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# M/M/1/K Loss and Delay Interdependent Queueing Model with Vacation and Controllable Arrival Rates

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## Abstract

**Objectives:** In this study, we assume that the vacation is taken while there are no consumers in the queue. There are several servicemen who will take the synchronous multiple vacations in the system. **Methods:** Assumed some loss and delay in consumers (Elective and emergency) and solve the steady-state probability equations using recursive approach and acquired some obvious iterative expressions. **Findings:** Carried out some numerical analysis using MATLAB and investigated the movement of $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$, $(\mu-\varepsilon)E\left(W_{A_1}\right)$ and $(\mu-\varepsilon)E\left(W_{B_1}\right)$ through graph. Further, $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$, $(\mu-\varepsilon)E\left(W_{A_1}\right)$ and $(\mu-\varepsilon)E\left(W_{B_1}\right)$ increase when $J_1$ & $J_0$ increases; decrease when M increases. Additionally, when L increases $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$ remains constant and $(\mu-\varepsilon)E\left(W_{A_1}\right)$, $(\mu-\varepsilon)E\left(W_{B_1}\right)$ increase. **Novelty:** Expanded the preceding models in this study by including vacations and performing the numerical analysis. Using vacation with controllable arrival rates in an optimal way in order to benefit both the server and the customer will minimise waiting time and provide the most feasible, affordable service to the consumer.

**Keywords:** Markovian Queueing System; Vacation; Loss and Delay; Finite Capacity; Interdependent Arrival and Service Rates; Varying Arrival Rates; Bivariate Poisson Process

## 1 Introduction

Jain (1998) [1] examined the finite population loss and delay queuing system without the passing idea in prior work. M. Thiagarajan and A. Srinivasan's analysis of the M/M/c/K loss and delay interdependent queueing model with adjustable arrival rates and no passing was published in Thiagarajan and Srinivasan (2011) [2]. In actual life, the loss and delay issue happen frequently. This is a development of the M/M/1/K Interdependent Queueing Model with Controllable Arrival Rates and Vacation. S. P. Subhapriya and M. Thiagarajan discuss developments and applications in mathematics. (2022) [3].

A "vacation period" is when a server starts performing other uninterruptible work when it is idle. We recommend reading Doshi (1986), Ke et al. (2010) and Shweta Upadhyaya [4–6] for an in-depth and extensive examination of vacation queueing systems. The analysis of an M/M/1/N queue with working breakdowns and vacations was done by B. Deepa and K. Kalidass[7]. There have also been numerous more models that are similar[8].

In this study, we take into account a server vacation scenario with an M/M/1/K loss and delay queueing model. For different arrival rates and independent service processes, the steady state probabilities are determined, and the average waiting periods for two customer kinds (elective and emergency) are produced. Customers that choose to wait patiently to receive the services they require. However, if an emergency consumer finds the server busy and so lost, they quit the system.

## 2 Description of the Model

Consider finite capacity, loss and delay and vacation queuing system with following assumptions:

The arrival process and the service process are $(X_1(t)\}$ , $\{X_2(t)\}$ respectively are correlated and follow a bivariate Poisson process given by

$$P[X_1(t) = x_1 \text{ , } X_2(t) = x_2] = \frac{e^{-(\lambda_i+\mu-\varepsilon)t} \sum_{j=0}^{min(x_1,x_2)} (\varepsilon t)^j [(\lambda_i - \varepsilon)t]^{x_1-j} [(\mu-\varepsilon)t]^{x_2-j}}{j!(x_1-j)!(x_2-j)!} \tag{1}$$

Where,$x_1, x_2 = 0, 1, 2\ldots; \lambda_{01},\ \lambda_{02}, \lambda_{11},\ \mu > 0,\ 0 \le \varepsilon < min(\lambda_{ij}, \mu)$.

