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Reliability Analysis of the Shaft Subjected to Twisting Moment and Bending Moment for The Exponential and Weibull Distributed Strength and Stress

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Abstract

Objectives: The purpose of the present work is to design a shaft subjected to twisting moment, bending moment and combined twisting and bending moment by determine the reliability of the shaft. **Methods:** Probabilistic approach is considered to find the lifetime of shaft by taking stress as random variable. Exponential and Weibull distributions are used to find lifetime of the equipment stress is considered as exponential and Weibull random variable. When the shaft is subjected to combined twisting and bending moment, the stress is found by using the two theories: (i) The maximum shear stress theory is used for ductile materials such as mild steel. (ii) The maximum normal stress theory is used for brittle materials such as cast iron. **Findings:** Reliability of the shaft is derived subjected to twisting and bending moments. The reliability is computed and compared for changing of the twisting moment, bending moment and diameter of the shaft. **Novelty:** To design a shaft by using reliability theory and to find reliability of the shaft by using the exponential and Weibull distribution is the novel idea.

Keywords: Reliability; Weibull distribution; Shaft; Twisting moment; Bending moment; Combined twisting and bending moment; Exponential distribution; Maximum normal stress theory; Maximum shear stress theory

1 Introduction

Shafts are provided for supporting the rotating part such as pulleys and gears, and they are supported by bearings which rest in the rigid machine housing. The gears and pulley which are placed on the shaft helps in transmitting the motion. In general, shafts are subjected to a combination of torsional and bending stresses. Shafts have always circular cross-sections and can be either hollow or solid. Adekunle A. A., et al.[(1)] studied development of CAD software for shaft under various loading conditions. Dr Edward E. Osakue et al.[2] studied fatigue shaft design verification for bending and torsion. Dr Edward E. Osakue et al.[3] studied the probabilistic fatigue design of shaft for bending and torsion. Fatima K. et al.[4] discussed statistical properties of exponential Rayleigh distribution and its application to medical sciences and engineering. Frydrysek K. et al.[5] studied performance-based design applied for a shaft subjected to combined stress. Gowtham et al.[6] studied drive shaft design and analysis. Kamboh, M. S., et al.[7] discussed the design and analysis of the drive shaft with a critical review of advanced composite materials and the root causes of shaft failure. Misra, A., et al.[8] studied reliability analysis of drilled shaft behavior using the finite difference method and Monte Carlo simulation. Nadarajah S.[9] studied reliability for lifetime distributions mathematical and computer modelling. Nayek et al.[10] studied reliability approximation for a solid shaft under a gamma setup. Patel, B., et al.[11] studied a critical review of the design of a shaft with multiple discontinuities and combined loadings. Pina-Monarrez, M.R.[12] discussed Weibull stress distribution for static mechanical stress and its stress/strength analysis. T. S. Uma Maheswari et al.[13] obtained reliability analysis of unsymmetrical columns subjected to eccentric load for stress follows exponential distribution. Villa-Covarrubias, B., et al.[14] discussed the probabilistic method to determine the shaft diameter and designed reliability. The exponential distribution is used in life testing materials and reliability theory. Weibull distribution is used for non-linear failure rate models. Exponential distribution as the special case of Weibull distribution.

2 Methodology

The probability density function for exponentially distributed strength ξ and for exponentially stress χ is given by[15]

$$f_{\xi}(\xi) = \lambda_{\xi} e^{-\lambda_{\xi} \times \xi} \quad for\, 0 \le \xi < \infty$$

$$f_{\chi}(\chi) = \lambda_{\chi} e^{-\lambda_{\chi} \times \chi} \quad for\, 0 \le \chi < \infty$$

Reliability of the stress-strength system is the probability that the strength ξ is greater than the stress χ applied on it is given by[15]

$$R_1(t) = \int_0^{\infty} f_{\chi}(\chi) \left[\int_{\chi}^{\infty} f_{\xi}(\xi)\, d\xi \right] d\chi = \int_0^{\infty} \lambda_{\chi} \times exp\left[-\left(\lambda_{\xi} + \lambda_{\chi}\right)\chi\right] d\chi$$

Then the reliability is

$$R_1(t) = \frac{\bar{\xi}}{\bar{\xi} + \bar{\chi}} \tag{1}$$

where the mean strength $\bar{\xi} = \frac{1}{\lambda_{\xi}}$ and the mean stress $\bar{\chi} = \frac{1}{\lambda_{\chi}}$

The probability density function of a Weibull random variable is given by[12]

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\}$$

where β is the shape parameter and η is the scale parameter.

