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RESEARCH ARTICLE

Characterisation of PrasNikh–N Distribution and Its Biomedical Application

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Abstract

Objectives: A new class of Two-parameter Akash distribution is proposed and termed PrasNikh–N Distribution (P2ND). Its various structural properties are derived and researched. **Methods:** The concept of weighted distributions is applied. The distribution parameters have been calculated by maximum likelihood estimation. **Findings:** The characterisation sketch of the distribution is analysed. To examine the significance and supremacy of the distribution, this is applied to a real lifetime medical data, consists of the weight loss (kilograms (Kg)) after the first cycle of chemotherapy, of randomly selected 60 patients from a hospital in Thrissur district, Kerala who were suffering from any type of gastrointestinal (GI) cancer. The goodness of fit is tested for the same. **Novelty:** The results are compared with the known distributions & indicate that the proposed distribution shows a better fit than the other distributions, and hence clarifies the significance of the new distribution.

Keywords: Akash distribution; Length biased distribution; Weighted distribution; Reliability; Data

1 Introduction

In many situations the real data set – from various fields of bio medical, engineering, economical, business etc., cannot identify a best distribution fitting with the conventional distributions, hence a modification or generalisation of the known distribution is significant for the same. There are many methods to modify a distribution. By adding an extra parameter or using the weighted distribution we can formulate one new distribution which satisfies all the properties of probability distribution but its characteristics are completely different from the parent one.

The idea of weighted distributions (WD) plays a significant role in fitting a model to the unknown function of weight while the samples are from the proposed distribution. These distributions provide a remarkable approach that deals with the issue of data interpretation and model specification. Also, it is significant in analysing lifetime data in many subjects like medicine, engineering, finance, and insurance. While the standard distribution does not fit well, the WD is suitable for modelling the statistical data.

The idea of WD, to research how the method of ascertainment can affect the distribution of recorded data was introduced by Fisher[(1)]. Later this concept is detailed- while the common method utilizing the standard distributions was found to be inappropriate- executed by Rao[(2)]. If the weight functions emphasize only the length of units of interest the WD reduces to length biased (LB) distribution. In the context of renewal theory, LB distribution was introduced by Cox[(3)]. LB sampling situation occurs when a proper sampling frame is absent. LB sampling implies the probability of selecting an element and its magnitude is proportional. LB distribution is the resulting distribution of observations that are selected with probability proportional to their lengths.

A lot of researchers have studied various weighted probability models having examples and applications in various areas. The LB power hazard rate distribution was executed by Mustafa and Khan[(4)]. Al-Kadim and Hussein[(5)] proposed LB-weighted Rayleigh and exponential distribution. Mathew[(6)] discussed some LB distributions with an overview. Abd-Elfattah et al.[(7)] presented LB Burr-XII distribution. For application to hydrological data, the LB Weibull-Rayleigh distribution was introduced by Chaito and Khamkong[(8)]. Osowoleet al.[(9)] detailed the area-biased quasi-transmuted uniform distribution. Fazal[(10)] attained the area-biased Poisson Exponential Distribution. Nanuwong & Bodhisuwan[(11)] proposed LB Beta-Pareto distribution. Rasul and Bashir[(12)] described the Poisson area-biased Lindley distribution. Mir et al.[(13)] examined the structural parameters of LBbeta distribution of 1[st] type. Bashir & Rasul[(14)] represented the area-biased Rayleigh distribution. Bashir & Mahmood[(15)] executed the multivariate area biased Lindley distribution. Aijaz et al.[(16)] proposed the Poisson area-biased Ailamujia distribution. Abouammoh[(17)] showed a new renewal, is better than used classes of life distribution

The '2-parameter' Akash distribution (TAD) is a freshly proposed life-time distribution introduced by Shanker & Shukla[(18)] of which the Akash distribution with one parameter is a specific case. Shanker[(19)] explained Akash distribution and applications.

