Pseudo-code for LIF-ANN:

C_n : n Matrix itemset of size k x m

Input {Linguistic Intuitionistic Fuzzy Decision Matrices}

A_n = {Collection of n Matrices of size k };

//* Aggregation Phase*//

Compute {Lin-IFOWA & the Initial Weight Vector}

For (n=1; $A_n\varnothing$; n++) **do begin**

Generate {Individual Preference Linguistic Intuitionistic Fuzzy Decision Matrices, X_n}

//* X_N is the collection of Individual Preference Linguistic Intuitionistic Fuzzy Decision Matrices *//

Generate {Fully Linguistic Intuitionistic Fuzzy Attribute Weight Vector}

While $i \leq m$ **do**

$w_i = \left\langle l_{\mu_{i+1}}, (\alpha(\sigma_{i+1}), \gamma(\sigma_{i+1}))\right\rangle$

Generate {Collective Overall Preference Linguistic Intuitionistic Fuzzy Decision Matrices using Fully Linguistic Intuitionistic Fuzzy Weight Vector, W^T}

//* Learning Phase*//
Generate {Weight Matrix by Fully Linguistic Intuitionistic Fuzzy Hebbian Rule}
$\Delta w_i = cf(w_i^t x)x$
//*Activation function*//
Fix {The Fully Linguistic Intuitionistic Fuzzy Threshold Value}
While Activated Linguistic Intuitionistic Fuzzy values $\geq$ *Threshold* **do**
Generate {Binary Matrix for final Decision with Fully Linguistic Intuitionistic Fuzzy values exceeding the Threshold}
Output{Best Alternative(s) to be chosen}{the final decision variable is can be converted into crisp variable and computations can be performed(method 2)}
End

3 Results and Discussion

3.1 Numerical Illustration: The Linguistic Intuitionistic Fuzzy ANN based on Hebbian Learning Rule

Assume there are four industries (alternatives) $\{L_1, L_2, L_3, L_4\}$ to be weighed against certain criteria. Evaluate industries in terms of their technological innovation capability, evaluating 'factors' such as resource ability for Digitalization (C_1), Organizational innovation (C_2), Innovation Centers (C_3), and Innovative products (C_4). The Experts assessment of the four industries are listed in Tables 1, 2 and 3.

Table 1. Decision Matrix I

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_5, (0.2, 0.7)\rangle$	$\langle l_2, (0.4, 0.6)\rangle$	$\langle l_5, (0.5, 0.5)\rangle$	$\langle l_3, (0.2, 0.6)\rangle$
L_2	$\langle l_4, (0.4, 0.6)\rangle$	$\langle l_5, (0.4, 0.5)\rangle$	$\langle l_3, (0.1, 0.8)\rangle$	$\langle l_4, (0.5, 0.5)\rangle$
L_3	$\langle l_3, (0.2, 0.7)\rangle$	$\langle l_4, (0.2, 0.7)\rangle$	$\langle l_4, (0.3, 0.7)\rangle$	$\langle l_5, (0.2, 0.7)\rangle$
L_4	$\langle l_6, (0.5, 0.4)\rangle$	$\langle l_2, (0.2, 0.8)\rangle$	$\langle l_3, (0.2, 0.6)\rangle$	$\langle l_3, (0.3, 0.6)\rangle$

Table 2. Decision Matrix II

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_4, (0.1, 0.7)\rangle$	$\langle l_3, (0.2, 0.7)\rangle$	$\langle l_3, (0.2, 0.8)\rangle$	$\langle l_6, (0.4, 0.5)\rangle$
L_2	$\langle l_5, (0.4, 0.5)\rangle$	$\langle l_3, (0.3, 0.6)\rangle$	$\langle l_4, (0.2, 0.6)\rangle$	$\langle l_3, (0.2, 0.7)\rangle$
L_3	$\langle l_3, (0.2, 0.7)\rangle$	$\langle l_4, (0.2, 0.7)\rangle$	$\langle l_2, (0.4, 0.6)\rangle$	$\langle l_3, (0.3, 0.7)\rangle$
L_4	$\langle l_5, (0.3, 0.6)\rangle$	$\langle l_4, (0.4, 0.5)\rangle$	$\langle l_2, (0.3, 0.6)\rangle$	$\langle l_4, (0.2, 0.6)\rangle$

