

## RESEARCH ARTICLE



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# Method for Solving Intuitionistic Fuzzy Transportation Problems - 1 using Singularly Perturbed Delay Differential Equations

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## Abstract

**Objectives:** To propose a new method to find the solution of Intuitionistic fuzzy transportation Problem I by applying Singularly Perturbed Delay Differential Equations. **Methods:** Intuitionistic fuzzy transportation Problem-I is converted to an Intuitionistic fuzzy linear programming problem and then to multi objective linear programming problem. The trader's weights are provided in the form of Singularly Perturbed Delay Differential equations, and by applying the trader's weights, a multi-objective linear programming problem is converted to crisp linear programming, which is then solved by using LINGO. **Findings:** The Proposed method to solve Intuitionistic Fuzzy Transportation Problems-1 by applying Singularly Perturbed Delay Differential Equations is illustrated by considering a numerical example discussed in the existing article, and the Optimal solution to Intuitionistic fuzzy transportation Problem-I is found and compared with the solution of the existing article. **Novelty:** Introducing Singularly Perturbed Delay Differential Equations in Solving the Intuitionistic Fuzzy Transportation Problem-I.

**Keywords:** Intuitionistic Fuzzy number; Intuitionistic Fuzzy Transportation problem; Singular perturbed Delay differential equation; Shish kin mesh; Finite difference scheme 1

## 1 Introduction

Transportation problem having uncertainty and hesitation in supply and demand is considered here. The objective of the fuzzy transportation problem, a transportation problem whose decision parameters are fuzzy numbers is to determine the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. In <sup>(1)</sup>, it is discussed a method for solving fuzzy transportation problem with LR flat fuzzy numbers. In the case of transportation problems, the decision maker can decide about the level of acceptance and non-acceptance of the transportation cost. An Intuitionistic Fuzzy Set (IFS) is characterized by a membership degree as well as a non-membership degree. Methods for Optimal Solution of

Intuitionistic Fuzzy Transportation Problems were introduced in<sup>(2)</sup> and<sup>(3)</sup>. Singularly Perturbed Problems or Singular Perturbation Problems (SPPs) always play a prominent role in the theory of differential equations and in their applications to the physical world. In<sup>(4)</sup>, described a technique for quantitatively resolving singular perturbation issues in delay differential equations. In<sup>(5)</sup> and<sup>(6)</sup>, introduced an initial value problem for a system of singularly perturbed delay differential equations. In<sup>(7)</sup>, discussed a method to solve LR Flat Fuzzy Transportation Problems using Singularly Perturbed Differential Equations and a method for Solving Type-2 Intuitionistic Fuzzy Transportation Problems using Singularly Perturbed Delay Differential Equations is discussed in<sup>(8)</sup>. In this study, a transportation problem with an intuitionistic fuzzy set is investigated and a new technique is proposed to solve Intuitionistic Fuzzy Transportation Problem -1 (IFTP -1) using the Singularly Perturbed Delay Differential Equations (SPDDEs). To overcome shortcomings such as obtaining negative solutions for the variables and obtaining negative objective function value using the existence methods, this novel approach is suggested. A Numerical example is provided to illustrate the suggested method and the solutions are compared with the existing methods and showed that the proposed method gives a better result than the existing methods.

## 2 Methodology

Here, Intuitionistic fuzzy set theory, Intuitionistic Fuzzy Transportation Problem-1 (IFTP-1),

Singularly Perturbed Delay Differential Equations (SPDDES) are given. Also, Weight Determination for Solving IFTP-1 using SPDDES is introduced and, a method to Solve Intuitionistic Fuzzy Transportation Problem-1 using SPDDES is proposed.