2 Methodology

2.1 The proposed P2N Distribution

We know that the pdf (probability density function) of TAD is,

$$f(x;\theta,\alpha)=\frac{\theta^3}{\alpha\theta^2+2}\left(\alpha+x^2\right)e^{-\theta x};x>0,\theta>0,\alpha>0 \tag{1}$$

The cdf (cumulative distribution function) of TAD is,

$$F(x;\theta,\alpha)=1-\left(1+\frac{\theta x(\theta x+2)}{\alpha\theta^2+2}\right)e^{-\theta x};x>0,\theta>0,\alpha>0 \tag{2}$$

The pdf for the weighted random variable X_w is$f_w(x)=\frac{w(x)f(x)}{E(w(x))},x>0$. If X represented as a random variable follows non-negative condition having a pdf $f(x)$ and $w(x)$ be the non - negative weight - function and $E(w(x))=\int w(x)f(x)dx<\infty$.

With respect to the many choices of $w(x)$, weighted models of several types while $w(x)=x^c$, the formulated distribution is called as WD. Here we detailed the TAD area-biased version. So, the weight function at $w(x)=x^2$, the resultant distribution termed as area biased distribution having a pdf as,

$$f_a(x)=\frac{x^2f(x)}{E\left(x^2\right)} \tag{3}$$

here $E\left(x^2\right)=\int_0^\infty x^2f(x,\theta,\alpha)dx$

$$E\left(x^2\right)=\frac{2\alpha\theta^2+24}{\theta^2\left(\alpha\cdot\theta^2+2\right)} \tag{4}$$

In Equation (3), apply the Equations (1) and (4) we get the pdf of P2ND as,

$$f_a(x)=\frac{x^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x} \tag{5}$$

and the cdf of P2ND as,

$$F_a(x)=\int_0^x f_a(x)dx=\int_0^x\frac{x^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x}dx=\frac{1}{2\alpha\cdot\theta^2+24}\int_0^x x^2\theta^5\left(\alpha+x^2\right)e^{-\theta x}dx$$

$$ie.,\ F_a(x) = \frac{1}{2\alpha \cdot \theta^2 + 24}\left(\alpha\theta^5 \int_0^x x^2 e^{-\theta x} dx + \theta^5 \int_0^x x^4 e^{-\theta x} dx\right) \tag{6}$$

Following equation simplificationEquation (6), we obtain the cdf of $P2ND$ as

$$F_a(x) = \frac{1}{2\alpha\theta^2 + 24}\left(\alpha\theta^2\gamma(3,\theta x) + \gamma(5,\theta x)\right) \tag{7}$$

Here $\gamma(3,\theta x)\&\gamma(5,\theta x)$ represents the Incomplete Gamma function

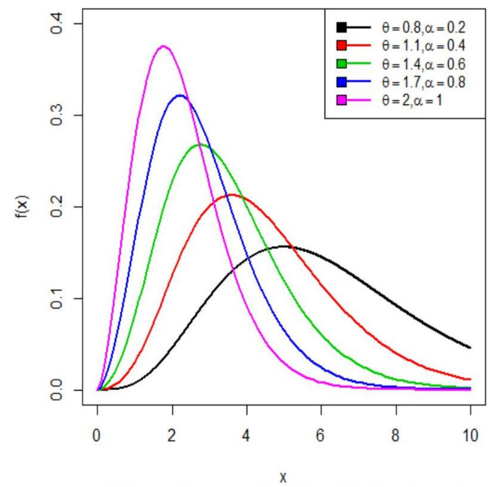

Fig 1. pdf of P2ND

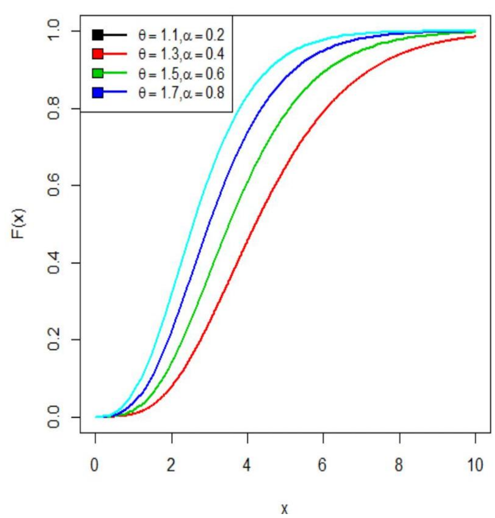

Fig 2. cdf of P2ND

Figures 1 and 2 shows the nature of pdf and cdf of P2ND for the different values of its parameters.
From the graphs of pdf (Figure 1) the symmetric nature of the distribution is noted.

