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Even Triangular Graceful Number o n Special Graphs

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Abstract

Objectives: To explore and detect some new types of graphs that exhibit even triangular graceful labeling. **Methods:** The methodology entails developing a mathematical formulation for labeling a given graph's vertices and demonstrating that these formulations result in Even triangular graceful labeling. **Findings:** Here we describe even triangular graceful labeling which is a new version of triangular graceful labeling. In the present paper, we establish even triangular graceful labeling for multi-star graph $K_{1,n,n,n}(n \geq 1)$. **Novelty:** Odd triangular graceful labeling was introduced by Lina and Asha S, here we find the even triangular graceful number for cycle C_n and some special graphs like generalized theta graph $\theta(P_1, P_2, ..., P_n)$, Golomb graph, Soifer graph, Moser Spindle graph. This is the first attempt of its sort, involving the investigation of even triangular graceful numbers for special graphs.

Keywords: Triangular graceful graphs; Triangular graceful numbers; Even triangular graceful labeling; Even triangular graceful graphs; Even triangular graceful numbers 1

1 Introduction

One of the most popular graph theory research areas at the moment is the investigation of graph labeling. There is a plethora of material on graph labeling strategies. Graph labeling strategies include elegant labeling, incidence labeling, gracious labeling, radio labeling, antimagic labeling, and prime labeling. One of these most often-used graph labeling approaches is elegant labeling [1]. A graceful labeling of a graph G is an injective function from the vertex set of G to the set, $(0, 1, \ldots, (E(G)|\}$ such that all of the induced edge labels are distinct, where each induced edge label is described as the absolute value of variation between the labels of its end vertices [2]. Rosa originally utilized this sort of graph labeling in 1967 as β - valuation, and it was a useful method for breaking down a full graph into isomorphic subgraphs. Despite Graham and Sloane's assertion that the majority of graphs are not graceful, it is still a challenging task to determine which graphs are graceful [3]. In 2000, Suresh Singh and Devaraj established the term triangular elegant labeling.

Let $V(G)$, $E(G)$ be the vertex as well as edge set of the graph G correspondingly. Suppose an injective function $f : V(G) \to \{0,\ 1,\ 2,\ \ldots, T_q\}$ here T_q indicates the q^{th} triangular number and q presents the number of edges of G. That is $T_1 = 1$, $T_2 = 3$, $T_3 = 6$, and so on and $T_n = \frac{n(n+1)}{2}$. Describe the function $f^{\star}(E(G) = \{1,\ 2,\ 3,\ \ldots, T_q\}$ so that $f^{\star}(u,\ v) = |f(u) - f(v)|$ for all the edges (uv). If $f^{\star}(E(G))$ is a series of successive triangular numbers such as $\{T_1, T_2, \ldots, T_q\}$ then the function is considered to be triangular graceful labeling and a graph that accepts such labeling is referred to as a triangular graceful graph[(4)]. Lina and Asha S introduced odd triangular graceful labeling of some graphs[(4)]. Asha S and Akshaya V introduced the odd hexagonal graceful labeling[(5)] on some graphs. The polygonal graceful labeling on some simple graphs[(6)] was introduced by M P Syed Ali Nisaya and A Rama Lakshmi, inspired by their research work, we establish even triangular gracefulness of multi-star graph $K_{1,n,n,n}$ $(n \geq 1)$[(7)] and we found even triangular graceful numbers for some special graphs.

2 Methodology

It includes the basic definitions of some special graphs discussed below.

2.1 Definition

The generalized theta graph $\theta(P_1, P_2, \ldots, P_n)$[(7)] consists of n $\geq$ 3 paired internally disjoint length paths $P_1, P_2, \ldots, P_n$ that shares a pair of common end vertices $u_0$and v_0. The theta graph $\theta(P_1, P_2, \ldots, P_n)$, when all P_i's are of the same length say, paths of length three then it can be denoted by $\theta(3P_4)$. The graph $\theta(3P_4)$ is shown below in the figure.

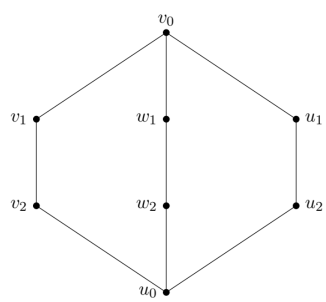

Fig 1. Theta graph $\theta(3P_4)$

2.2 Definition

A polyhedral graph with 18 edges and 10 vertices is called a golomb graph. It is a unit distance graph.