Table 3. Decision Matrix III

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_5, (0.2, 0.6)\rangle$	$\langle l_3, (0.3, 0.7)\rangle$	$\langle l_4, (0.4, 0.5)\rangle$	$\langle l_4, (0.2, 0.7)\rangle$
L_2	$\langle l_4, (0.3, 0.7)\rangle$	$\langle l_5, (0.3, 0.6)\rangle$	$\langle l_2, (0.1, 0.8)\rangle$	$\langle l_3, (0.4, 0.6)\rangle$
L_3	$\langle l_4, (0.2, 0.7)\rangle$	$\langle l_5, (0.3, 0.6)\rangle$	$\langle l_1, (0.1, 0.8)\rangle$	$\langle l_4, (0.2, 0.7)\rangle$
L_4	$\langle l_3, (0.2, 0.7)\rangle$	$\langle l_3, (0.1, 0.7)\rangle$	$\langle l_4, (0.3, 0.6)\rangle$	$\langle l_4, (0.4, 0.5)\rangle$

Let us consider the initial weights as given below:

$$\omega = (0.245925, 0.308033, 0.279993, 0.166049)^T.$$

In order to apply the Gaussian method of weights determination, we initiate the following computations:

$$\mu_1 = 1, \mu_2 = 1.5, \mu_3 = 2, \mu_4 = 2.5, \mu_5 = 3.$$

$$\sigma_1 = 0, \sigma_2 = 0.3535, \sigma_3 = 0.8164, \sigma_4 = 0.935, \sigma_5 = 1.2247.$$

The membership and non-membership grades are as follows:

$$\alpha(\sigma_2) = 0.1061; \alpha(\sigma_3) = 0.2452; \alpha(\sigma_4) = 0.2808; \alpha(\sigma_5) = 0.3678;$$

$$\gamma(\sigma_2) = 0.5773; \gamma(\sigma_3) = 0.1640; \gamma(\sigma_4) = 0.0580; \gamma(\sigma_5) = 0.2007;$$

The weights computed from the proposed Gaussian method are as follows:

$$w_1 = \langle l_{1.5}, (\alpha(\sigma_2), \gamma(\sigma_2)) \rangle = \langle l_{1.5}, (0.1061, 0.5773) \rangle; w_2 = \langle l_2, (0.2452, 0.1640) \rangle;$$

$$w_3 = \langle l_{2.5}, (0.2808, 0.0580) \rangle;\ w_4 = \langle l_3, (0.3678, 0.2007) \rangle.$$

Hence, the weight vector is:

$$W = \begin{pmatrix} \langle l_{1.5}, (0.1061, 0.5773) \rangle, & \langle l_2, (0.2452, 0.1640) \rangle \\ \langle l_{2.5}, (0.2808, 0.0580) \rangle, & \langle l_3, (0.3678, 0.2007) \rangle \end{pmatrix}.$$

The aggregated matrices after applying LinIFOWA operator are as follows:

$$X_1 = \begin{pmatrix} < l_{150}, (0.35812, 0.59217) > \\ < l_{240}, (0.34791, 0.59648) > \\ < l_{240}, (0.22936, 0.7) > \\ < l_{108}, (0.30295, 0.59338) > \end{pmatrix}; X_2 = \begin{pmatrix} < l_{216}, (0.2149, 0.68718) > \\ < l_{180}, (0.28468, 0.58857) > \\ < l_{96}, (0.2781, 0.64549) > \\ < l_{160}, (0.3175, 0.56723) > \end{pmatrix};$$

$$X_3 = \begin{pmatrix} < l_{240}, (0.27084, 0.61337) > \\ < l_{120}, (0.26795, 0.67546) > \\ < l_{80}, (0.2065, 0.69297) > \\ < l_{180}, (0.23816, 0.634) > \end{pmatrix}.$$