2.2 Survival (reliability) function of P2ND

$$S(x) = 1 - F_a(x) = 1 - \frac{1}{2\alpha\theta^2 + 24}\left(\alpha\theta^2\gamma(3,\theta x) + \gamma(5,\theta x)\right)$$

2.3 Hazard function of P2ND

$$h(x) = \frac{f_a(x)}{1 - F_a(x)} = \frac{x^2\theta^5\left(\alpha + x^2\right)e^{-\theta x}}{(2\alpha\theta^2 + 24) - (\alpha\theta^2\gamma(3,\theta x) + \gamma(5,\theta x))}$$

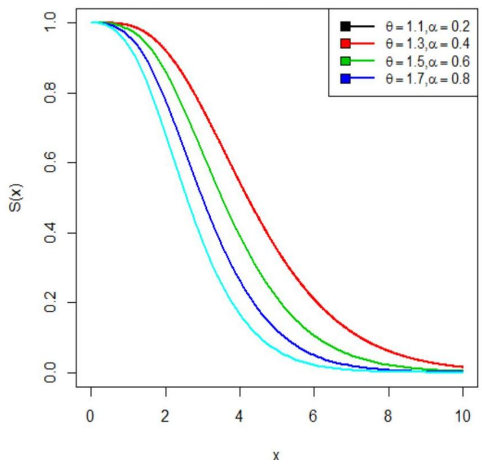

Fig 3. Reliability Function of P2ND

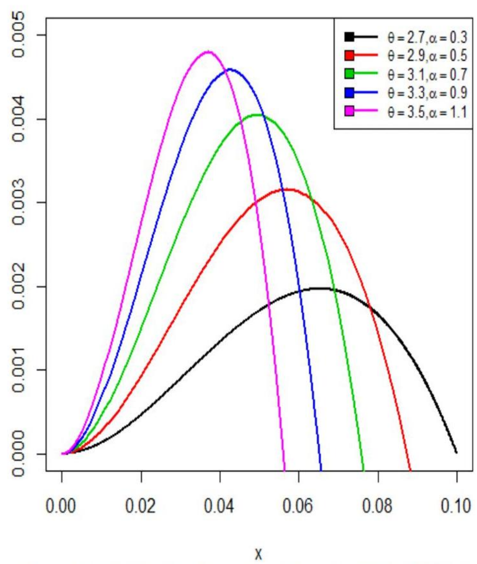

Fig 4. Hazard Function of P2ND

Figures 3 and 4 shows the nature of reliability function and hazard function of P2ND for the different values of its parameters.

3 Results and Discussions

The structural properties and other characteristics of the P2ND are discussed here.

3.1 Moments

The z^{th} order raw moment of the random variable X follows P2ND having the parameters θ and α, is,

$$E\left(X^r\right)=\mu_r{}'=\int_0^\infty x^r f_a(x)dx=\int_0^\infty x^r\frac{x^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x}dx=\int_0^\infty\frac{x^{r+2}\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x}dx$$

$$=\frac{\theta^5}{2\alpha\theta^2+24}\int_0^\infty x^{r+2}\left(\alpha+x^2\right)e^{-\theta x}dx\ ie.,\ E\left(X^r\right)=\frac{\theta^5}{2\alpha\theta^2+24}\left(\alpha\int_0^\infty x^{(r+3)-1}e^{-\theta x}dx+\int_0^\infty x^{(r+5)-1}e^{-\theta x}x\right)\tag{8}$$