2.3 Definition

The soifer graph is a planar graph with 9 nodes and 20 edges. It is an undirected graph.

2.4 Definition

Moser Spindle graph is an undirected graph with 11 edges and 7 vertices. It is a unit distance graph sometimes called hajos graph.

3 Results and Discussion

3.1 Even triangular graceful labeling

Suppose G is considered to be a graph with edge set E(G), vertex set V(G), as well as the number of edges q[(8)]. Let an injective function $f: V(G) \to \{0,1,2,\ldots,T_{2q}\}$. Here q indicates the number of edges of G and $T_i$represents the i^{th} triangular number. That is T_1=1, T_2=3, T_3=6 and so on and $T_n = \frac{n(n+1)}{2}$. If the function f induces the function $f^{\star}$ on $E(G)$ so that $f*(uv) = |f(u) - f(v)|$ for every edges $(uv) \in E(G)$ with $f*(E(G) = \{T_2\ , T_4, T_6, \ldots, T_{2q}\}$ we refer to f as an even triangular graceful labeling, and an even triangular graceful graph permits such labeling.

3.1.1 Theorem

The multi-star graph $K_{1,n,n,n}\,(n \geq 1)$ is even triangular graceful.

Proof.

Consider $K_{1,n,n,n}(n \geq 1)$

Then, $|V(K_{1,n,n,n})| = 3n + 1$ and $|E(K_{1,n,n,n})| = 3n$

Let $V(K_{1,n,n,n}) = \{v_0, v_1, \ldots, v_n, w_1, w_2, \ldots, w_n, u_1, u_2, \ldots, u_n\}$.Define $f : V(K_{1,n,n,n}) \to \{0, 1, 2, \ldots, T_{2q}\}$ as follows

$f(v_0) = 0$

$f(v_{i+1}) = T_{2n^2-2i} - f(v_0),\ 0 \leq i \leq n-1$

$f(w_i) = f(v_i) - T_{2n^2-(n+1)-2i},\ 1 \leq i \leq n f(u_{i+1}) = f(w_{i+1}) - T_{(2n^2-4n)-2i},\ 0 \leq i \leq n-1$

Clearly, f is injective.

Also, f induces the function $f^{\star}$ on $E(K_{1,n,n,n})$ such that $f^{\star}(uv) = (f(u) - f(v)|$ for *all*

$(uv) \in E(K_{1,n,n,n}) = \{T_2, T_4, T_6, \ldots, T_{2q}\}$

Thus $f^{\star}(E(K_{1,n,n,n})) = (T_2, T_4, T_6, \ldots, T_{2q}\}$.

Therefore f is an even triangular graceful labeling.

Therefore $K_{1,n,n,n}$ is an even triangular graceful graph.

3.1.2 Example

Fig illustrates the even triangular graceful labeling of the multi-star graph $K_{1,3,3,3}$.

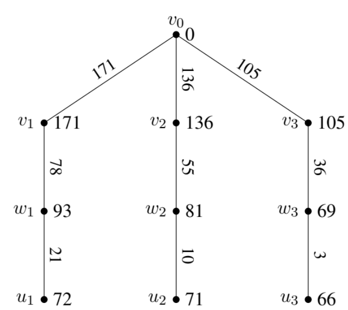

Fig 2. Multi-star graph $K_{1,3,3,3}$

3.2 Even triangular graceful number

In the case of some simple and special graphs G that are not even triangular graceful, the even triangular graceful number of G represented by $E_g^t n(G)$is defined as the minimum number of vertices removed from G to make the resulting graph even triangular and graceful.

3.2.1 Theorem

The even triangular graceful number of cycle $C_n(n \geq 3)$ is $E_g^t n(G) = 1$.

Proof.

As cycles $C_n(n \geq 3)$ are not even triangular graceful, when any one vertex is removed from C_n It will result in a path i e., $C_n - \{v\}$ for any vertex v it becomes a path which is even triangular graceful, as all the paths are even triangular graceful. Thus, the minimum number of vertices removed from C_n to make it an even triangular graceful graph is one.

