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# Fixed Point and Common Fixed Point Theorems for Single Valued and Multivalued Mappings in Partial b-Metric Spaces

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## Abstract

**Objectives:** To prove some fixed point theorems and common fixed point theorems by using partial b-metric spaces. **Methods:** Ciric type contraction for single-valued mapping is also used to prove fixed point theorem and Nadler's type Banach contraction is used to produce fixed point and common fixed point theorems. **Findings:** We have to find fixed point and common fixed point theorems for single valued mapping and a fixed point theorem for multivalued mapping. **Novelty :** We have to use a new type of space called partial b-metric space to prove all the theorems in this paper. No one has proven these theorems before in this space.

**2020 Mathematics Subject Classification :** 47H10, 54H25.

**Keywords:** Partial b-metric space; Single valued; Multi valued; Common fixed point; Fixed point

## 1 Introduction

An essential part of functional and nonlinear analysis is fixed point theory. Banach[(1)] presented an important finding regarding contraction maps. Since then, other writers have contributed numerous papers that deal with fixed point results (see, for example,[(2–4)]).

More recently, Shukla[(5)] expanded upon the notions of $b-$metric spaces and partial metric spaces, introducing the concept of partial $b-$metric spaces. In this context, Shukla not only formulated the Banach contraction principle but also established a Kannan-type fixed point theorem within partial $b-$metric spaces.

The aforementioned spaces have seen a great deal of development. The aforementioned spaces have recently been the focus of investigation into fixed point and common fixed point results for single-valued as well as multi-valued mappings; for examples, see Ali et al.[(6)], Khan et al.[(7)], Kanwel et al.[(8–10)], Tassaddiq et al.[(11)], Karapinar et al.[(12,13)], Qawaqneh et al.[(14)], Shoaib et al.[(15)], and the references within it.

This work is dedicated to the formulation and proof of contractive mappings fixed point theorems in partial b-metric spaces. The application of contraction of the Ciric[(16)] type is used in Theorem 3.1 to determine a fixed point of a single-valued map and provide an extension in partial b-metric space. Theorem 3.2 uses Banach[(1)] type contraction in the setting of complete partial b-metric spaces to expand Nadler's fixed point theorem [17]. A fixed point theorem for multi-valued mapping in partial b-metric spaces was demonstrated by Theorem 3.3.

## 2 Preliminaries

Firstly, we recall some basic definitions,

**Definition 2.1** [5] Let $X$ be a non-empty set and the self mapping $d : X \times X \to R^+$ ($R^+$ stands for non-negative reals) satisfies:

(Pb1) $x = y$ if and only if $d(x,x) = d(x,y) = d(y,y)$;

(Pb2) $d(x,x) \leq d(x,y)$;

(Pb3) $d(x,y) = d(y,x)$;

(Pb4) there exist a real number $s \geq 1$ such that $d(x,y) \leq s[d(x,z) + d(z,y)] - d(z,z)$.

Then $d$ is called a partial $b-$metric on $X$ and $(X,d)$ is called a partial $b-$metric space with coefficient $s$.

**Definition 2.2** Let $\{x_n\}$ be a sequence in a partial $b-$metric space $(X,d)$. Then:

(1) The sequence $\{x_n\}$ is said to be a convergent in $(X,d)$, if there exists $x^* \in X$ such that $lim_{n\to\infty} d(x_n,x^*) = 0$.

(2) The sequence $\{x_n\}$ is said to be a Cauchy sequence in $(X,d)$, if for every $\varepsilon > 0$ there exists a positive $n_0 \in N$ such that $d(x_n,x_m) < \varepsilon$ for all $n,m > n_0$ (or, equivalently, $lim_{n,m\to\infty} d(x_n,x_m) = 0$).

(3) $(X,d)$ is called a complete partial $b-$metric space if every Cauchy sequence is convergent in $X$.

**Definition 2.3** Let X be a non-empty set. Then a self-mapping $T : X \to X$ is said to be a fixed point if for all $x \in X$ such that $T(x) = x$.

**Example 2.4** The mapping $T : R \to R$ defined by $T(x) = sinx$ has 0 as a fixed point.

**Definition 2.5** A pair of self-mappings have a common fixed point $S,T : X \to X$ is a point $a \in X$ for which

$S(a) = T(a) = a$.