After the simplification of Equation (8), we get,

$$E\left(X^r\right)=\mu_r{}'=\frac{\alpha\theta^2(r+2)!+(r+4)!}{\theta^r\left(2\alpha\theta^2+24\right)}\tag{9}$$

The initial set of moments of P2ND by letting $r=1,2,3,$ and Equation (4) in Equation (9).

$$E(X)=\mu_1'=\frac{6\alpha\cdot\theta^2+120}{\theta\left(2\cdot\alpha\cdot\theta^2+24\right)},\ E\left(X^2\right).=\mu_2{}'=\frac{24\alpha\theta^2+720}{\theta^2\left(2\alpha\theta^2+24\right)}$$

$$E\left(X^3\right)=\mu_3{}'=\frac{120\alpha\theta^2+5040}{\theta^3\left(2\alpha\theta^2+24\right)},\ E\left(X^4\right)=\mu_4{}'=\frac{720\alpha\theta^2+40320}{\theta^4\left(2\alpha\theta^2+24\right)}$$

$$\text{Variance}=\frac{\left(24\alpha\theta^2+720\right)\left(2\alpha\theta^2+24\right)-\left(6\alpha\theta^2+120\right)^2}{\theta^2\left(2\alpha\theta^2+24\right)^2}$$

$$S.D(\sigma)=\sqrt{\left(\frac{\left(24\alpha\theta^2+720\right)\left(2\alpha\theta^2+24\right)-\left(6\alpha\theta^2+120\right)^2}{\theta^2\left(2\alpha\theta^2+24\right)^2}\right)}$$

3.2 Harmonic mean (HM) of P2ND is,

$$H.M=E\left(\frac{1}{x}\right)=\int_0^\infty\frac{1}{x}f_a(x)dx=\int_0^\infty\frac{x\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x}dx=\frac{\theta^5}{2\alpha\theta^2+24}\int_0^\infty x\left(\alpha+x^2\right)e^{-\theta x}dx$$

$$\text{imples, }H.M=\frac{\theta^5}{2\alpha\theta^2+24}\left(\alpha\int_0^\infty x^{(3)-2}e^{-\theta x}dx+\int_0^\infty x^{(4)-1}e^{-\theta x}dx\right)\tag{10}$$

Simplifying Equation (10) we get,

$$H.M=\frac{\theta(2\alpha\theta+6)}{2\alpha\theta^2+24}$$

3.3 Moment generating function and characteristic function

$$M_X(t)=E\left(e^{tx}\right)=\int_0^\infty e^{tx}f_a(x)dx$$

By Taylor's series, we obtain,

$$=\int_0^\infty\sum_{j=0}^\infty\frac{t^j}{j!}x^j f_a(x).dx=\sum_{j=0}^\infty\frac{t^j}{j!}\mu_j{}'=\sum_{j=0}^\infty\frac{t^j}{j!}\left(\frac{\alpha\theta^2(j+2)!+(j+4)!}{\theta^j\left(2\alpha\theta^2+24\right)}\right)$$

$$M_X(t)=\frac{1}{2\alpha\theta^2+24}\sum_{j=0}^\infty\frac{t^j}{j!\theta^j}\left(\alpha\theta^2(j+2)!+(j+4)!\right)\tag{11}$$

Also, the characteristic function of P2ND is,

$$\varphi_x(t)=M_X(it)=\frac{1}{2\alpha\theta^2+24}\sum_{j=0}^\infty\frac{it^j}{j!\theta^j}\left(\alpha\theta^2(j+2)!+(j+4)!\right)\tag{12}$$

3.4 Order Statistics (OS)

OS plays a key role in statistics and has a wide range of applicability in reliability.