### 2.1 Intuitionistic Fuzzy Set Theory

**Definition 2.1.1:** An IFS  $\tilde{F}^I$  in  $U$  is defined by  $\tilde{F}^I = \{(\tilde{U}, \mu_{\tilde{F}^I}(\tilde{U}), \nu_{\tilde{F}^I}(\tilde{U})) / \tilde{U} \in U\}$  where  $\mu_{\tilde{F}^I}, \nu_{\tilde{F}^I} : U \rightarrow (0, 1]$  are functions such that  $0 \leq \mu_{\tilde{F}^I}(\tilde{U}) + \nu_{\tilde{F}^I}(\tilde{U}) \leq 1 \forall \tilde{U} \in U$ .  $\mu_{\tilde{F}^I}(\tilde{U})$  represents the membership function and  $\nu_{\tilde{F}^I}(\tilde{U})$  represents the non-membership function of the element  $\tilde{U} \in U$  being in  $\tilde{F}^I$ . The hesitation degree for the element  $\tilde{U} \in U$  being in  $\tilde{F}^I$  is given by  $h(\tilde{U}) = 1 - \mu_{\tilde{F}^I}(\tilde{U}) - \nu_{\tilde{F}^I}(\tilde{U}) \leq 1 \forall \tilde{U} \in U$ .

**Definition 2.1.2:** An IF subset  $\tilde{F}^I = \{(\tilde{U}, \mu_{\tilde{F}^I}(\tilde{U}), \nu_{\tilde{F}^I}(\tilde{U})) / \tilde{U} \in U\}$ , of the real line  $\mathbf{R}$  is called an IFN if

(i)  $\nexists \tilde{U} \in \mathbf{R}$  such that  $\mu_{\tilde{F}^I}(\tilde{U}) = 1$  and  $\nu_{\tilde{F}^I}(\tilde{U}) = 0$

(ii) " $\mu_{\tilde{F}^I}$  and  $\nu_{\tilde{F}^I}$  are piece wise continuous mappings from  $\mathbf{R}$  to the closed interval  $[0, 1]$ ",  $0 \leq \mu_{\tilde{F}^I}(\tilde{U}) + \nu_{\tilde{F}^I}(\tilde{U}) \leq 1 \forall \tilde{U} \in U$  holds.

**Definition 2.1.3:** A Triangular Intuitionistic Fuzzy Number (TIFN)  $\tilde{F}^I$  denoted by  $\tilde{F}^I = (\rho_1, \rho_2, \rho_3; \rho'_1, \rho'_2, \rho'_3)$  is defined as an IFS in  $\mathbf{R}$  where

$$\mu_{\tilde{F}^I}(\tilde{U}) = \begin{cases} \frac{\tilde{U} - \rho_1}{\rho_2 - \rho_1}, & \text{if } \rho_1 < \tilde{U} \leq \rho_2 \\ \frac{\rho_3 - \tilde{U}}{\rho_3 - \rho_2}, & \text{if } \rho_2 \leq \tilde{U} < \rho_3 \\ 0, & \text{otherwise} \end{cases}$$

is the membership function

$$\nu_{\tilde{F}^I}(\tilde{U}) = \begin{cases} \frac{\rho_2 - \tilde{U}}{\rho_2 - \rho'_1}, & \text{if } \rho'_1 < \tilde{U} \leq \rho_2 \\ \frac{\tilde{U} - \rho'_3}{\rho'_3 - \rho'_2}, & \text{if } \rho_2 \leq \tilde{U} < \rho'_3 \\ 1, & \text{otherwise} \end{cases}$$

is the non-membership function, where  $\rho'_1 \leq \rho_1 < \rho_2 < \rho_3 \leq \rho'_3$ .

**Definition 2.1.4:** Let  $\tilde{F}_1^I = (\tau_1, \tau_2, \tau_3; \tau'_1, \tau'_2, \tau'_3)$  and  $\tilde{F}_2^I = (v_1, v_2, v_3; v'_1, v'_2, v'_3)$  be two TIFNs. Then the operations are defined as follows

Addition:  $\tilde{F}_1^I \oplus \tilde{F}_2^I = (\tau_1 + v_1, \tau_2 + v_2, \tau_3 + v_3; \tau'_1 + v'_1, \tau'_2 + v'_2, \tau'_3 + v'_3)$