The Weibull reliability function is given by[12]

$$R(t) = exp\left\{-\left(\frac{t}{\eta}\right)^{\beta}\right\} \tag{2}$$

The Weibull stress and strength analysis to estimate the reliability index $R_2(t)$ of an element that is operated under variable stress is given by

$$R_2(t) = \int_0^{\infty} f(\chi)\left\{\int_{\chi}^{\infty} f(\xi)\, d\xi\right\} d\chi \tag{3}$$

In Equation (3), $f(\chi)$ is the Weibull stress distribution and $f(\xi)$ is the Weibull strength distribution that represents the strength behavior of the used material.

By replacing Equation (2) into Equation (3), the stress and strength $R_2(t)$ index is given by

$$R_2(t) = \int_0^{\infty} \frac{\beta}{\eta}\left(\frac{\chi}{\eta}\right)^{\beta-1} exp\left\{-\left(\frac{\chi}{\eta}\right)^{\beta}\right\}\left[\int_{\chi}^{\infty} \frac{\beta}{\eta_{\xi}}\left(\frac{\xi}{\eta_{\xi}}\right)^{\beta-1} exp\left\{-\left(\frac{\xi}{\eta_{\xi}}\right)\right\} d\xi\right] d\chi$$

By solving the integral with respect to the strength variable ξ and after some algebra, the $R_2(t)$ index is estimated as[12]

$$R_2(t) = \frac{\eta_{\xi}^{\beta}}{\eta_{\xi}^{\beta} + \eta^{\beta}} \tag{4}$$

2.1Shaft subjected to twisting m oment only

If the shaft is subjected to a twisting moment only, then torsion equation is given by[16].

$$\frac{T}{J} = \frac{\sigma_t}{r}$$

where T = Twisting moment acting upon the shaft

J = Polar moment of inertia of the shaft about the axis of rotation

σ_t = Torsional shear stress and

r = Distance from neutral axis to the outer-most fibre

$= \frac{d}{2}$ where d is the diameter of the shaft

For round solid shaft, polar moment of inertia is given by[16]

$$J = \frac{\pi}{32} \times d^4$$

From the torsion equation twisting moment is

$$T = \frac{\pi}{16} \times \sigma_t \times d^3 \; and$$

The shear stress due to twisting moment is the mean stress

$$\bar{\chi}_t = \sigma_t = \frac{16 \times T}{\pi \times d^3}$$

Then reliability of the shaft subjected to twisting moment for exponentially distributed strength and stress is

$$R_{1t} = \frac{\bar{\xi}}{\bar{\xi} + \left(\frac{16 \times T}{\pi \times d^3}\right)} \tag{5}$$

And reliability of the shaft subjected to twisting moment for Weibull distributed strength and stress is

$$R_{2t} = \frac{\eta_{\xi}^{\beta}}{\eta_{\xi}^{\beta} + \left(\frac{16 \times T}{\pi \times d^3}\right)^{\beta}} \tag{6}$$

2.2 Shaft subjected to bending moment only

If the shaft is subjected to a bending moment only, then bending equation is given by[16]

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

where M = Bending moment
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation
σ_b = Bending stress and
y = Distance from neutral axis to the outer-most fibre
For solid shaft, moment of inertia is given by[16]

$$I = \frac{\pi}{64} \times d^4 \ \ and \ y = \frac{d}{2}$$

From the bending equation bending moment is given by[16]

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

The shear stress due to bending moment is the mean stress

$$\bar{\chi}_b = \sigma_b = \frac{32 \times M}{\pi \times d^3}$$

Then reliability of the shaft subjected to bending moment for exponentially distributed strength and stress is

$$R_{1b} = \frac{\bar{\xi}}{\bar{\xi} + \left(\frac{32 \times M}{\pi \times d^3}\right)} \tag{7}$$

And reliability of the shaft subjected to twisting moment for Weibull distributed strength and stress is

$$R_{2b} = \frac{\eta_\xi^\beta}{\eta_\xi^\beta + \left(\frac{32 \times M}{\pi \times d^3}\right)^\beta} \tag{8}$$

2.3 Shaft subjected to combined twisting moment and bending moment

When the shaft is subjected to a combined twisting moment and bending moment, then the shaft must be designed based on the two moments simultaneously.

(i) According to maximum shear stress theory, the maximum shear stress in the shaft is

$$\sigma_{t\,max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4(\sigma_t)^2}$$

where σ_t = shear stress induced due to twisting moment
σ_b = bending stress induced due to bending moment then

$$\sigma_{t\,max} = \frac{16}{\pi \times d^3}\left[\sqrt{M^2 + T^2}\right]$$

The expression $\sqrt{M^2 + T^2}$ is known as equivalent twisting moment and is denoted by T_e.