**Definition 2.6** Consider a metric space $(X,d)$. Let $CB(X)$ denote the family of all non-empty bounded and closed subsets of $X$. Suppose that a map $H : CB(X) \times CB(X) \to R$ for $U,V \in CB(X)$, define

$$H(U,V) = max\{\sup_{u\in U} d(u,V), \sup_{v\in V} d(v,U)\}$$

where $d(u,V) = inf\{d(u,v) : v \in V\}$ is the distance of a point $u$ to the set $V$. This $H$ is a metric on $CB(X)$, called Housdorff metric induced by the metric $d$.

**Definition 2.7** [18] Let a multi-valued mapping $T : X \to CB(X)$ on a non-empty set $X, CB(X)$ be the family of all non-empty closed and bounded subsets of $X$. A point $y \in X$ is called fixed point of $T$ if $y \in Ty$.

**Lemma 2.8** [18] Let partial $b-$metric space $(X,d)$ and let $CB(X)$ is the family of all non-empty closed and bounded subsets of $X$. Then, for $U,V \in CB(X)$,

(1) $d(a,U) \leq H(U,V), a \in U$ ;

(2) For $\varepsilon > 0$ and $a \in U, \exists b \in V$ such that

$d(a,b) \leq H(U,V) + \varepsilon$

## 3 Result and Discussion

In this paper, we begin with the proof of the following result in partial $b-$metric space with the help of single valued mapping:

**Theorem 3.1** Let a $(X,d,s)$ be a complete partial $b-$metric space with $s \geq 1$ and $T : X \to X$ is a single valued mapping, such that

$$d(Tx,Ty) \leq \alpha_1 d(x,y) + \alpha_2 d(x,Tx) + \alpha_3 d(y,Ty) + \alpha_4 [d(y,Tx) + d(x,Ty)],$$

where $\alpha_1 + (1+s)\alpha_2 + \alpha_3 + (1+s)\alpha_4 < 1, \forall x,y \in X$. Then, $\exists x^* \in X$ such that $x_n \to x^*$ and $x^*$ is the unique fixed point.

**Proof.** Let $x_0 \in X$ and $\{x_n\}$ be a sequence in $X$ define as

$x_n = Tx_n = T^n x_0, n = 1,2,3,...$

Now,

$$\begin{aligned}
d(x_n,x_{n+1}) &= d(Tx_{n-1},Tx_n) \\
&\leq \alpha_1 d(x_{n-1},x_n) + \alpha_2 d(x_{n-1},x_n) + \alpha_3 d(x_n,x_{n+1}) \\
&\quad + \alpha_4 [d(x_n,x_n) + d(x_{n-1},x_{n+1})] \\
&\leq \alpha_1 d(x_{n-1},x_n) + \alpha_2 d(x_{n-1},x_n) + \alpha_3 d(x_n,x_{n+1}) + \alpha_4 d(x_n,x_{n+1}) \\
&\quad + s\alpha_4 d(x_{n-1},x_n) + s\alpha_4 d(x_n,x_{n+1}) - \alpha_4 d(x_n,x_n) \\
&\leq \alpha_1 d(x_{n-1},x_n) + \alpha_2 d(x_{n-1},x_n) + \alpha_3 d(x_n,x_{n+1}) + \alpha_4 d(x_n,x_{n+1}) \\
&\quad + s\alpha_4 d(x_{n-1},x_n) + s\alpha_4 d(x_n,x_{n+1})
\end{aligned}$$

$$(1 - \alpha_3 - \alpha_4 - s\alpha_4) d(x_n,x_{n+1}) \leq (\alpha_1 + \alpha_2 + s\alpha_4) d(x_{n-1},x_n)$$