Hence, the even triangular graceful number of the cycles $C_n(n \geq 3)$ *is* $E_g^t n(G) = 1$.

3.2.2 Theorem

The even triangular graceful number of the theta graph $\theta(mP_n)$ is $E_g^t n(\theta(mP_n)) = 1$.

Proof.

Theta graph $\theta(mP_n)$ is not even triangular graceful as it does not admit even triangular graceful labeling. If one of the end vertices u_0 or v_0 is removed from the theta graph it becomes a multi-star graph which is even triangular graceful.

Hence, the even triangular graceful number of the theta graph is $E_g^t n(\theta(mP_n)) = 1$.

3.2.3 Theorem

The even triangular graceful number of the Golomb graph is three.

Proof.

Let G denote the Golomb graph which contains 10 vertices and 18 edges having the vertex set $V(G)$ as follows.

$V(G) = \{v_1, v_2, v_3,\ u_1, u_2,\ u_3,\ u_4, u_5,\ u_6,\ w\}$ where v_1, v_2 *and* v_3 are the vertices of K_3 and $u_1, u_2,\ u_3,\ u_4, u_5$ *and* u_6 be the vertices of the wheel and w be the central vertex of the wheel. Define $f : V(G) \to \{0,\ 1,\ 2,\ \ldots, T_{2q}\}$ and the function f defined is shown in the figure below. Let $E = \{wu_1, wu_2,\ wu_3, wu_4, wu_5,\ wu_6, u_1u_2,\ u_2u_3,\ u_3u_4, u_4u_5,\ u_5u_6, u_6u_1,\ u_1v_1,\ u_3v_2,$

$u_5v_3,\ v_1v_2, v_3v_2, v_3v_1\}$The function f does not induce even triangular graceful labeling on G and the edge labels of G, $f^{\star}$ are given in the figure below. Clearly, f is not an injective function and f does not induce the function $f^{\star}$ on $E(G)$ such that $f^{\star}(uv) = |f(u) - f(v)|$ for all $(uv) \in E(G) = \{T_2,\ T_4,\ T_6,\ \ldots,\ T_{2q}\}$ .Therefore f is not an even triangular graceful labeling. Therefore, G is not an even triangular graceful graph. Now we find the even triangular graceful number by considering the following steps so that the Golomb graph becomes an even triangular graceful graph.

Step:1

Find the minimum number of required vertices removed from G to make it an even triangular graceful graph.

Remove any one vertex from K_3 and the central vertex w. To eliminate the vertex from the wheel we consider the following cases.

Case:1 If v_{i+1}, where $i = 0,\ 1$ is removed from K_3 then eliminate the vertex adjacent to v_3 from the wheel.

Case:2 If v_{i+1}, is removed from K_3 then eliminate the vertex adjacent to v_i from the wheel where $i = 2$.

Only removal of these vertices as mentioned in the above cases results in the minimum number of required vertices removed from G to make it an even triangular graceful graph.

Removal of other vertices results in the highest number of vertices eliminated from the graph G to make it an even triangular graceful.

Step:2

Reconstruct the resulting graph with the remaining vertices such that the vertices adjacent to the graph G are adjacent to the corresponding vertices in the reconstructed graph.

Step:3

Rename the vertices of the resulting graph as $u,\ v, v_1, v_2,\ v_3, v_4$ *and* v_5 .

Step:4

Let us denote the new graph obtained from the graph G as G'.

Let G' be a $F-tree$ graph with 6 edges and 7 vertices i e.,$G' = FP_5$

Let $q' = |E(G')| = 6$Define $f'\ : V(G') \to \{0,\ 1,\ 2,\ \ldots, T_{2q}\}$ as follows:

$f'(v_1) = 0 f'(v_{i+1}) = f'(v_i) + T_{2q-2i+2},\ for\ i\ is\ odd\ ,\ for\ 1\ \leq\ i\ \leq\ 4\ and f'(v) = f'(v_n) - T_2 f'(u) = f'(v_{n-1}) - T_{n-1}$Clearly f' indicates injective and f' induce the function $(f')^{\star}$ on $E(G')$ so that$f'(v_{i+1}) = f'(v_i) - T_{2q-2i+2},\ for\ i\ is\ even,\ for\ 1\ \leq\ i\ \leq\ 4$

$\left(f'\right)^{\star}(uv) = \left(f'(u) - f'(v)\right|$ for all $(uv) \in E(G') = \{T_2\ ,\ T_4,\ T_6,\ \ldots,\ T_{2q}\}$.