Parameters $\lambda_{01},\ \lambda_{02}, \lambda_{11},\ \mu$ and $\varepsilon$ are mean arrival rate of elective consumers when the system is in faster rate of arrivals, mean arrival rate of emergency consumers when the system is in faster rate of arrivals, mean arrival rate of elective consumers when the system is in slower rate of arrivals, mean service rate of consumers of type B and co-variance between arrival and service processes respectively.

$$\lambda_n = \begin{cases} \lambda_{0j}; & 0 \le n < 1,\ j = 1,2 \\ \lambda_{01}; & 1 \le n \le R-1 \\ \lambda_{11}; & r+1 \le n \le K \end{cases}$$

$P_{0,n}$ and $P_{1,n}$ are the probability at vacation and at normal state respectively.

n=0,1,…,r-1,r,r+1,…,R-1,R,R+1,…,K-1,K.

## 3 Steady State Equations

We observe that only $P_{0,n}(0)$ and $P_{1,n}(0)$ exists when n=0,1,….r-1,r; $P_{0,n}(0)$, $P_{1,n}(0)$, $P_{0,n}(1)$ and $P_{1,n}(1)$ exists when n=r+1,r+2,…,R-2,R-1; only $P_{0,n}(1)$ and $P_{1,n}(1)$ exists when n=R,R+1,…,K. Further $P_{0,n}(0) = P_{1,n}(0) = P_{0,n}(1) = P_{1,n}(1) = 0$ when n > K.

$$(\lambda_0 - 2\varepsilon) P_{0,0}(0) = (\mu - \varepsilon) P_{1,1}(0); \tag{2}$$

$$(\lambda_{01} + v - \varepsilon) P_{0,n}(0) = (\lambda_{01} - \varepsilon) P_{0,n-1}(0); (1 \le n \le R-1) \tag{3}$$

$$(\lambda_{11} + v - \varepsilon) P_{0,r+1}(1) = 0; \tag{4}$$

$$(\lambda_{11} + v - \varepsilon) P_{0,n}(1) = (\lambda_{11} - \varepsilon) P_{0,n-1}(1); (r+2 \le n \le R-1) \tag{5}$$

$$(\lambda_{11} + v - \varepsilon) P_{0,R}(1) = (\lambda_{11} - \varepsilon) P_{0,R-1}(1) + (\lambda_{01} - \varepsilon) P_{0,R-1}(0) \tag{6}$$

$$(\lambda_{11} + v - \varepsilon) P_{0,n}(1) = (\lambda_{11} - \varepsilon) P_{0,n-1}(1); (R+1 \le n \le K) \tag{7}$$

$$(\lambda_{01}+\mu-2\varepsilon)P_{1,n}(0)=(\lambda_{01}-\varepsilon)P_{1,n-1}(0)+(\mu-\varepsilon)P_{1,n+1}(0)+vP_{0,n}(0);(1\leq n\leq r-1) \tag{8}$$

$$(\lambda_{01}+\mu-2\varepsilon)P_{1,r}(0)=(\lambda_{01}-\varepsilon)P_{1,r-1}(0)+(\mu-\varepsilon)P_{1,r+1}(0)+(\mu-\varepsilon)P_{1,r+1}(1)+vP_{0,r}(0); \tag{9}$$

$$(\lambda_{01}+\mu-2\varepsilon)P_{1,n}(0)=(\lambda_{01}-\varepsilon)P_{1,n-1}(0)+(\mu-\varepsilon)P_{1,n+1}(0)+vP_{0,n}(0);(r+1\leq n\leq R-2) \tag{10}$$

$$(\lambda_{01}+\mu-2\varepsilon)P_{1,R-1}(0)=(\lambda_{01}-\varepsilon)P_{1,R-2}(0)+vP_{0,R-1}(0); \tag{11}$$

$$(\lambda_{11}+\mu-2\varepsilon)P_{1,r+1}(1)=(\mu-\varepsilon)P_{1,r+2}(1)+vP_{0,r+1}(1); \tag{12}$$