Let $X_{(i)}, i = 1,2,3,\ldots,n.$ be the OS of a random sample $X_i, i = 1,2,3,\ldots,n$ with pdf $f_x(x)$ & cdf $F_X(x)$, then the pdf of $r^{th} OS, X_{(r)}$ is

$$f_{x(r)}(x) = \frac{n!}{(r-1)!.(n-r)!} f_X(x) (F_X(x))^{r-1} (1-F_X(x))^{n-r} \tag{13}$$

By applying Equations (5) and (7) in Equation (13), the pdf of r^{th} OS of P2ND is,

$$\begin{aligned} f_{x(r)}(x) \quad &= \tfrac{n!}{(r-1)!(n-r)!}\left(\tfrac{x^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x}\right) \\ &\times\left(\tfrac{1}{2\alpha\theta^2+24}\left(\alpha\theta^2\gamma(3,\theta x)+\gamma(5,\theta x)\right)\right)^{r-1} \\ &\times\left(1-\tfrac{1}{2\alpha\theta^2+24}\left(\alpha\theta^2\gamma(3,\theta x)+\gamma(5,\theta x)\right)\right)^{n-r} \end{aligned}$$

Then, the pdf of $X_{(n)}$ of P2ND is,

$$f_{x(n)}(x) = \frac{nx^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x} \times \left(\frac{1}{2.\alpha.\theta^2+24}\left(\alpha\theta^2\gamma(3,\theta x)+\gamma(5,\theta x)\right)\right)^{n-1}$$

and the pdf of $X_{(l)}$ of P2ND is,

$$f_{x(1)}(x) = \frac{nx^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x^2\right)e^{-\theta x} \times \left(1-\frac{1}{2\alpha\theta^2+24}\left(\alpha\theta^2\gamma(3,\theta x)+\gamma(5,\theta x)\right)\right)^{n-1}$$

3.5 Likelihood Ratio Test

Consider the random sample, $X_i, i = 1.2.3,\ldots,n$ from $P2ND$. To test its significance, the hypothesis is,

$$H_o : f(x) = f(x;\theta,\alpha) \; against \; H_1 : f(x) = f_a(x;\theta,\alpha)$$

To analyze and examine that the random sample comes from the P2ND, the test statistic used is,

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n}\frac{f_a(x;\theta,\alpha)}{f(x;\theta,\alpha)} = \prod_{i=1}^{n}\left(\frac{x_i^2\theta^2\left(\alpha\theta^2+2\right)}{2\alpha\theta^2+24}\right) = \left(\frac{\theta^2\left(\alpha\theta^2+2\right)}{2\alpha\theta^2+24}\right)^n\prod_{i=1}^{n}x_i^2$$

We refuse to retain the H_0, if

$$\Delta = \left(\frac{\theta^2.\left(\alpha/\theta^2+2\right)}{2.\alpha.\theta^2+24}\right)^n\prod_{i=1}^{n}x_i^2 > k$$

Equivalently, we should also refuse to retain the H_0 where

$$\Delta^* = \prod_{i=1}^{n}x_i^2 > k^*, \; here \; k^* = k\left(\frac{2\alpha\theta^2+24}{\theta^2\left(\alpha\theta^2+2\right)}\right)^n$$

Whether, if the sample n is large, $2log\Delta$ is Chi-square having 1 degree of freedom. Then, we refused to accept H_0, while the probability is,$p(\Delta^* > \beta^*)$, Where $\beta^* = \prod_{i=1}^{n} x_i{}^2$ is less than a specified level of significance and $\prod_{i=1}^{n} x_i{}^2\cdot$ is the observed statistic Δ^*.

3.6 Bonferroni Curve (BoC) and Lorenz Curve (LoC)

The BoC and LoC are called classical curves and are being utilized to calculate the distribution of inequality in poverty or income. The BoC & LoC are defined as

$$B(p) = \frac{1}{p\mu_1^{\prime\prime}}\int^q xf(x)dx$$

and $L(p) = pB(p) = \frac{1}{\mu_1{}'} \int_0^q x f(x) dx$ Where $\mu_1{}' = \frac{6\alpha\theta^2+120}{\theta(2\alpha\theta^2+24)}$ and $q = F^{-1}(p)$