Hence, f' admit even triangular graceful labeling.

Therefore, the resulting graph G' is an even triangular graceful graph.

Hence, the lowest number of required vertices removed from the graph G to make it an even triangular graceful graph is three.

Therefore, the even triangular graceful number of the Golomb graph is $E_g^t n(G) = 3$.

Hence proved.

3.2.4 Example

The below figures illustrate the even triangular graceful number of the Golomb graph.

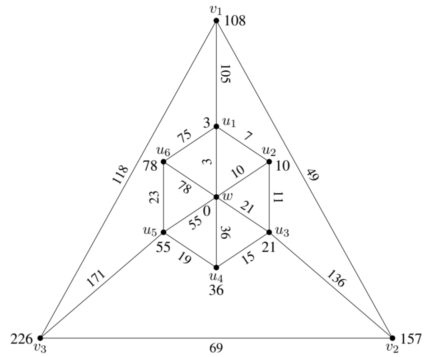

Fig 3. Golomb graph which is not an even triangular graceful graph with $E_g^t n(G) = 3$

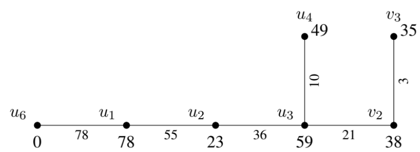

Fig 4. F-tree FP_5 obtained by removing vertices v_1, u_5, w

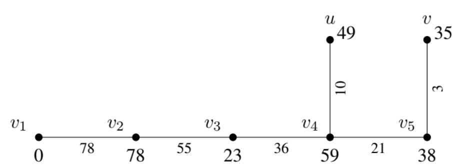

Fig 5. F-tree which is an even triangular graceful graph

3.2.5 Theorem

The even triangular graceful number of the Soifer graph is four.

Proof.

Let G denote the Soifer graph which contains 9 vertices and 20 edges having the vertex set $V(G)$ as follows.

$V(G) = \{v_1, v_2, v_3, v_4,\ u_1, u_2,\ u_3,\ u_4, u_5\}$ where v_1, v_2, v_3 *and* v_4 are the vertices of the outer cycle C_4 and $u_1, u_2,\ u_3,\ u_4$ *and* u_5 are the vertices of the inner cycle C_5 of G.

Define $f : V(G) \rightarrow \{0,\ 1,\ 2, \ldots, T_{2q}\}$ and the function f defined is shown in the figure below. Let $E = \{u_1u_2, u_2u_3,\ u_3u_4, u_4u_5, u_5u_1,\ u_1v_1, u_3v_2,\ u_5v_1,\ u_2v_2, u_2v_1,\ u_3v_2, u_4v_3,\ u_4v_4,\ u_5v_4,$

$u_1u_3,\ u_1u_4, v_1v_2, v_3v_2, v_3v_4, v_1v_4\}$The function f does not induce even triangular graceful labeling on G and the edge labels of G, f^* are given in the figure below.

f is not an injective function and f does not induce the function f^* on $E(G)$ such that $f^*(uv) = (f(u) f(v)|$ for all $(uv) \in E(G) = \{T_2\ ,\ T_4,\ T_6,\ \ldots,\ T_{2q}\}$.

Therefore, f is not an even triangular graceful labeling.

Therefore, G is not an even triangular graceful graph.

Now we find the even triangular graceful number by considering the following steps, such that the Soifer graph becomes an even triangular graceful graph.

Step:1

Here we find the minimum number of required vertices removed from G to make it an even triangular graceful graph.

We take into account the next four cases.

Case:1 If v_i is removed from the outer cycle then either remove $u_{i+1},\ u_{i+2},\ u_{i+3}$ *or*

$u_{i+2},\ u_{i+3},\ u_{i+4}$ from the inner cycle for $i = 1$.

Case:2 If v_i and v_{i+1} are removed from the outer cycle we remove the vertex that is not adjacent to v_i from the inner cycle for $i = 1$.