$$(\lambda_{11}+\mu-2\varepsilon)P_{1,n}(1)=(\mu-\varepsilon)P_{1,n+1}(1)+(\lambda_{11}-\varepsilon)P_{1,n-1}(1)+vP_{0,n}(1);(r+2\leq n\leq R-1) \tag{13}$$

$$(\lambda_{11}+\mu-2\varepsilon)P_{1,R}(1)=(\mu-\varepsilon)P_{1,R+1}(1)+(\lambda_{11}-\varepsilon)P_{1,R-1}(1)+(\lambda_{01}-\varepsilon)P_{1,R-1}(0)+vP_{0,R}(1); \tag{14}$$

$$(\lambda_{11}+\mu-2\varepsilon)P_{1,n}(1)=(\mu-\varepsilon)P_{1,n+1}(1)+(\lambda_{11}-\varepsilon)P_{1,n-1}(1)+vP_{0,n}(1);(R+1\leq n\leq K-1) \tag{15}$$

$$(\mu-\varepsilon)P_{1,K}(1)=(\lambda_{11}-\varepsilon)P_{1,K-1}(1)+vP_{0,K}(1); \tag{16}$$

Let

$$J_1=\frac{\lambda_{01}-\varepsilon}{\mu-\varepsilon},\ J_2=\frac{\lambda_{02}-\varepsilon}{\mu-\varepsilon}\ ,\ L=\frac{\lambda_{11}-\varepsilon}{\mu-\varepsilon}\ ,M=\frac{v}{\mu-\varepsilon}\ ,T=\frac{L}{L+M}$$

$J_0=J_1+J_2=\frac{\lambda_0-2\varepsilon}{\mu-\varepsilon}$, where $\lambda_0=\lambda_{01}+\lambda_{02} and\ Q=\frac{J_1}{J_1+L}$

From Equation (2)

$$P_{1,1}(0)=J_0P_{0,0}(0) \tag{17}$$

From Equation (3)

$$P_{0,n}(0)=Q^nP_{0,0}(0);(1\leq n\leq R-1) \tag{18}$$

From Equations (4) and (5)

$$P_{0,n}(1)=0;(r+1\leq n\leq R-1) \tag{19}$$

From Equation (6)

$$P_{0,R}(1)=\frac{J_1}{L+M}Q^{R-1}P_{0,0}(0)=SP_{0,0}(0) \tag{20}$$

From Equation (7)

$$P_{0,n}(1) = T^{n-R} S P_{0,0}(0); (R+1 \le n \le K) \tag{21}$$

From Equation (8)

$$\begin{aligned} &P_{0,n}(0) = [\left(1 + J_1 + \cdots + J_1^{(n-1)}\right) J_0 - \left(1 + J_1 + \cdots + J_1^{(n-2)}\right) \\ &MQ - \left(1 + J_1 + \cdots + J_1^{(n-3)}\right) MQ^2 - \cdots - MQ^{(n-1)}] P_{0,0}(0); (2 \le n \le r) \end{aligned} \tag{22}$$

From Equation (9)

$$\begin{aligned} &P_{1,r+1}(0) = [\left(1 + J_1 + \cdots + J_1^{(r)}\right) J_0 - \left(1 + J_1 + \cdots + J_1^{(r-1)}\right) MQ - \left(1 + J_1 + \cdots + J_1^{(r-2)}\right) \\ &MQ^2 - \cdots - MQ^{(r)}] P_{0,0}(0) - P_{1,r+1}(1); (2 \le n \le r) \end{aligned} \tag{23}$$

From Equation (10)

$$\begin{aligned} &P_{1,n}(0) = [\left(1 + J_1 + \cdots + J_1^{(n-1)}\right) J_0 - \left(1 + J_1 + \cdots + J_1^{(n-2)}\right) MQ \\ &- \left(1 + J_1 + \cdots + J_1^{(n-3)}\right) MQ^2 - \cdots - MQ^{(n-1)}] P_{0,0}(0) \\ &- \left(1 + J_1 + \cdots + J_1^{(n-r-1)}\right) P_{1,r+1}(1); (r+2 \le n \le R-1) \end{aligned} \tag{24}$$