Subtraction:  $\tilde{F}_1^I \ominus \tilde{F}_2^I = (\tau_1 - v_3, \tau_2 - v_2, \tau_3 - v_1; \tau'_1 - v'_3, \tau'_2 - v'_2, \tau'_3 - v'_1)$

Multiplication:

$\tilde{F}_1^I \otimes \tilde{F}_2^I = (f_1, f_2, f_3; f'_1, f'_2, f'_3)$ , where  $f_1 = \min\{\tau_1 v_1, \tau_1 v_3, \tau_3 v_1, \tau_3 v_3\}$

$f_2 = \tau_2 v_2; f_3 = \max\{\tau_1 v_1, \tau_1 v_3, \tau_3 v_1, \tau_3 v_3\}$

$f'_1 = \min\{\tau'_1 v'_1, \tau'_1 v'_3, \tau'_3 v'_1, \tau'_3 v'_3\}$

$f'_3 = \max\{\tau'_1 v'_1, \tau'_1 v'_3, \tau'_3 v'_1, \tau'_3 v'_3\}$

Scalar Multiplication:

$$k\tilde{F}_1^I = \left( k\tau_1, k\tau_2, k\tau_3; k\tau'_1, k\tau'_2, k\tau'_3 \right) : k > 0$$

$$k\tilde{F}_1^I = \left( k\tau_3, k\tau_2, k\tau_1; k\tau'_3, k\tau'_2, k\tau'_1 \right) : k < 0$$

**Definition 2.1.5:** Let  $\tilde{T}^I = \left( v_1, v_2, v_3; v'_1, v'_2, v'_3 \right)$  be a TIFN. The score function for  $\mu_{\tilde{T}^I}$  is defined by  $sf(\mu_{\tilde{T}^I}) = \frac{v_1 + 2v_2 + v_3}{4}$ . The score function for  $v_{\tilde{T}^I}$  is defined by  $sf(v_{\tilde{T}^I}) = \frac{v'_1 + 2v'_2 + v'_3}{4}$ . The accuracy ranking function of  $\tilde{T}^I$  is defined by  $\hat{R}(\tilde{T}^I) = \frac{sf(\mu_{\tilde{T}^I}) + sf(v_{\tilde{T}^I})}{2} = \frac{(v_1 + 2v_2 + v_3) + (v'_1 + 2v'_2 + v'_3)}{8}$

## 2.2 Intuitionistic Fuzzy Transportation Problem-1 (IFTP-1)

Here, the mathematical model of an Intuitionistic fuzzy transportation problem-1 with m origins and n destinations is given.

Let  $c_{ij}$  be the “cost of transporting one unit of the product from the  $i^{\text{th}}$  origin to the  $j^{\text{th}}$  destination”,  $\tilde{a}_i^I$  be the “IF quantity available at the  $i^{\text{th}}$  origin”,  $\tilde{b}_j^I$  be the “IF quantity needed at the  $j^{\text{th}}$  destination”,  $\tilde{X}_{ij}^I$  be the “IF quantity transported from the  $i^{\text{th}}$  origin to the  $j^{\text{th}}$  destination”. Then the balanced IFTP – 1 is given by

$$\begin{aligned} \text{Min } \tilde{Z}^I &= \sum_{i=1}^m \sum_{j=1}^n c_{ij} \times \tilde{X}_{ij}^I \\ \text{Subject to } \sum_{j=1}^n \tilde{X}_{ij}^I &= \tilde{a}_i^I, \quad i = 1 \text{ to } m \\ \sum_{i=1}^m \tilde{X}_{ij}^I &= \tilde{b}_j^I, \quad j = 1 \text{ to } n \end{aligned}$$

$$\tilde{X}_{ij}^I \geq \tilde{0}; \quad \text{for all } i, j. \quad (\text{I})$$