The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces same shear stress (σ_t) as the actual twisting moment.

By limiting the maximum shear stress $(\sigma_{t_{max}})$ equal to the allowable shear stress (σ_{ts}) for the material is the mean stress

$$\bar{\chi}_{ts} = \sigma_{ts} = \frac{16 \times T_e}{\pi \times d^3}$$

Then reliability of the shaft subjected to combined twisting and bending moment for exponentially distributed strength and stress is

$$R_{1ts} = \frac{\bar{\xi}}{\bar{\xi} + \left(\frac{16 \times T_e}{\pi \times d^3}\right)} \tag{9}$$

And reliability of the shaft subjected to twisting moment for Weibull distributed strength and stress is

$$R_{2ts} = \frac{\eta_{\xi}^{\beta}}{\eta_{\xi}^{\beta} + \left(\frac{16 \times T_e}{\pi \times d^3}\right)^{\beta}} \tag{10}$$

(ii) According to maximum normal stress theory, the maximum normal stress in the shaft is

$$\sigma_{bmax} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4(\sigma_t)^2}$$

where σ_t = shear stress induced due to twisting moment
σ_b = bending stress induced due to bending moment then

$$\sigma_{bmax} = \frac{32}{\pi \times d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

The expression $\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$ is known as equivalent bending moment and is denoted by M_e.

The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.

By limiting the maximum normal stress $(\sigma_{b_{max}})$ equal to the allowable bending stress (σ_{bn}) for the material is the mean stress

$$\bar{\chi}_{bn} = \sigma_{bn} = \frac{32 \times M_e}{\pi \times d^3}$$

Then reliability of the shaft subjected to combined twisting and bending moment for exponentially distributed strength and stress is

$$R_{1bn} = \frac{\bar{\xi}}{\bar{\xi} + \left(\frac{32 \times M_e}{\pi \times d^3}\right)} \tag{11}$$

And reliability of the shaft subjected to twisting moment for Weibull distributed strength and stress is

$$R_{2bn} = \frac{\eta_{\xi}^{\beta}}{\eta_{\xi}^{\beta} + \left(\frac{32 \times M_e}{\pi \times d^3}\right)^{\beta}} \tag{12}$$

3 Results and Discussion

3.1 Shaft subjected to twisting moment only

The reliability of the shaft increases when the diameter increases and reliability decreases when twisting moment increase.

Table 1. T=100000 N-mm, $\xi = 106 N/mm^2$, $\beta = 2$

d	R_{1t}	R_{2t}
14	0.363600494	0.246096322
16	0.460289655	0.421077127
18	0.548391176	0.595884221
20	0.624866024	0.735071552
22	0.689157733	0.830949164
24	0.742158677	0.892298519
26	0.785388119	0.930519290
28	0.820489991	0.954320131
30	0.848983334	0.969329363
32	0.872167973	0.978969564

Table 2. d=20.5 mm, $\xi = 106 N/mm^2$, $\beta = 2$

T	R_{1t}	R_{2t}
12000	0.937297264	0.995544679
24000	0.881993838	0.982413771
36000	0.832852948	0.961282113
48000	0.788898906	0.933180400
60000	0.749351667	0.899376416
72000	0.713580132	0.861245316
84000	0.681068221	0.820150949
96000	0.651389805	0.777353126
108000	0.624189933	0.733947073
120000	0.599170563	0.690833874

3.2 Shaft subjected to bending moment only

The reliability of the shaft increases when the diameter increases and reliability decreases when bending moment increase.

Table 3. M=100000 N-mm, $\xi = 106 N/mm^2$, $\beta = 2$

d	R_{1b}	R_{2b}
14	0.222195431	0.07545007
16	0.298945614	0.15385924
18	0.377781649	0.26934479
20	0.454403741	0.40955959
22	0.525736582	0.55133715
24	0.590025676	0.67439764
26	0.646616530	0.77001560
28	0.695619354	0.83930245
30	0.737594302	0.88765457
32	0.773313713	0.92087063

Table 4. d=20.5 mm, $\xi = 106 N/mm^2$, $\beta = 2$

M	R_{1b}	R_{2b}
12000	0.881993838	0.982413771
24000	0.788898906	0.933180400
36000	0.713580132	0.861245316
48000	0.651389805	0.777353126

Continued on next page

Table 4 continued

60000	0.599170563	0.690833874
72000	0.554702357	0.608110694
84000	0.516378657	0.532722202
96000	0.483008217	0.466055635
108000	0.453689029	0.408165887
120000	0.427725566	0.358409575