$$\begin{aligned} B(p) &= \tfrac{\theta(2.\alpha\theta^2+24)}{p(6.\alpha\theta^2+120)} \int_0^q \tfrac{x^3\theta^5}{2\alpha\theta^2+24} \left(\alpha+x^2\right) e^{-\theta x} dx \\ &= \tfrac{\theta^6}{p(6\alpha\theta^2+120)} \int_0^q x^3 \left(\alpha+x^2\right) e^{-\theta x} dx \\ &= \tfrac{\theta^6}{p(6\alpha\theta^2+120)} \left(\alpha \int_0^q x^{4-1} e^{-\theta x} dx + \int_0^q x^{6-1} e^{-\theta x} dx\right) \end{aligned}$$

Simplifying, we get

$$\begin{aligned} B(p) &= \tfrac{\theta^6}{p(6\alpha\theta^2+120)} (\alpha\gamma(4,\theta q) + \gamma(6,\theta q)) \\ L(p) &= \tfrac{\theta^6}{(6\alpha\theta^2+120)} (\alpha\gamma(4,\theta q) + \gamma(6,\theta q)) \end{aligned}$$

3.7 Renyi Entropy and Tsallis Entropy

Entropy plays a key role in various areas of research.

The Renyi Entropy, denoted as e(β). That is,

$$\begin{aligned} e(\beta) &= \tfrac{1}{1-\beta} log \left(\int f^{\beta}(x) dx\right) \ Where\ \beta > 0\ and\ \beta \neq 1 \\ &= \tfrac{1}{1-\beta} log \int_0^\infty \left(\tfrac{x^2\theta^5}{2\alpha\theta^2+24} \left(\alpha+x^2\right) e^{-\theta x}\right)^{\beta} dx \end{aligned}$$

$$= \frac{1}{1-\beta} log \left(\left(\frac{\theta^5}{2\alpha\theta^2+24}\right)^{\beta} \int_0^\infty x^{2\beta} e^{-\theta\beta x} \left(\alpha+x^2\right)^{\beta} dx \right) \tag{14}$$

By binomial expansion, Equation (14),

$$\begin{aligned} &= \tfrac{1}{1-\beta} log \left(\left(\tfrac{\theta^5}{2\alpha\theta^2+24}\right)^{\beta} \textstyle\sum_{j=0}^{\infty} \begin{pmatrix} \beta \\ j \end{pmatrix} \alpha^{\beta-j} x^{2j} \int_0^\infty x^{2\beta} e^{-\theta\beta x} dx \right) \\ &= \tfrac{1}{1-\beta} log \left(\left(\tfrac{\theta^5}{2\alpha\theta^2+24}\right)^{\beta} \textstyle\sum_{j=0}^{\infty} \begin{pmatrix} \beta \\ j \end{pmatrix} \alpha^{\beta-j} \int_0^\infty x^{(2\beta+2j+1)-1} e^{-\theta\beta x} dx \right) \\ e(\beta) &= \tfrac{1}{1-\beta} log \left(\left(\tfrac{\theta^5}{2\alpha\theta^2+24}\right)^{\beta} \textstyle\sum_{j=0}^{\infty} \begin{pmatrix} \beta \\ j \end{pmatrix} \alpha^{\beta-j} \tfrac{\Gamma(2\beta+2j+1)}{(\theta\beta)^{2\beta+2j+1}} \right) \end{aligned}$$

The Tsallis Entropy for the continuous random variable, it is expressed as,

$$S_{\lambda} = \frac{1}{\lambda-1} \cdot \left(1 - \int_0^\infty f^{\lambda} \cdot (x) dx\right) = \frac{1}{\lambda-1} \cdot \left(1 - \int_0^\infty \left(\frac{x^2 \cdot \theta^5}{2\alpha\theta^2+24} \left(\alpha+x^2\right) e^{-\theta x}\right)^{\lambda} dx\right)$$

$$= \frac{1}{\lambda-1} \left(1 - \left(\frac{\theta^5}{2\alpha\theta^2+24}\right)^{\lambda} \int_0^\infty x^{2\lambda} e^{-\lambda\theta x} \left(\alpha+x^2\right)^{\lambda} dx\right) \tag{15}$$