## 2.3 Singularly Perturbed Delay Differential Equations (SPDDES)

“A singularly perturbed delay differential equation is an ordinary differential equation in which the highest derivative is multiplied by a small parameter and involving at least one delay term”. “Delay Differential Equations (DDEs) with constant lags  $\tau_j > 0$  for  $j = 1, 2, \dots, k$  have the form:  $Y' = f(t, Y(t), Y(t - \tau_1), \dots, Y(t - \tau_k))$ ”. An initial value  $Y(0) = \varphi(0)$  is not enough to define a unique solution of  $Y' = f(t, Y(t), Y(t - \tau_1), \dots, Y(t - \tau_k))$  on an interval  $a \leq t \leq b$ . The function  $Y(t) = \varphi(t)$  must be specified for  $t \leq a$  so that  $Y(t - \tau_j)$  is defined when  $a \leq t \leq a + \tau_j$ . The function  $\varphi(t)$  is called the history of the solution. “A hybrid finite difference scheme with an appropriate piece wise uniform Shishkin-type mesh is suggested to solve a second-order SPDDE of reaction-diffusion type”. Consider

$$Lu(x) = -\varepsilon u''(x) + a(x)u(x) + b(x)u(x-1) = f(x) \text{ on } (0, 2) \quad (\text{II})$$

$$\text{with } u = \emptyset \text{ on } [-1, 0] \text{ and } u(2) = l \quad (\text{III})$$

where  $\emptyset$  is sufficiently smooth on  $[-1, 0]$ .

The Novel Hybrid Scheme for SPDDEs is given as:

$$D_0^+ U(x_j) = \frac{3U(x_{j+1}) - 4U(x_j) + U(x_{j-1}))}{2h_{j+1}}, \quad D_0^- U(x_j) = \frac{3U(x_j) - 4U(x_{j-1}) + U(x_{j-2}))}{2h_j}$$

This is used to compute numerical approximations to the solution of Equations (II) and (III).

## 2.4 Weight Determination for Solving IFTP-1 Using SPDDEs

The weighting vector for solving IFTP -1 is represented in the form of the following SPDDE by the decision maker.

$$\begin{aligned} -\varepsilon u''(x) + 4u(x) - u(x-1) &= 2 \text{ for } x \in (0, 2), \\ u(x) &= x^2 \text{ where } x \in [-1, 0], \quad u(2) = 0. \end{aligned}$$

Based on the parameter, four separate points are found, and the numerical solutions at those locations are picked and normalised to produce the weighted vector. Table 1 contains information on the trader’s weight vector.

**Table 1. Numerical Solution of  $-\varepsilon u''(x) + 4u(x) - u(x-1) = 2$**

Continued on next page

Table 1 continued

| X         | Values of U(X) | Weight Vector |
|-----------|----------------|---------------|
| 0.2500000 | 0.6408602      | 0.261998543   |
| 0.7500000 | 0.5161290      | 0.211005530   |
| 1.2500000 | 0.6601004      | 0.269864384   |
| 1.7500000 | 0.6289553      | 0.257131543   |

The trader's weighting vector is therefore constructed as

$$w = (0.261998543, 0.211005530, 0.269864384, 0.257131543)^T$$

## 2.5 Method to Solve Intuitionistic Fuzzy Transportation Problem-1 Using SPDDes

In this section, a method to solve IFTP-1 using SPDDes is proposed.

The proposed approach follows these steps:

**Step 1:** The balanced IFTP-1 with triangular fuzzy intuitionistic parameters and variables is expanded as

$$\text{Min } \tilde{Z}^I = \sum_{i=1}^m \sum_{j=1}^n c_{ij} \times (X_{ij}^1, X_{ij}^2, X_{ij}^3; X_{ij}^4, X_{ij}^2, X_{ij}^5) \quad (1)$$