From Equation (11)

$$\begin{aligned} P_{1,r+1}(1) &= \frac{\begin{array}{c} [(1+J_1)\left\{\begin{bmatrix} (1 + J_1 + \cdots + J_1^{(R-2)}) J_0 - (1 + J_1 + \cdots + J_1^{(R-3)}) MQ \\ -(1 + J_1 + \cdots + J_1^{(R-4)}) MQ^2 - \cdots - MQ^{(R-2)} \end{bmatrix}\right\} \\ -J_1 \{[(1 + J_1 + \cdots + J_1^{(R-3)}) J_0 - (1 + J_1 + \cdots + J_1^{(R-4)}) MQ \\ -(1 + J_1 + \cdots + J_1^{(R-5)}) MQ^2 - \cdots - MQ^{(R-3)}]\} - MQ^{(R-1)}] \end{array}}{\left(1 + J_1 + \cdots + J_1^{(R-r-1)}\right)} P_{0,0}(0) \\ &= F P_{0,0}(0) \end{aligned} \tag{25}$$

From Equations (12) and (13)

$$P_{1,n}(1) = \left(1 + L + \cdots + L^{(n-r-1)}\right) F P_{0,0}(0); (r+2 \le n \le R) \tag{26}$$

From Equations (14), (15) and (16)

$$P_{1,n}(1) = (B+1) P_{1,n-1}(1) - B P_{1,n-2}(1) - M P_{0,n-1}(1); (n = R+1, R+2, \ldots K)\ P_{0,0}(0); (R+1 \le n \le K) \tag{27}$$

## 4 Characteristics of the Model

$$P(0) = \sum_{n=0}^{K} P_{1,n}(0)$$

P(0) exists only when n=1,2,….r-1,r,r+1,….R-1, we get

$$P(0) = \textstyle\sum_{n=1}^{R-1} P_{0,n}(0) + \sum_{n=1}^{r} P_{1,n}(0) + \sum_{n=r+1}^{R-1} P_{1,n}(0) \tag{28}$$

Now,

$$P(1) = \sum_{n=0}^{K} P_{1,n}(1)$$

P(1) exists only when n=r+1,r+2….L

$$P(1) = \sum_{n=r+1}^{K} P_{0,n}(1) + \sum_{n=r+1}^{R} P_{1,n}(1) + \sum_{n=R+1}^{K} P_{1,n}(1) \tag{29}$$

The system is empty can be calculated from the normalizing condition

$$P(0) + P(1) = 1 \tag{30}$$

The expected waiting time for the consumers of type A when the system is in faster, slower rate of arrivals, consumers of type B when the system is in faster, slower rate of arrivals are given by $E\left(W_{A_0}\right)$, $E\left(W_{A_1}\right)$, $E\left(W_{B_0}\right)$ and $E\left(W_{B_1}\right)$ respectively.

$$E\left(W_{A_0}\right) = \frac{1}{\mu-\varepsilon}\{[\sum_{n=1}^{r}(P_{0,n}(0)+P_{1,n}(0)) + \sum_{n=r+1}^{R-1}(P_{0,n}(0)+P_{1,n}(0))](n)\} \tag{31}$$

$$E\left(W_{A_1}\right) = \frac{1}{\mu-\varepsilon}\{[\sum_{n=r+1}^{R-1}(P_{0,n}(1)+P_{1,n}(1)) + \sum_{n=R}^{K}(P_{0,n}(1)+P_{1,n}(1))](n)\} \tag{32}$$

$$E\left(W_{B_0}\right) = \frac{1}{\mu-\varepsilon}\{P_{00}(0) + [\sum_{n=1}^{r}(P_{0,n}(0)+P_{1,n}(0)) + \sum_{n=r+1}^{R-1}(P_{0,n}(0)+P_{1,n}(0))](n+1)\} \tag{33}$$

$$E\left(W_{B_1}\right) = \frac{1}{\mu-\varepsilon}\{[\sum_{n=r+1}^{R-1}(P_{0,n}(1)+P_{1,n}(1)) + \sum_{n=R}^{K}(P_{0,n}(1)+P_{1,n}(1))](n+1)\} \tag{34}$$