By binomial expansion in Equation (13),

$$\begin{aligned} &= \tfrac{1}{\lambda-1} \left(1 - \left(\tfrac{\theta^5}{2\alpha\theta^2+24}\right)^{\lambda} \textstyle\sum_{k=0}^{\infty} \begin{pmatrix} \lambda \\ k \end{pmatrix} \alpha^{\lambda-k} x^{2k} \int_0^\infty x^{2\lambda} e^{-\lambda\theta x} dx\right) \\ &= \tfrac{1}{\lambda-1} \left(1 - \left(\tfrac{\theta^5}{2\alpha\theta^2+24}\right)^{\lambda} \textstyle\sum_{k=0}^{\infty} \begin{pmatrix} \lambda \\ k \end{pmatrix}^{\lambda-k} \int_0^\infty x^{(2\lambda+2k+1)-1} e^{-\lambda\theta x} dx\right) \\ S_{\lambda} &= \tfrac{1}{\lambda-1} \left(1 - \left(\tfrac{\theta^5}{2\alpha\theta^2+24}\right)^{\lambda} \textstyle\sum_{k=0}^{\infty} \begin{pmatrix} \lambda \\ k \end{pmatrix} \alpha^{\lambda-k} \tfrac{\Gamma(2\lambda+2k+1)}{(\lambda\theta)^{2\lambda+2k+1}}\right) \end{aligned}$$

3.8 Estimation of Parameters

The MLE (Maximum Likelihood Estimate) of the parameters of P2ND is estimated. For all X_i, i=1, 2,…,n. a random sample of the n size by the P2ND, the probability function is,

$$L(x)=\prod_{i=1}^{n} f_a(x)=\prod_{i=1}^{n}\left(\frac{x_i^2\theta^5}{2\alpha\theta^2+24}\left(\alpha+x_i^2\right)e^{-\theta x_i}\right)=\frac{\theta^{5n}}{\left(2\alpha\theta^2+24\right)^n}\prod_{i=1}^{n}\left(x_i^2\left(\alpha+x_i^2\right)e^{-\theta x_i}\right)$$

$$logL=5nlog\theta-nlog\left(2\alpha\theta^2+24\right)+2\sum_{i=1}^{n}logx_i+\sum_{i=1}^{n}log\left(\alpha+x_i^2\right)-\theta\sum_{i=1}^{n}x_i \tag{16}$$

$$\frac{\partial logL}{\partial\theta}=\frac{5n}{\theta}-n\left(\frac{4\theta\alpha}{2\alpha\theta^2+24}\right)-\sum_{i=1}^{n}x_i=0\ and\ \frac{\partial logL}{\partial\alpha}=-n\left(\frac{2\theta^2}{2\alpha\theta^2+24}\right)+\sum_{i=1}^{n}\left(\frac{1}{\left(\alpha+x_i^2\right)}\right)=0$$

The solution of these systems of equations by using R program results the MLE of α and θ.

By the asymptotic normality outcomes, attain the CI (Confidence Interval). If $\widehat{\beta}=(\widehat{\theta},\ \widehat{\alpha})$ shows the MLE of $\beta=(\theta,\ \alpha)$. $\sqrt{n}(\widehat{\beta}-\beta)\to N_2\left(0,I^{-1}(\beta)\right)$, Where $I(\beta)$ is FEM (Fisher's Information Matrix),

$$I(\beta)=-\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 logL}{\partial\theta^2}\right) & E\left(\frac{\partial^2 logL}{\partial\theta\partial\alpha}\right)\\ E\left(\frac{\partial^2 logL}{\partial\alpha\cdot\partial\theta}\right) & E\left(\frac{\partial^2 logL}{\partial\alpha^2}\right)\end{pmatrix},$$