Subject to

$$\sum_{j=1}^n (X_{ij}^1, X_{ij}^2, X_{ij}^3; X_{ij}^4, X_{ij}^2, X_{ij}^5) = (a_i^1, a_i^2, a_i^3; a_i^4, a_i^2, a_i^5), \quad i = 1 \text{ to } m \quad (2)$$

$$\sum_{i=1}^m (X_{ij}^1, X_{ij}^2, X_{ij}^3; X_{ij}^4, X_{ij}^2, X_{ij}^5) = (b_j^1, b_j^2, b_j^3; b_j^4, b_j^2, b_j^5), \quad j = 1 \text{ to } n \quad (3)$$

$$(X_{ij}^1, X_{ij}^2, X_{ij}^3; X_{ij}^4, X_{ij}^2, X_{ij}^5) \geq 0 \text{ for all } i, j. \quad (4)$$

**Step 2:** Convert the problem consider in step 1 into the following fuzzy linear programming problem.

$$\text{Min } \tilde{Z}^I = \sum_{i=1}^m \sum_{j=1}^n (c_{ij} X_{ij}^1, c_{ij} X_{ij}^2, c_{ij} X_{ij}^3; c_{ij} X_{ij}^4, c_{ij} X_{ij}^2, c_{ij} X_{ij}^5) \quad (5)$$

Subject to

$$\sum_{j=1}^n X_{ij}^k = a_i^k \text{ for all } i, k = 1 \text{ to } 5 \quad (6)$$

$$\sum_{i=1}^m X_{ij}^k = b_j^k \text{ for all } j, k = 1 \text{ to } 5 \quad (7)$$

$$X_{ij}^4 \geq 0, \text{ for all } i, j \quad (8)$$

$$X_{ij}^2 - X_{ij}^1 \geq 0, \text{ for all } i, j \quad (9)$$

$$X_{ij}^3 - X_{ij}^2 \geq 0, \text{ for all } i, j \quad (10)$$

$$X_{ij}^1 - X_{ij}^4 \geq 0, \text{ for all } i, j \quad (11)$$

$$X_{ij}^5 - X_{ij}^3 \geq 0, \text{ for all } i, j \quad (12)$$

**Step 3:** The problem in step 2 is converted into the multi-objective linear programming problem (MOLPP) as follows:

$$\text{Minimize } (f_1(x), f_2(x), \dots, f_k(x))$$

Subject to Equations (6), (7), (8), (9), (10), (11) and (12) where,  $f_i : R^n \rightarrow R^i$ , where “is the n-dimensional Euclidean space”.

**Step 4:** The MOLPP is defined as the following when the weighting component is taken into account:

$$\text{Minimize } w(x)_{x \in X} = \sum_{m=1}^k w_m f_m(x)$$

Subject to Equations (6), (7), (8), (9), (10), (11) and (12).

Give weights derived from the Singular Perturbation Problem to the MOLPP obtained in step 3 to create a crisp linear programming problem and then find the optimal solution  $X_{ij}^{1*}, X_{ij}^{2*}, X_{ij}^{3*}, X_{ij}^{4*}, X_{ij}^{2*}, X_{ij}^{5*}$  using the method to solve crisp linear programming problem (using LINGO Software).

### 3 Results and Discussion

Consider the illustration suggested in <sup>(2)</sup> given in Table 2. The Singularly Perturbed Delay Differential Problem discussed in 2.4 is used to calculate the weights in the suggested method presented above, which solves the problem.

Table 2. IFTP-1

| Source                                    | Destination   |               |                    |               | Supply ( $\tilde{a}_i^I$ )                |
|---|---------------|---------------|--------------------|---------------|---|
|   | $D_1$         | $D_2$         | $D_3$              | $D_4$         |   |
| $S_1$                                     | 16            | 1             | 8                  | 13            | (2,4,5;1,4,6)                             |
| $S_2$                                     | 11            | 4             | 7                  | 10            | (4,6,8;3,6,9)                             |
| $S_3$                                     | 8             | 15            | 9                  | 2             | (3,7,12;2,7,13)                           |
| $S_4$                                     | 6             | 12            | 5                  | 14            | (8,10,13;5,10,16)                         |
| <b>Demand(<math>\tilde{b}_j^I</math>)</b> | (3,4,6;1,4,8) | (2,5,7;1,5,8) | (10,15,20;8,15,22) | (2,3,5;1,3,6) | $\sum \tilde{a}_i^I = \sum \tilde{b}_j^I$ |