Now, applying the Hebbian learning rule with initially assuming

$$W^T = w^1 = \begin{pmatrix} < l_{1.5}\ (0.1062, 0.5773) > \\ < l_2\ (0.2452, 0.1640) > \\ < l_{2.5}\ (0.2808, 0.0580) > \\ < l_3\ (0.3678, 0.2007) > \end{pmatrix},$$

and using the above algorithm for the Learning Phase, we find:

$$w^2 = \begin{pmatrix} < l_{151.5}\ (0.42629, 0.34186) > \\ < l_{242}\ (0.5078, 0.09782) > \\ < l_{242.5}\ (0.44575, 0.0406) > \\ < l_{111}\ (0.55933, 0.11909) > \end{pmatrix}; w^3 = \begin{pmatrix} < l_{367.5}\ (0.54958, 0.23492) > \\ < l_{422}\ (0.64792, 0.05757) > \\ < l_{338.5}\ (0.59989, 0.02621) > \\ < l_{271}\ (0.69924, 0.06755) > \end{pmatrix};$$

$$w^4 = \begin{pmatrix} < l_{607.5}\ (0.67157, 0.14409) > \\ < l_{542}\ (0.74226, 0.03889) > \\ < l_{418.5}\ (0.68251, 0.01816) > \\ < l_{451}\ (0.77087, 0.04283) > \end{pmatrix}.$$

Method 1:

Now, to determine the threshold value, the average $\langle l_{504.75}\ (0.71987, 0.04569) \rangle$ of $w^4$is taken and the binary step function for Activation Phase is given as:

$$f(x) = \begin{cases} 1\ for\ x \geq \langle l\_{504.75}\ (0.71987, 0.04569) \rangle, \\ 0\ for\ \ x < \langle l_{504.75}\ (0.71987, 0.04569) \rangle. \end{cases}$$

The Binary decision matrix after applying the threshold is $(0\ 1\ 0\ 0)^T$.

Hence, L_2 is the best alternative.

Method 2:

Using Linguistic Median Membership function crisp values of w^4.

$$w^4 = \begin{pmatrix} 304.51374 \\ 271.85169 \\ 210.08218 \\ 226.36402 \end{pmatrix}$$

By ranking these crisp values, we get $L_1 \succ L_2 \succ L_4 \succ L_3$.

From Table 4 it is clear that $L_1$and L_2 are best options from method 1 and 2. And the proposed LIF-ANN gives the closer prediction of the same decision alternative, even though the linguistic intuitionistic fuzzy nature is preserved in method 1 without defuzzifying the LIFNs, and crisp in nature in method 2.

Table 4. Ranking of Best Alternative obtained from the Fully Fuzzy and Crisp Methods

Sl. No	Proposed LIF-ANN	Ranking of Alternatives
1	Method 1	L_2−best Alternative
2	Method 2	$L_1 \succ L_2 \succ L_4 \succ L_3$

4 Conclusion

Many bottle-neck problems in the real world are linguistic in nature. The linguistic intuitionistic fuzzy sets has gained the attention of researchers in many of the research areas including ANN and Machine learning. In this paper Some basic theorems are introduced and proved for the proposed aggregation operator and the operators are used in the LIF-ANN to aggregate the LIF matrices. A new algorithm based on the attribute weight determination for ANN as a fully linguistic intuitionistic fuzzy sets is proposed and numerical illustration is provided with LIFN decision data. The linguistic intuitionistic fuzzy format is maintained throughout the problem and the final ranking results are also derived as a linguistic intuitionistic fuzzy number. Comparison of the proposed method is done with changing the final decision variable into a crisp data and the results reveal their consistency. The novelty of this study is proposing linguistic fuzzy decision-making problems in ANN method and on retaining the linguistic intuitionistic fuzzy format throughout and until the end of the problem, with the weight vector and the ultimate solution being retained until the process's conclusion as linguistic intuitionistic fuzzy numbers. Future proposals from us will include more decision-making problems with new operators in various ANN methods.

5 Declaration

Presented in 'International Conference on Applied Mathematical Sciences' (ICAMS 2022) during 21 & 22 Dec. 2022, organized virtually by the Department of Mathematics, JJ College of Arts and Science, Pudukkottai, Tamil Nadu, India. The Organizers claim the peer review responsibility.

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