$$E\left(\frac{\partial^2 logL}{\partial\theta^2}\right)=-\frac{5n}{\theta^2}-n\left(\frac{4\alpha\left(2\alpha\theta^2+24\right)-16\alpha^2\theta^2}{\left(2\alpha\theta^2+24\right)^2}\right)E\left(\frac{\partial^2 logL}{\partial\alpha^2}\right)=n\left(\frac{4\theta^4}{\left(2\alpha\theta^2+24\right)^2}\right)-\sum_{i=1}^{n}\left(\frac{1}{\left(\alpha+x_i^2\right)^2}\right)\&$$

$$E\left(\frac{\partial^2 logL}{\partial\theta\cdot\partial\alpha}\right)=-n\cdot\left(\frac{4\theta\left(2\alpha\theta^2+24\right)-8\alpha\theta^3}{\left(2\alpha\theta^2+24\right)^2}\right)$$

Since β is not known, estimate $I^{-1}(\beta)$ by $I^{-1}(\widehat{\beta})$ obtain asymptotic CI for θ and α.

3.9 Application

Here, the fitting of a lifetime medical real data in P2NDis considered. It shows that the P2ND fits quite satisfactorily over Lindley, TAD, Akash, and exponential distributions.

The real lifetime medical data (Table 1) consists of the weight loss (kilograms (Kg)) after the first cycle of chemotherapy, of randomly selected 60 patients from a hospital in Thrissur district, Kerala who were suffering from any type of gastrointestinal (GI) cancer (involves all cancers in digestive tract organs for example the small & large intestine, stomach, colon, pancreas, anus, rectum, biliary system, and liver).

Table 1. Weight loss (kilograms (Kg))

4.390	4.395	4.645	3.765	3.750	3.855	3.985	4.050
0.320	0.490	0.620	1.150	1.210	1.260	1.410	2.025
2.910	3.190	3.265	3.350	3.350	4.975	5.075	5.380
2.035	2.160	2.210	2.370	2.530	2.690	2.800	2.910
2.910	3.190	3.265	3.350	3.350	3.430	3.500	3.535
3.765	3.750	3.855	3.985	4.050	4.245	4.325	4.380
4.390	4.395	4.645	4.755	4.930	4.975	5.075	5.380
3.350	3.430	3.500	3.535				

To compare the performance of P2ND with Lindley, TAD, Akash & exponential distributions, consider the standard general criteria & notations. The lesser values of BIC, AIC, -2logL, and AICC imply the better distribution to which they correspond.

Table 2 implies that P2ND has lesser BIC, AIC, -2logL, and AICC values by comparing to the other distributions. Hence, it can be concluded that the P2NDis a better fit.

Table 2. Analysis of Fitted Distributions

Distributions	MLE	Standard error (S.E)	-2logL	AIC	BIC	AICC
P2ND	$\widehat{\alpha} = 1.6977$ $\widehat{\theta} == 1.4369$	$\widehat{\alpha} = 1.6893$ $\theta = 0.1293$	143.7944	147.7944	151.1927	148.1987
TAD	$\widehat{\alpha} = 0.02967$ $\theta = 0.8936$	$\widehat{\alpha} = 0.1596$ $\widehat{\theta} = 0.0894$	147.2922	151.2922	154.67	151.6165
Akash	$\widehat{\theta} = 0.8010$	$\widehat{\theta} = 0.0697$	152.6894	154.6894	156.3783	154.7946
Exponential	$\widehat{\theta} = 0.2931$	$\widehat{\theta} = 0.0495$	170.9553	172.9556	174.9245	172.9661
Lindley	$\widehat{\theta} = 0.4952$	$\widehat{\theta} = 0.0610$	159.9501	161.9501	163.919	161.9606

4 Conclusion

A generalized format of TAD distribution was suggested and termed asP2ND. Its several statistical properties involving the mean, harmonic mean, variance, moments, BoC, and LoC have been studied. The MLE of the distribution parameters are estimated. P2ND has been examined and investigated with medical data to demonstrate its significance. It is really important to study the characteristics of such bio-medical data. The findings show that the suggested P2ND fits across TAD, Akash, exponential, and Lindley distributions rather well.

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