3.3 Shaft subjected to combined twisting moment and bending moment

Table 5. M=100000 N-mm, T=120000 N-mm, ξ =106 N/mm^2 , β =2

d	$R_{1\mathrm{ts}}$	$R_{1\mathrm{bn}}$	$R_{2\mathrm{ts}}$	$R_{2\mathrm{bn}}$
14	0.267808519	0.182339259	0.117996648	0.047373632
16	0.353160483	0.249742833	0.229638901	0.099753306
18	0.437373978	0.321554914	0.376682569	0.183431363
20	0.516058228	0.393994130	0.532083364	0.297108046
22	0.586662574	0.463907107	0.668270045	0.428188409
24	0.648218978	0.529069931	0.772492547	0.557943997
26	0.700849799	0.588202398	0.845886469	0.671080986
28	0.745294549	0.640805752	0.895420271	0.760919289
30	0.782560458	0.686937758	0.928329045	0.828023447
32	0.813704916	0.727000505	0.950194051	0.876415423

The reliability of the shaft increases when the diameter increases for the exponential and Weibull-distributed random stress.

Table 6. M=100000 N-mm, d=20.5 mm, ξ =106 N/mm^2 , β =2

T	$R_{1\mathrm{ts}}$	$R_{1\mathrm{bn}}$	$R_{2\mathrm{ts}}$	$R_{2\mathrm{bn}}$
12000	0.919894756	0.875021777	0.992473991	0.980007804
24000	0.873404908	0.832852948	0.979423430	0.961282113
36000	0.827615170	0.791114914	0.958418845	0.934827104
48000	0.785339333	0.752398603	0.930481857	0.902286477
60000	0.746769798	0.716923699	0.896869576	0.865122702
72000	0.711623636	0.684469718	0.858946265	0.824737383
84000	0.679537866	0.654734711	0.818065796	0.782421537
96000	0.650163448	0.627422401	0.775479665	0.739303056
108000	0.623188131	0.602264626	0.732276768	0.696316883
120000	0.598339254	0.579025225	0.689353841	0.654198579

The reliability of the shaft decreases when the twisting moment increases for the exponential and Weibull-distributed random stress.

Table 7. T=100000 N-mm, d=20.5 mm, ξ =106 N/mm^2 , β =2

M	$R_{1\mathrm{ts}}$	$R_{1\mathrm{bn}}$	$R_{2\mathrm{ts}}$	$R_{2\mathrm{bn}}$
12000	0.919894756	0.866567449	0.992473991	0.976839806
24000	0.873404908	0.782020349	0.979423430	0.927905899
36000	0.827615170	0.70973167	0.958418845	0.856701833
48000	0.785339333	0.648961078	0.930481857	0.773634969
60000	0.746769798	0.597518740	0.896869576	0.687890208
72000	0.711623636	0.553519373	0.858946265	0.605826263
84000	0.679537866	0.515498452	0.818065796	0.530967151
96000	0.650163448	0.482333603	0.775479665	0.464711260
108000	0.623188131	0.453159542	0.732276768	0.407134085
120000	0.598339254	0.427301762	0.689353841	0.357613593

The reliability of the shaft decreases when the bending moment increases for the exponential and Weibull-distributed random stress.

Table 8. M=12000 N-mm, T=10000 N-mm, d=20.5 mm, β =2

ξ	R_{1ts}	R_{1bn}	R_{2ts}	R_{2bn}
20.5	0.689525891	0.556736985	0.831431605	0.612031415
30.5	0.767670777	0.651407416	0.916093171	0.777379970
40.5	0.814388361	0.712755916	0.950619659	0.860279509
50.5	0.845463174	0.755742566	0.967670277	0.905420279
60.5	0.867625164	0.787537937	0.977251491	0.932156441
70.5	0.884227934	0.812009017	0.983146224	0.949128188
80.5	0.897130121	0.831425221	0.987022453	0.960514055
90.5	0.907444900	0.847206171	0.989704071	0.968498421
100.5	0.915879659	0.860285405	0.991634788	0.974302420
110.5	0.922905462	0.871302021	0.993070326	0.978648290

The reliability of the shaft increases when strength increases for exponential and Weibull-distributed random stress.

4 Conclusion

The reliability of the shaft is derived subjected to the twisting moment, bending moment and combined twisting and bending moment. Reliability is computed and compared for various values of the twisting moment, bending moment, diameter and strength of the shaft for exponential and Weibull distribution. By the computations, the reliability of the shaft decreases when the twisting moment and the bending moment increase. Reliability increases when the diameter and strength of the shaft increase.

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