$d(x_n,x_{n+1}) \leq (\frac{\alpha_1+\alpha_2+s\alpha_4}{1-\alpha_3-\alpha_4-s\alpha_4})d(x_{n-1},x_n)$,
where $k = \frac{\alpha_1+\alpha_2+s\alpha_4}{1-\alpha_3-\alpha_4-s\alpha_4}$.
Therefore,
$d(x_n,x_{n+1}) \leq kd(x_{n-1},x_n)$
$\leq k^2d(x_{n-2},x_{n-1})$.
Continuing this process, we get
$d(x_n,x_{n+1}) \leq k^nd(x_0,x_1)$.
Consider $m,n \in N$ with $m > n$
$d(x_n,x_m) \leq s[d(x_n,x_{n+1})+d(x_{n+1},x_m)] - d(x_{n+1},x_{n+1})$
$\leq sd(x_n,x_{n+1})+sd(x_{n+1},x_m)$
$\leq sd(x_n,x_{n+1})+s\{s[d(x_{n+1},x_{n+2})+d(x_{n+2},x_m)] - d(x_{n+2},x_{n+2})\}$
$\leq sd(x_n,x_{n+1})+s^2d(x_{n+1},x_{n+2})+s^2d(x_{n+2},x_m)$
$\leq sd(x_n,x_{n+1})+s^2d(x_{n+1},x_{n+2})+s^3d(x_{n+2},x_{n+3})+...+s^md(x_{n+m-1},x_{n+m})$
$\leq sk^nd(x_0,x_1)+s^2k^{n+1}d(x_0,x_1)+s^3k^{n+2}d(x_0,x_1)+...+s^mk^{n+m-1}d(x_0,x_1)$
$\leq sk^nd(x_0,x_1)[1+sk+(sk)^2+...+(sk)^{m-1}]$
$\leq sk^nd(x_0,x_1)[\frac{1-(sk)^m}{1-sk}]$,
when $m,n \to \infty$, $d(x_n,x_m) \to 0$. Hence, $\{x_n\}$ is a Cauchy sequence in $X$. Since $X$ is complete, $\{x_n\}$ converges to an element of $X$, say $x^* \in X$.
Now,
$d(x^*,Tx^*) \leq s[d(x^*,x_{n+1})+d(x_{n+1},Tx^*)] - d(x_{n+1},x_{n+1})$
$\leq sd(x^*,x_{n+1})+sd(Tx_n,Tx^*)$
$\leq sd(x^*,x_{n+1})+s\alpha_1d(x_n,x^*)+s\alpha_2d(x_n,Tx_n)+s\alpha_3d(x^*,Tx^*)$
$+s\alpha_4d(x^*,Tx_n)+s\alpha_4d(x_n,Tx^*)$
$\leq sd(x^*,x_{n+1})+s\alpha_1d(x_n,x^*)+s\alpha_2d(x_n,x_{n+1})+s\alpha_3d(x^*,Tx^*)$
$+s\alpha_4d(x^*,x_{n+1})+s\alpha_4\{s[d(x_n,x^*)+d(x^*,Tx^*)] - d(x^*,x^*)\}$
$\leq sd(x^*,x_{n+1})+s\alpha_1d(x_n,x^*)+s\alpha_2d(x_n,x_{n+1})+s\alpha_3d(x^*,Tx^*)$
$+s\alpha_4d(x^*,x_{n+1})+s^2\alpha_4d(x_n,x^*)+s^2\alpha_4d(x^*,Tx^*)$
$d(x^*,Tx^*) \leq \frac{(s+s\alpha_4)}{(1-s\alpha_3-s^2\alpha_4)}d(x^*,a_{n+1})+\frac{(s\alpha_1+s^2\alpha_4)}{(1-s\alpha_3-s^2\alpha_4)}d(a_n,x^*)+\frac{s\alpha_2}{(1-s\alpha_3-s^2\alpha_4)}d(a_n,a_{n+1})$,
taking $n \to \infty$, we have
$d(x^*,Tx^*) \leq \frac{(s+s\alpha_4)}{(1-s\alpha_3-s^2\alpha_4)}d(x^*,x^*)+\frac{(s\alpha_1+s^2\alpha_4)}{(1-s\alpha_3-s^2\alpha_4)}d(x^*,x^*)+\frac{s\alpha_2}{(1-s\alpha_3-s^2\alpha_4)}d(x^*,x^*)$,
implies that $x^* = Tx^*$. Hence, $x^*$ is an fixed point of $T$.
For uniqueness, assume that $y^*$ is another fixed point of $T$. Then, we have $Ty^* = y^*$.
Consider,
$d(x^*,y^*) = d(Tx^*,Ty^*)$
$\leq \alpha_1d(x^*,y^*)+\alpha_2d(x^*,Tx^*)+\alpha_3d(y^*,Ty^*)+\alpha_4[d(y^*,Tx^*)+d(x^*,Ty^*)]$
$\leq \alpha_1d(x^*,y^*)+\alpha_2d(x^*,x^*)+\alpha_3d(y^*,y^*)+\alpha_4[d(y^*,x^*)+d(x^*,y^*)]$
$\leq \alpha_1d(x^*,y^*)+\alpha_2d(x^*,y^*)+\alpha_3d(y^*,x^*)+\alpha_4[d(y^*,x^*)+d(x^*,y^*)]$
$\leq (\alpha_1+\alpha_2+\alpha_3+2\alpha_4)d(x^*,y^*)$,
this implies that $x^* = y^*$.