The difference between the expected waiting time of consumers of type A & B when the system is in faster and slower rate of arrivals are

$D_0 = E\left(W_{B_0}\right) - E\left(W_{A_0}\right)$ and $D_1 = E\left(W_{B_1}\right) - E\left(W_{A_1}\right)$ respectively

## 5 Numerical Illustrations

For various values of $J_1$, $J_0$, $L$ and M the values of $P_{0,0}(0)$, $P(0)$, $P(1)$, $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{A_1}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$ and $(\mu-\varepsilon)E\left(W_{B_1}\right)$ are computed.

**Table 1. For single server by assuming r=5, R=8, K=10**

| $J_1$ | $J_0$ | L | M | $P_{0,0}(0)$ | P(0) | P(1) | $(\mu-\varepsilon)E(W_{A_0})$ | $(\mu-\varepsilon)E(W_{A_1})$ | $(\mu-\varepsilon)E(W_{B_0})$ | $(\mu-\varepsilon)E(W_{B_1})$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.8 | 1.4 | 0.4 | 4 | 0.0522 | 0.6441 | 0.3553 | 345.76 | 272.23 | 359.63 | 279.03 |
| 1.2 | 1.4 | 0.4 | 4 | 0.0313 | 0.6268 | 0.3732 | 561.01 | 477.25 | 581.95 | 489.18 |
| 1.6 | 1.4 | 0.4 | 4 | 0.0139 | 0.5073 | 0.4927 | 1020.6 | 1416 | 1057.4 | 1451.4 |
| 1.8 | 1.4 | 0.4 | 4 | 0.0089 | 0.4466 | 0.5534 | 1398.9 | 2476.3 | 14449.1 | 2538.2 |
| 1.2 | 1.1 | 0.3 | 3.6 | 0.0622 | 0.7409 | 0.2591 | 333.65 | 166.67 | 347.37 | 170.83 |
| 1.2 | 1.3 | 0.3 | 3.6 | 0.0385 | 0.6742 | 0.3258 | 490.4 | 338.53 | 509.03 | 346.99 |
| 1.2 | 1.4 | 0.3 | 3.6 | 0.0440 | 0.6829 | 0.3170 | 434.31 | 288.03 | 451.1 | 295.24 |
| 1.2 | 1.6 | 0.3 | 3.6 | 0.0288 | 0.6026 | 0.3974 | 585.54 | 551.7 | 607.29 | 565.5 |
| 2.5 | 2 | 0.5 | 10 | 0.0011 | 0.2655 | 0.7345 | 6630 | 26200 | 6866.8 | 26855 |
| 2.5 | 2 | 0.6 | 10 | 0.0010 | 0.2379 | 0.7621 | 6630 | 30346 | 6866.8 | 31105 |
| 2.5 | 2 | 0.9 | 10 | 0.0007 | 0.1631 | 0.8360 | 6630 | 48285 | 6866.8 | 49492 |
| 2.5 | 2 | 1 | 10 | 0.0006 | 0.1434 | 0.8566 | 6630 | 56569 | 6866.8 | 57983 |
| 1.3 | 1 | 0.8 | 0.6 | 0.0231 | 0.5075 | 0.4925 | 613.75 | 850.86 | 636.34 | 872.14 |
| 1.3 | 1 | 0.8 | 1.3 | 0.0355 | 0.5545 | 0.4455 | 437.57 | 502.14 | 454.23 | 514.69 |
| 1.3 | 1 | 0.8 | 1.9 | 0.0441 | 0.5814 | 0.4187 | 369.22 | 379.71 | 383.69 | 389.2 |
| 1.3 | 1 | 0.8 | 2.5 | 0.0513 | 0.6025 | 0.3975 | 328.76 | 309.87 | 341.99 | 317.62 |