This IFTP-1 is balanced. Using Steps 1 and 2 of the proposed method, the above-balanced IFTP-1 can be transformed to the following fuzzy linear programming problem

$$\text{Min } \tilde{Z}^I = 16 \tilde{X}_{11} + \tilde{X}_{12} + 8\tilde{X}_{13} + 13\tilde{X}_{14} + 11\tilde{X}_{21} + 4\tilde{X}_{22} + 7\tilde{X}_{23} + 10\tilde{X}_{24} + 8\tilde{X}_{31} + 15\tilde{X}_{32} + 9\tilde{X}_{33} + 2\tilde{X}_{34} + 6\tilde{X}_{41} + 12\tilde{X}_{42} + 5\tilde{X}_{43} + 14\tilde{X}_{44}$$

Subject to

$$\begin{aligned} X_{11}^1 + X_{12}^1 + X_{13}^1 + X_{14}^1 &= 2 \\ X_{11}^2 + X_{12}^2 + X_{13}^2 + X_{14}^2 &= 4 \\ X_{11}^3 + X_{12}^3 + X_{13}^3 + X_{14}^3 &= 5 \\ X_{11}^4 + X_{12}^4 + X_{13}^4 + X_{14}^4 &= 1 \\ X_{11}^2 + X_{12}^2 + X_{13}^2 + X_{14}^2 &= 4 \\ X_{11}^5 + X_{12}^5 + X_{13}^5 + X_{14}^5 &= 6 \\ X_{21}^1 + X_{22}^1 + X_{23}^1 + X_{24}^1 &= 4 \\ X_{21}^2 + X_{22}^2 + X_{23}^2 + X_{24}^2 &= 6 \\ X_{21}^3 + X_{22}^3 + X_{23}^3 + X_{24}^3 &= 8 \\ X_{21}^4 + X_{22}^4 + X_{23}^4 + X_{24}^4 &= 3 \\ X_{21}^2 + X_{22}^2 + X_{23}^2 + X_{24}^2 &= 6 \\ X_{21}^5 + X_{22}^5 + X_{23}^5 + X_{24}^5 &= 9 \\ X_{31}^1 + X_{32}^1 + X_{33}^1 + X_{34}^1 &= 3 \\ X_{31}^2 + X_{32}^2 + X_{33}^2 + X_{34}^2 &= 7 \\ X_{31}^3 + X_{32}^3 + X_{33}^3 + X_{34}^3 &= 12 \end{aligned}$$