Case:3 For i = 2 if vertices v_{i+1} and v_{i+2} are removed from the outer cycle we remove the vertex that is not adjacent to v_{i+1} and v_{i+2} from the inner cycle.

Case:4 For i = 2 if vertices v_{i-1} and v_{i+2} are removed from the outer cycle the vertices u_{i+2} and u_{i+3} are removed from the inner cycle.

Only removal of these vertices as mentioned in the above cases results in the minimum number of required vertices removed from G to make it an even triangular graceful graph.

Removal of other vertices results in the highest number of vertices eliminated from the graph G to make it an even triangular graceful.

Step:2

Reconstruct the resulting graph with the remaining vertices such that the vertices adjacent to the graph G are adjacent to the corresponding vertices in the reconstructed graph.

Step:3

Rename the vertices of the resulting graph as $u,\ v, v_1, v_2$ *and* v_3 .

Step:4

Let us denote the new graph obtained from the graph G as G'.

Let G' be a $F-tree$ graph with 5 vertices and 4 edges $ie)$ $G' = FP_3$

Let $q' = \left(E\left(G'\right)\right| = 4$

Define $f'\ : V(G') \to \left(0,\ 1,\ 2,\ \ldots, T_{2q}\right\}$ as follows:

$f'(v_1) = 0 f'(v_{i+1}) = f'(v_i) + T_{2q-2i+2},\ for\ i\ is\ odd\ ,\ for\ 1 \le i \le 2\ and$Clearly f' indicates injective and f' induce the function $(f')^*$ on $E(G')$ such that $f'(v_{i+1}) = f'(v_i) - T_{2q-2i+2},\ for\ i\ is\ even,\ for\ 1 \le i \le 2 \quad f'(v) = f'(v_n) - T_2 \quad f'(u) = f'(v_{n-1}) - T_{n-1}$

$(f')^*(uv) = |f'(u) - f'(v)|$ for all $(uv) \in E(G') = \{T_2\ ,\ T_4,\ T_6,\ \ldots,\ T_{2q}\}$

Hence, f' admit even triangular graceful labeling.

Therefore, the resulting graph G' is an even triangular graceful graph.

Hence, the minimum number of required vertices removed from the graph G to make it an even triangular graceful graph is four.

Therefore, the even triangular graceful number of the Soifer graph is $E_g^t n(G) = 4$.

Hence proved.

3.2.6 Example

The below figures depict the even triangular graceful number of the Soifer graph.

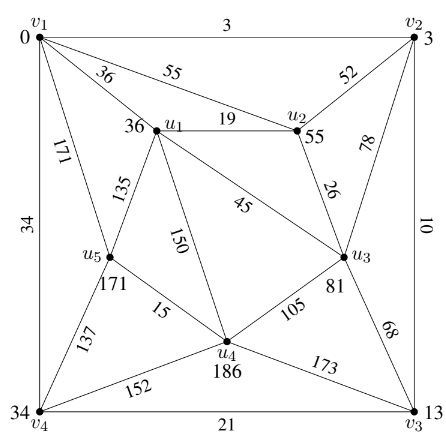

Fig 6. Soifer graph which is not an even triangular graceful graph having $E_g^t n(G) = 4$

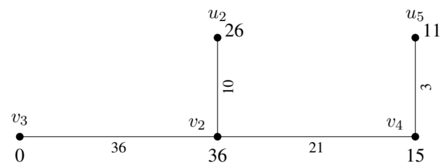

Fig 7. F-treeFP_3 obtained by removing vertices v_1, u_1, u_3, u_4

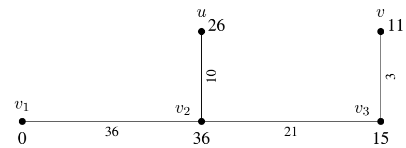

Fig 8. F-treeFP_3 which is an even triangular graceful graph

3.2.7 Theorem

The even triangular graceful number of the Moser Spindle graph is three.

Proof.