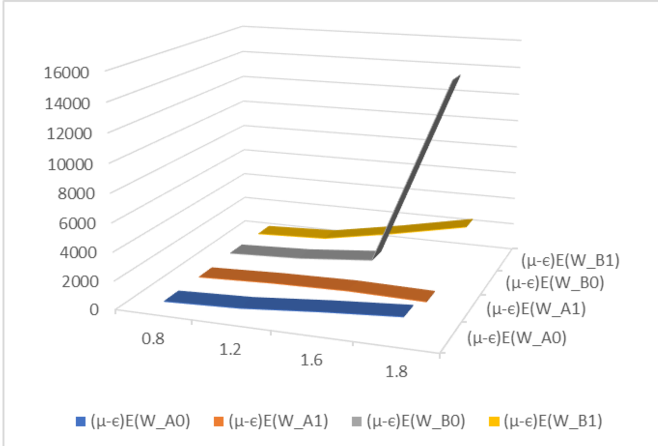


**Fig 1. By varying $J_1$ and keeping other parameters fixed**

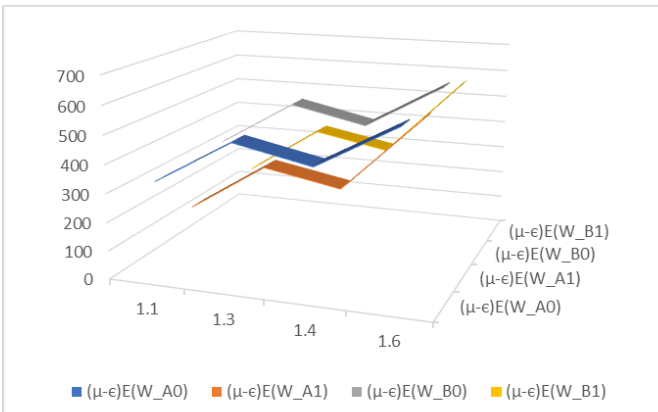


**Fig 2. By varying $J_0$ and keeping other parameters fixed**

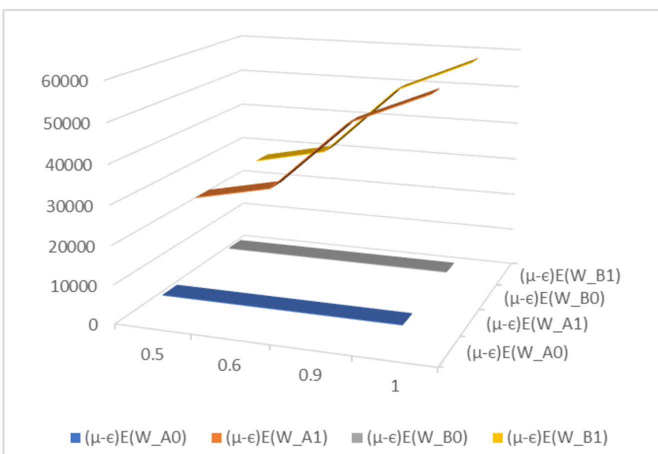


**Fig 3. By varying L and keeping other parameters fixed**

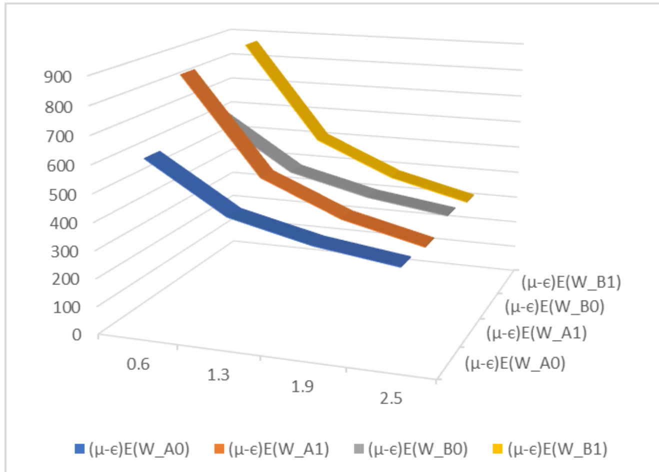


**Fig 4. By varying M and keeping other parameters fixed**

## 6 Conclusion

It is observed from the table that

- When $J_1$ increases and the other parameters are kept fixed, $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{A_1}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$ and $(\mu-\varepsilon)E\left(W_{B_1}\right)$ increase.
- When $J_0$ increases and the other parameters are kept fixed, $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{A_1}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$ and $(\mu-\varepsilon)E\left(W_{B_1}\right)$ increase.
- When L increases and the other parameters are kept fixed $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$ remains constant and $(\mu-\varepsilon)E\left(W_{A_1}\right)$, $(\mu-\varepsilon)E\left(W_{B_1}\right)$ increase.
- When M increases and the other parameters are kept fixed, $(\mu-\varepsilon)E\left(W_{A_0}\right)$, $(\mu-\varepsilon)E\left(W_{A_1}\right)$, $(\mu-\varepsilon)E\left(W_{B_0}\right)$ and $(\mu-\varepsilon)E\left(W_{B_1}\right)$ decrease.

The model includes the earlier models as particular cases. For example, when $\nu = 0$, this model reduces to the M/M/c/K loss and delay interdependent queueing model with controllable arrival rates and no passing[5]. When $\lambda_0$ tends to $\lambda_1$, $\varepsilon = 0$ and $\nu = 0$, this model reduces to the Finite population loss and delay queueing system with no passing[1]. When $\lambda_0$ tends to $\lambda_1$ and $\varepsilon = 0$, this model reduces to the conventional model.