$$\begin{aligned}
&X_{31}^4 + X_{32}^4 + X_{33}^4 + X_{34}^4 = 2 \\
&X_{31}^2 + X_{32}^2 + X_{33}^2 + X_{34}^2 = 7 \\
&X_{31}^5 + X_{32}^5 + X_{33}^5 + X_{34}^5 = 13 \\
&X_{41}^1 + X_{42}^1 + X_{43}^1 + X_{44}^1 = 8 \\
&X_{41}^2 + X_{42}^2 + X_{43}^2 + X_{44}^2 = 10 \\
&X_{41}^3 + X_{42}^3 + X_{43}^3 + X_{44}^3 = 13 \\
&X_{41}^4 + X_{42}^4 + X_{43}^4 + X_{44}^4 = 5 \\
&X_{41}^5 + X_{42}^5 + X_{43}^5 + X_{44}^5 = 10 \\
&X_{41}^1 + X_{42}^1 + X_{43}^1 + X_{44}^1 = 3 \\
&X_{11}^2 + X_{21}^2 + X_{31}^2 + X_{41}^2 = 4 \\
&X_{11}^3 + X_{21}^3 + X_{31}^3 + X_{41}^3 = 6 \\
&X_{11}^4 + X_{21}^4 + X_{31}^4 + X_{41}^4 = 1 \\
&X_{11}^5 + X_{21}^5 + X_{31}^5 + X_{41}^5 = 8 \\
&X_{12}^1 + X_{22}^1 + X_{32}^1 + X_{42}^1 = 2 \\
&X_{12}^2 + X_{22}^2 + X_{32}^2 + X_{42}^2 = 5 \\
&X_{12}^3 + X_{22}^3 + X_{32}^3 + X_{42}^3 = 7 \\
&X_{12}^4 + X_{22}^4 + X_{32}^4 + X_{42}^4 = 1 \\
&X_{12}^5 + X_{22}^5 + X_{32}^5 + X_{42}^5 = 8 \\
&X_{13}^1 + X_{23}^1 + X_{33}^1 + X_{43}^1 = 10 \\
&X_{13}^2 + X_{23}^2 + X_{33}^2 + X_{43}^2 = 15 \\
&X_{13}^3 + X_{23}^3 + X_{33}^3 + X_{43}^3 = 20 \\
&X_{13}^4 + X_{23}^4 + X_{33}^4 + X_{43}^4 = 8 \\
&X_{13}^5 + X_{23}^5 + X_{33}^5 + X_{43}^5 = 15 \\
&X_{14}^1 + X_{24}^1 + X_{34}^1 + X_{44}^1 = 2 \\
&X_{14}^2 + X_{24}^2 + X_{34}^2 + X_{44}^2 = 3 \\
&X_{14}^3 + X_{24}^3 + X_{34}^3 + X_{44}^3 = 5 \\
&X_{14}^4 + X_{24}^4 + X_{34}^4 + X_{44}^4 = 1 \\
&X_{14}^5 + X_{24}^5 + X_{34}^5 + X_{44}^5 = 6
\end{aligned}$$

$$X_{ij}^4 \geq 0, i = 1, 2, \dots, 4; j = 1, 2, \dots, 4$$

$$X_{ij}^2 - X_{ij}^1 \geq 0, i = 1, 2, \dots, 4; j = 1, 2, \dots, 4$$

$$X_{ij}^3 - X_{ij}^2 \geq 0, i = 1, 2, \dots, 4; j = 1, 2, \dots, 4$$

$$X_{ij}^1 - X_{ij}^4 \geq 0, i = 1, 2, \dots, 4; j = 1, 2, \dots, 4$$

$$X_{ij}^5 - X_{ij}^3 \geq 0, i = 1, 2, \dots, 4; j = 1, 2, \dots, 4$$

Using Steps 3 and 4 of the suggested method, the above fuzzy linear programming problem can be transformed to the multi-objective linear programming problem and then to the crisp linear programming problem by giving weights (0.261998543, 0.211005530, 0.269864384, 0.257131543)<sup>T</sup> found from the Singularly Perturbed delay differential problem. The optimal solution attained by using the suggested method is as follows

$$\begin{aligned}
&\tilde{X}_{11} = (0, 0, 0; 0, 0, 0); \tilde{X}_{12} = (2, 4, 5; 1, 4, 6); \tilde{X}_{13} = (0, 0, 0; 0, 0, 0); \tilde{X}_{14} = (0, 0, 0; 0, 0, 0); \\
&\tilde{X}_{21} = (0, 0, 0; 0, 0, 0); \tilde{X}_{22} = (0, 1, 2; 0, 1, 2); \tilde{X}_{23} = (4, 5, 6; 3, 5, 7); \tilde{X}_{24} = (0, 0, 0; 0, 0, 0); \\
&\tilde{X}_{31} = (1, 2, 4; 1, 2, 4); \tilde{X}_{32} = (0, 0, 0; 0, 0, 0); \tilde{X}_{33} = (0, 2, 3; 0, 2, 3); \tilde{X}_{34} = (2, 3, 5; 1, 3, 6); \\
&\tilde{X}_{41} = (2, 2, 2; 0, 2, 4); \tilde{X}_{42} = (0, 0, 0; 0, 0, 0); \tilde{X}_{43} = (6, 8, 11; 5, 8, 12); \tilde{X}_{44} = (0, 0, 0; 0, 0, 0).
\end{aligned}$$

And the objective value is (84, 135, 191; 57, 135, 218)

### 3.1 Comparative Study

The solution obtained from the proposed method, the solution of methods in<sup>(2)</sup> are represented in Table 3.