Let G denote the Moser Spindle graph which comprises 11 edges and 7 vertices having the vertex set $V(G)$ as follows. $V(G)=\{u_0,u_1,u_2,\ u_3,\ u_4,u_5,u_6\}$ where $u_2,\ u_3,\ u_4,u_5,u_6,u_2$ forms the cycle C_5 and the vertex u_0 is adjacent to $u_2,\ u_3$ *and* u_4 and u_1 is adjacent to u_2,u_5 *and* u_6. Define $f:V(G)\rightarrow\left(0,\ 1,\ 2,\ \ldots,T_{2q}\right\}$ and the function f defined is shown in the figure below. Let $E=\{u_1u_2,u_2u_3,\ u_3u_4,u_4u_5,u_5u_6,u_6u_2,\ u_1u_6,\ u_1u_5,u_0u_2,u_0u_3\}$The function f does not induce even triangular graceful labeling on G and the edge labels of G,$f^{\star}$ is given in the figure below. Clearly, f is not an injective function and f does not induce the function $f^{\star}$ on $E(G)$ such that$f^{\star}(uv)=|f(u)-f(v)|$ for all $(uv)\in E(G)=\{T_2\ ,\ T_4,\ T_6,\ \ldots,\ T_{2q}\}$ Therefore, f is not an even triangular graceful labeling. Therefore, G is not an even triangular graceful graph. Now we find the even triangular graceful number by considering the following steps, such that the Moser Spindle graph becomes an even triangular graceful graph.

Step:1

Find the minimum number of required vertices removed from G to make it an even triangular graceful graph. We remove the vertices u_0,u_1 and any one vertex from the cycle C_5. Only the removal of these vertices results in the minimum number of required vertices removed from G to make it an even triangular graceful graph. Removal of other vertices results in the highest number of vertices eliminated from the graph G to make it an even triangular graceful.

Step:2

Now we reconstruct the resulting graph with the remaining vertices such that the vertices adjacent in the graph G are adjacent to the corresponding vertices in the reconstructed graph.

Step:3

Rename the vertices of the resulting graph as$u_1,u_2,\ u_3$ *and* u_4.

Step:4

Let us denote the new graph obtained from the graph G as G'.Let G' be a path graph with 3 edges and 4 vertices i e., $G'=P_4$Let $q'=|E(G')|=3$

Define $f':V(G')\rightarrow\left(0,\ 1,\ 2,\ \ldots,T_{2q}\right\}$ as follows:

Let $f^{'}(u_1)=T_{2q}\ \ f^{'}(u_2)=0$

Clearly $f^{'}$ indicates injective and $f^{'}$induce the function $\left(f^{'}\right)^{\star}$ on $E\left(G^{'}\right)$ so that $f^{'}(u_3)=3f^{'}(u_{i+3})=f^{'}(u_{i+2})+T_{2q-2i},\ for\ 1\le i\le q-2$

$(f')^{\star}(uv)=|f'(u)-f'(v)|$ for all $(uv)\in E(G')=\left(T_2\ ,\ T_4,\ T_6,\ \ldots,\ T_{2q}\right\}$.

Hence f' admit even triangular graceful labeling.

Therefore, the resulting graph G' is an even triangular graceful graph.

Hence, the minimum number of required vertices removed from the graph G to make it an even triangular graceful graph is three. Therefore, the even triangular graceful number of the Moser Spindle graph is $E_g^t n(G)=3$.

Hence proved.

3.2.8 Example

The below figures illustrate the even triangular graceful number obtained from the Moser Spindle graph.

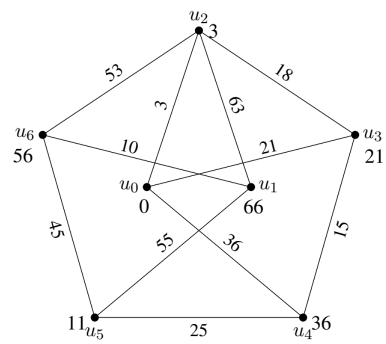

Fig 9. Moser Spindle graph which is not an even triangular graceful graph having $E_g^t n(G) = 4$.

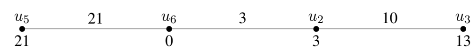

Fig 10. PathP_4 which is an even triangular graceful graph

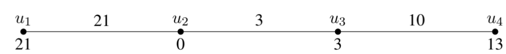

Fig 11. PathP_4 with renamed vertices

4 Conclusion

In the present paper, we presented even triangular graceful labeling on multi-star graphs and found the even triangular graceful number for some special graphs. We have also found various other graphs that admit even triangular graceful labeling. Future works can be carried out for similar other graphs using this concept.

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