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## References

1) Jain M. Finite Population Loss And Delay Queueing System With Nopassing. *OPSEARCH*. 1998;35(3):261–276. Available from: https://doi.org/10.1007/BF03399219.
2) Thiagarajan M, Srinivasan A. M/M/c/K loss and delay interdependent queueing model with controllable arrival rates and no passing. *Sankhya B*. 2011;73(2):316–330. Available from: https://doi.org/10.1007/s13571-011-0027-7.
3) Subhapriya SP, Thiagarajan M. M/M/1/K Interdependent Queueing Model with Vacation and Controllable Arrival Rates. *Advances and Applications in Mathematical Sciences*. 2022;21(8):4671–4679. Available from: https://www.mililink.com/upload/article/1274572531aams_vol_218_june_2022_a41_p4671-4679_s._p._subhapriya_and_m._thiagarajan.pdf.
4) Doshi BT. Queueing systems with vacations ? A survey. *Queueing Systems*. 1986;1(1):29–66. Available from: https://doi.org/10.1007/BF01149327.
5) Ke JC, Wu CH, Zhang Z. Recent Developments in Vacation queueing models: a short survey. *International Journal of Operations Research*. 2010;7(4):3–8. Available from: http://www.orstw.org.tw/ijor/vol7no4/2-Vol.%207,%20No.4%20pp.3-8.pdf.
6) Upadhyaya S. Queueing systems with vacation: an overview. *International Journal of Mathematics in Operational Research*. 2016;9(2):167–213. Available from: https://doi.org/10.1504/IJMOR.2016.077997.
7) Deepa B, Kalidass K. An M/M/1/N Queue with Working Breakdowns and Vacations. *International Journal of Pure and Applied Mathematics*. 2018;119(10):859–873. Available from: https://acadpubl.eu/jsi/2018-119-10/articles/10a/76.pdf.
8) Manoharan P, Ashok A. Impatient customers in an M/M/1 Queue with working vacation and Multiple vacation. *International Journal of Future generation communication and Networking*. 2000;13(3):2268–2272. Available from: http://sersc.org/journals/index.php/IJFGCN/article/view/28829/16069.