**Table 3. The Intuitionistic fuzzy solution and the objective value for the Numerical Example**

| Sources | Approach  | Destinations  |  |   |   |
|---------|---|---|--|---|---|
|         |   | 1   | 2  | 3   | 4   |
| 1       | The proposed approach<br>Singh and Yadav method<br>and Ali Mahmoodirad &<br>et al method <sup>(2)</sup> |   | (2, 4, 5; 1, 4, 6) (2,<br>4, 5; 1, 4, 6) (2, 4, 5;<br>1, 4, 6)   |   |   |
| 2       | The proposed approach<br>Singh and Yadav method<br>and Ali Mahmoodirad &<br>et al method <sup>(2)</sup> |   | (0, 1, 2; 0, 1, 2) (−3,<br>1, 5; −5, 1, 7) (0, 1,<br>2; 0, 1, 2) | (4, 5, 6; 3, 5, 7)<br>(−19, 5, 29; −30, 5,<br>40) (4, 5, 6; 3, 5, 7)          |   |
| 3       | The proposed approach<br>Singh and Yadav method<br>and Ali Mahmoodirad &<br>et al method <sup>(2)</sup> | (1, 2, 4; 1, 2, 4) (−2,<br>4, 10; −3, 4, 11) (1,<br>2, 4; 1, 2, 4)  |  | (0, 2, 3; 0, 2, 3) (0,<br>2, 3; 0, 2, 3)                                      | (2, 3, 5; 1, 3, 6) (2, 3, 5; 1,<br>3, 6) (2, 3, 5; 1, 3, 6) |
| 4       | The proposed approach<br>Singh and Yadav method<br>and Ali Mahmoodirad &<br>et al method <sup>(2)</sup> | (2, 2, 2; 0, 2, 4)<br>(−10, 0, 11; −15, 0,<br>16) (2, 2, 2; 0, 2, 4)  |  | (6, 8, 11; 5, 8, 12)<br>(−19, 10, 39; −33,<br>10, 53) (6, 8, 11; 5,<br>8, 12) |   |
|         | The proposed approach<br>Singh and Yadav method<br>and Ali Mahmoodirad &<br>et al method <sup>(2)</sup> | $(84, 135, 191; 57, 135, 218) R(\tilde{Z}^I) = 136.25$ (−310, 131, 579; −506, 131, 775)<br>$R(\tilde{Z}^I) = 132.75$ (84, 135, 191; 57, 135, 218) $R(\tilde{Z}^I) = 136.25$ |  |   |   |

## 4 Conclusion

Here, the needs and availabilities of IFTP-1 are represented using triangular intuitionistic fuzzy numbers (TIFN). A new method is projected to solve IFTP -1 where weights of the decision maker (trader) are obtained from singularly perturbed delay differential equations and normalized and used in transportation problems. This novel approach was suggested to overcome shortcomings such as obtaining negative solutions for the variables and obtaining negative objective function value in the existence of positive unit transportation costs. The new method is then illustrated using a numerical example, and the solutions are contrasted with those of the existing approaches. The findings of the Comparative study table, where some of the produced solutions are negative, some of the demand values are not respected, and the objective function value is negative, show the limitations of Singh and Yadav's method in<sup>(2)</sup>. On the other hand, the outputs of the suggested approach address all of these shortcomings. The obtained rank for the intuitionistic cost of the projected approach is not better than that of Singh and Yadav's method in<sup>(2)</sup> because the intuitionistic cost of Singh and Yadav's method in<sup>(2)</sup> contains some negative values, and it is found that the solution obtained using the proposed method and that obtained by Ali Mahmoodirad et al.'s method in<sup>(2)</sup> are identical.

## 5 Declaration

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