**Theorem 3.2** Consider a complete partial b metric space $(X,d,s)$ with constant $s \geq 1$ and suppose that $F,G : X \to X$ be two maps, for which $\eta_1,\eta_2 \in [0,\frac{1}{3})$ such that
$d(Fa,Gb) \leq \eta_1d(a,b)+\eta_2[d(a,Fb)+d(a,Gb)]$.
Then there exists a common fixed point of $F$ and $G$.

**Proof.** Let $x_0 \in X$. Consider the sequence $\{x_n\}$ so that $x_{2n+2} = Gx_{2n+1}, x_{2n+1} = Fx_{2n}$. Then
$d(x_{2n+1},x_{2n+2}) = d(Fx_{2n},Gx_{2n+1})$
$\leq \eta_1d(x_{2n},x_{2n+1})+\eta_2[d(x_{2n},Fx_{2n})+d(x_{2n+1},Gx_{2n+1})]$
$\leq \eta_1d(x_{2n+1},x_{2n})+\eta_2d(x_{2n+1},x_{2n+2})+\eta_2d(x_{2n},x_{2n+1})$,
$(1-\eta_2)d(x_{2n+1},x_{2n+2}) \leq (\eta_1+\eta_2)d(x_{2n},x_{2n+1})$,
$d(x_{2n+1},x_{2n+2}) \leq [\frac{\eta_1+\eta_2}{(1-\eta_2)}]d(x_{2n},x_{2n+1}) \leq kd(x_{2n},x_{2n+1})$,
where $k = [\frac{\eta_1+\eta_2}{(1-\eta_2)}]$. As $\eta_1,\eta_2 \in [0,\frac{1}{3})$. So $\eta_1+2\eta_2 < 1 \Rightarrow \eta_1+\eta_2 < 1-\eta_2$.
This implies that $\frac{\eta_1+\eta_2}{(1-\eta_2)} < 1$, i.e. $k < 1$. So,

$d(x_{2n+1},x_{2n+2}) \leq kd(x_{2n},x_{2n+1})$
$\leq k^2 d(x_{2n-1},x_{2n})$.
Continuing this process, we obtain
$d(x_{2n+1},x_{2n+2}) \leq k^n d(x_0,x_1)$.
In general,
$d(x_n,x_{n+1}) \leq k^n d(x_0,x_1)$.
Now, let $m,n \in N$ with $m > n$
$d(x_n,x_m) \leq s[d(x_n,x_{n+1}) + d(x_{n+1},x_m)] - d(x_{n+1},x_{n+1})$
$\leq sd(x_n,x_{n+1}) + sd(x_{n+1},x_m)$
$\leq sd(x_n,x_{n+1}) + s\{s[d(x_{n+1},x_{n+2}) + d(x_{n+2},x_m)] - d(x_{n+2},x_{n+2})\}$
$\leq sd(x_n,x_{n+1}) + s^2 d(x_{n+1},x_{n+2}) + s^2 d(x_{n+2},x_m)$
$\leq sd(x_n,x_{n+1}) + s^2 d(x_{n+1},x_{n+2}) + s^3 d(x_{n+2},x_{n+3}) + \ldots + s^{m-n} d(x_{m-1},x_m)$
$\leq sk^n d(x_0,x_1) + s^2 k^{n+1} d(x_0,x_1) + s^3 k^{n+2} d(x_0,x_1) + \ldots + s^{m-n} k^{m-1} d(x_0,x_1)$
$d(x_n,x_m) \leq sk^n d(x_0,x_1) \frac{1-(sk)^m}{1-sk}$.
When $m,n \to \infty, lim_{n\to\infty} d(x_n,x_m) = 0$.
Hence, $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in $X$. Since $X$ is complete, $\{x_n\}$ converges to $y \in X$.
Now,
$d(y,Gy) \leq s[d(y,Fx_{2n}) + d(Fx_{2n},Gy)] - d(Fx_{2n},Fx_{2n})$
$\leq sd(y,Fx_{2n}) + sd(Fx_{2n},Gy)$
$\leq sd(y,x_{2n+1}) + s[\eta_1 d(x_{2n},y) + \eta_2 d(y,Gy) + \eta_2 d(x_{2n},Fx_{2n})]$
$\leq sd(y,x_{2n+1}) + s\eta_1 d(x_{2n},y) + s\eta_2 d(y,Gy) + s\eta_2 d(x_{2n},x_{2n+1})$
$(1-s\eta_2)d(y,Gy) \leq sd(y,x_{2n+1}) + s\eta_1 d(x_{2n},y) + s\eta_2 d(x_{2n},x_{2n+1})$
$d(y,Gy) \leq \frac{s}{1-s\eta_2} d(y,x_{2n+1}) + \frac{s\eta_1}{1-s\eta_2} d(a_{2n},y) + \frac{s\eta_2}{1-s\eta_2} d(x_{2n},x_{2n+1})$.
When $n \to \infty$,
$d(y,Gy) \leq \frac{s}{1-s\eta_2} d(y,y) + \frac{s\eta_1}{1-s\eta_2} d(y,y) + \frac{s\eta_2}{1-s\eta_2} d(y,y)$
$d(y,Gy) \leq \frac{s}{1-s\eta_2} d(y,Gy) + \frac{s\eta_1}{1-s\eta_2} d(y,Gy) + \frac{s\eta_2}{1-s\eta_2} d(y,Gy)$
$d(y,Gy) \leq 0$,
this implies that $y = Gy$.
Now,
$d(y,Fy) \leq s[d(y,Gx_{2n+1}) + d(Gx_{2n+1},Fy)] - d(Gx_{2n+1},Gx_{2n+1})$
$d(y,Fy) \leq sd(y,Gx_{2n+1}) + sd(Gx_{2n+1},Fy)$
$d(y,Fy) \leq sd(y,x_{2n+2}) + s\eta_1 d(x_{2n+1},y) + s\eta_2 d(x_{2n+1},Gx_{2n+1}) + s\eta_2 d(y,Fy)$
$(1-s\eta_2)d(y,Fy) \leq sd(y,x_{2n+2}) + s\eta_1 d(x_{2n+1},y) + s\eta_2 d(x_{2n+1},x_{2n+2})$
$d(y,Fy) \leq \frac{s}{1-s\eta_2} d(y,x_{2n+2}) + \frac{s\eta_1}{1-s\eta_2} d(x_{2n+1},y) + \frac{s\eta_2}{1-s\eta_2} d(x_{2n+1},x_{2n+2})$,
when $n \to \infty$,
$d(y,Fy) \leq \frac{s}{1-s\eta_2} d(y,y) + \frac{s\eta_1}{1-s\eta_2} d(y,y) + \frac{s\eta_2}{1-s\eta_2} d(y,y)$
$d(y,Fy) \leq \frac{s}{1-s\eta_2} d(y,Fy) + \frac{s\eta_1}{1-s\eta_2} d(y,Fy) + \frac{s\eta_2}{1-s\eta_2} d(y,Fy)$,
this implies that $y = Fy$.
Thus, $Gy = Fy = y$. Hence, $y$ is a common fixed point of $G$ and $F$.

Now, we will prove a fixed point theorem for multi-valued mappings in partial $b-$metric space:

**Theorem 3.3** Let $(X,d)$ be a complete partial $b-$metric space with constant $s \geq 1$. Let $G : X \to CB(X)$ is a multivalued mapping defined as

$H(Gx,Gy) \leq \alpha d(x,y), \forall x,y \in X \ \ and \ \ \alpha \in [0,1), s \geq 1$.

Then, there exists $x \in X$ such that $y \in Gy$.

**Proof.** Suppose that $x_0 \in X, Gx_0 \neq 0$ is closed and bounded subset of $X$. Also, let $x_1 \in Gx_0$, $Gx_1 \neq \phi$ be closed and bounded subset of $X$. By lemma 2.8, there exists $x_2 \in Ga_1$ such that

$d(x_1,x_2) \leq H(Gx_0,Gx_1) + \alpha$.

Now, $Gx_2 \neq \phi$ closed and bounded subsets of $X$, there exists $x_3 \in Gx_2$ such that

$$d(x_2,x_3) \leq H(Gx_1,Gx_2) + \alpha^2. \tag{3.3.1}$$

By using contraction condition,
$d(x_2,x_3) \leq d(x_1,x_2) + \alpha^2$,

$d(x_3,x_4) \leq H(Gx_2,Gx_3)+\alpha^3$
$\leq \alpha d(x_2,x_3)+\alpha^3.$
Using Equation (3.3.1), we have
$d(x_3,x_4) \leq \alpha[d(x_1,x_2)+\alpha^2]+\alpha^3$
$\leq \alpha^2 d(x_1,x_2)+2\alpha^3$
$\leq \alpha^2[H(Gx_0,Gx_1)+\alpha]+2\alpha^3$
$\leq \alpha^2[\alpha d(x_0,x_1)+\alpha]+2\alpha^3$
$\leq \alpha^3 d(x_0,x_1)+\alpha^3+2\alpha^3$
$\leq \alpha^3 d(x_0,x_1)+3\alpha^3.$
In general,
$d(x_n,x_{n+1}) \leq \alpha^n d(x_0,x_1)+n\alpha^n.$
For convenience, we set $d(x_n,x_{n+1})=d_n$, so the above result can be written as
$d_n \leq \alpha^n d_0+n\alpha^n.$
For $m,n \in N, m \geq n$, we have
$d(x_n,x_m) \leq s[d(x_n,x_{n+1})+d(x_{n+1},x_m)]-d(x_{n+1},x_{n+1})$
$\leq sd(x_n,x_{n+1})+sd(x_{n+1},x_m)$
$\leq sd(x_n,x_{n+1})+s\{s[d(x_{n+1},x_{n+2})+d(x_{n+2},x_m)]-d(x_{n+2},x_{n+2})\}$
$\leq sd(x_n,x_{n+1})+s^2d(x_{n+1},x_{n+2})+s^2d(x_{n+2},x_m)$
$\leq sd(x_n,x_{n+1})+s^2d(x_{n+1},x_{n+2})+s^3d(x_{n+2},x_{n+3})+\ldots+s^{m-n}d(x_{m-1},x_m)$
$\leq s\alpha^n d(x_0,x_1)+s^2\alpha^{n+1}d(x_0,x_1)+s^3\alpha^{n+2}d(x_0,x_1)+\ldots+s^{m-n}\alpha^{m-1}d(x_0,x_1)$
$+sn\alpha^n+s(n+1)\alpha^{n+1}+s^2(n+2)\alpha^{n+2}+\ldots+s^{m-n}(m-1)\alpha^{m-1}$
$\leq s\alpha^n d(x_0,x_1)[1+s\alpha+(s\alpha)^2+\ldots s^{m-n-1}\alpha^{m-n-1}]+\sum_{i=n}^{m-1} is^{i-n+1}\alpha^i$
$\leq s\alpha^n d(x_0,x_1)[\frac{1-(s\alpha)^{m-n-1}}{1-s\alpha}]+\sum_{i=n}^{m-1} is^{i-n+1}\alpha^i.$
In the limiting case when $m,n \to \infty$,
$d(x_n,x_m)=0,$
this implies that $\{x_n\}$ is a Cauchy sequence in $X$, the completeness of $X$ implies that there exists $y \in X$ such that,
$x_n \to y.$
Now we will prove that $y$ is a fixed point of $G$.
$d(y,Gy) \leq s[d(y,x_n)+d(x_n,Gy)]-d(x_n,x_n) \leq sd(y,x_n)+sd(x_n,Gy).$
By Lemma 2.8,
$d(y,Gy) \leq d(y,x_n)+sH(Gx_{n-1},Gy)$
$\leq sd(y,x_n)+s\alpha d(x_{n-1},y).$
In the limiting case when $n \to \infty$,
$d(y,Gy) \leq sd(y,y)+s\alpha d(y,y)$
$\leq sd(y,Gy)+s\alpha d(y,Gy)$
$(1-s-s\alpha)d(y,Gy) \leq 0.$
This implies that $y \in Gy$. Hence, $y$ is a fixed point of $G$.

## 4 Conclusion

The use of fixed point techniques is both appealing and extremely helpful. This theory has potential applications to discrete dynamics for set-valued operators, functional inclusions, optimization theory, fractal graphics, and other nonlinear functional analysis domains. We have generalized and demonstrated fixed point and common fixed point theorems for single-valued mappings obeying Ciric type contractions in partial $b-$metric space. One fixed point theorem for multi-valued mappings with Nadler's type contractions has also been established in these spaces. Future studies could find these generalizations to be helpful.

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