INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 06-07-2023

Accepted: 13-01-2024

Published: 08-02-2024

Citation: Savita , Kumar R (2024) Dynamic Weighted Cumulative Residual Entropy Estimators for Laplace Distribution: Bayesian Approach. Indian Journal of Science and Technology 17(6): 556-565. https://doi.org/10.17485/IJST/v17i6.1661

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Dynamic Weighted Cumulative Residual Entropy Estimators for Laplace Distribution: Bayesian Approach

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Abstract

Objectives: To develop Bayesian estimators of dynamic weighted cumulative residual entropy (DWCRE) for Laplace distribution and to investigate posterior risks using various priors and loss functions. **Methods:** Weighted entropy measure of information is provided by a probabilistic experiment whose basic events are described by their objective probabilities and some qualitative (objective or subjective) weights. In this paper, we have used priors (Jeffrey's, Hartigan, Uniform and Gumble Type II) and several loss functions. **Findings:** Bayesian estimators and associated posterior risks for Laplace distribution have been derived for different priors and loss functions. Monte Carlo Simulation study and graphical analyses have also been presented along with the conclusion. Through the comprehensive simulation study in the paper, it has been observed that Hartigan prior is better than other priors in terms of the posterior risk whereas Uniform prior has always higher posterior risk. **Novelty:** The introduction of new Bayesian estimators and their posterior risks for dynamic weighted cumulative residual entropy (DWCRE) of Laplace distribution.

Keywords: Bayesian estimators; Laplace distribution; Fisher information matrix; Loss functions; Priors

1 Introduction

Laplace distribution is a continuous probability distribution and also known as double exponential distribution. It has numerous applications in finance, ocean engineering, image and speech recognition and hydrology. Its prominent feature is the method by which it models the errors, i.e., the probability of deviations from a central value. This distribution is popularly practiced for modeling lifetime in engineering field. To delineate the financial data this distribution is more convenient than normal distribution due to its peak and thick tail. Thus, this distribution has various applications in the financial field. The probability density function (pdf) of the Laplace distribution is given by,

$$f(x) = \frac{1}{2\sigma} exp\left(-\frac{|x-\mu|}{\sigma}\right) \tag{1.1}$$

where $\sigma > 0$ and μ are respectively scale and location parameters of this distribution.

Information theory provides natural mathematical tools for measuring the uncertainty of random variables and the information shared by them. Entropy is a useful indicator of information content that has been used in a number of applications. The probability distribution of a random variable is associated with some sort of uncertainty and entropy is used to quantify it. The entropy of the random variable X having c.d.f (F) with p.d.f (f) is defined as

$$H(f) = -\int_0^\infty f(x) log\, f(x) dx$$

Weighted residual entropy is defined as

$$H^w(x,t) = -\int_t^\infty x \frac{f(x)}{\overline{F}(t)} log\left(\frac{f(x)}{\overline{F}(t)}\right) dx$$

In literature, the weighted cumulative residual entropy (WCRE) is defined as

$$E^w(X) = -\int_0^\infty x\overline{F}(x) log(\overline{F}(x)) dx$$

Dynamic weighted cumulative residual entropy (DWCRE) is defined as

$$\varepsilon^w(x,t) = -\int_t^\infty W(x) \frac{\overline{F}(x)}{\overline{F}(t)} log\left(\frac{\overline{F}(x)}{\overline{F}(t)}\right) dx \; for\; t\; such\; that\; \overline{F}(t) > 0.$$

Here, the case of $W(x) = x$ is considered and the DWCRE for Equation (1.1) is simplified as

$$\varepsilon^w(t) = \sigma(t+2). \tag{1.2}$$

Al-Babtain et al.[1] have studied the dynamic cumulative residual Renyi's entropy for Lomax distribution. Shah et al.[2] discussed the properties and applications of a new alpha skew Laplace distribution. The Bayesian and non- Bayesian estimation of dynamic cumulative residual Tsallis entropy for moment exponential distribution under progressive censored type II is discussed by Alyami et al.[3]. Helmy et al.[4] described the analysis of information measures using generalized type -I hybrid censored data. Entropy estimation is mainly inconvenient, when there are less samples relative to the total number of symbols. We refer to this unliterary as the "under-sampled" regime. In this regime, it is ordinary for many symbols with non-zero probability to stay undetected, and frequently we can only bound or estimate the reinforce of the distribution (i.e., the number of symbols with non-zero probability). In the estimation procedure, the Bayesian approach permits prior subjective knowledge on parameters to be included. To get the same quality of inferences Bayesian methods require less sample data than methods based on sampling theory. We can defeat this limitation by constructing a prior over the area of countable infinite discrete distributions and the resulting estimator is found to be compatible even when the support of the true distribution is finite.

The Bayesian estimation of exponentiated logistic distribution based on lower record value is described by Shaikh and Patel[5]. Al- Babtain et al.[6] discussed the estimation of different types of entropies for Kumaraswamy distribution. Estimation of entropy for inverse Lomax distribution under multiple censored data is described by Bantan et al.[7]. Shrahili et al.[8] discussed the estimation of entropy for Log- Logistic distribution under progressive type II censoring. Almarashi et al.[9] studied the Bayesian analysis of DCRE for Lindley distribution. Savita and Kumar[10] discussed Bayesian estimators and associated posterior risks for DCRE of the Laplace distribution under different loss functions. A connection between weighted generalized CRE and variance is described by Toomaj and Di Crescenzo[11]. The Bayesian estimators for the weighted version of dynamic cumulative residual entropy of Laplace distribution has not been reported in the literature.

Keeping above in view, in the present paper, Bayesian estimators are proposed for the weighted version of dynamic cumulative residual entropy, that is, for the dynamic weighted cumulative residual entropy for Laplace distribution (Equation (1.2)). Further the estimators and their posterior risks are obtained for the weighted case of dynamic cumulative residual entropy and hence are more general. This is the novelty of the paper.

In the paper, to obtain estimators of DWCRE for Laplace distribution using Bayesian techniques, posterior distributions using different priors and various loss functions have been discussed in section 2. In section 3, Bayesian estimators and associated posterior risks for the distribution have been calculated using different priors and loss functions. Numerical computation and graphical analysis have been done in section 4. The conclusions drawn has been presented in section 5.

2 Methodology

2.1 Prior and loss functions

Prior information is utilized in Bayesian analysis, and it is an important approach to statistics. In Bayesian estimation, uncertainty about the latent variable is described by prior probability distribution. We require appropriate choice of priors for the parameters in the Bayesian deduction. In literature, some authors have discussed that there is no method to check the superiority of one prior over other prior, i.e., one is better than other. If we have enough information about the parameter, then informative priors are mostly used. In this paper, we have used various informative and non-informative priors. The likelihood function for Equation (1.1) is given by,

$$L(x/\sigma) = \frac{1}{(2\sigma)^n} exp\left(-\sum_{i=1}^{n} |x_i - \mu| / \sigma\right)$$

The posterior distributions of scale parameter σ for Jeffrey's, Hartigan, Uniform and Gumbel Type II priors, respectively given by $\pi_1(\sigma/\underset{-}{x}), \pi_2(\sigma/\underset{-}{x}), \pi_3(\sigma/\underset{-}{x})$ and $\pi_4(\sigma/\underset{-}{x})$ are as under:

$$\pi_1(\sigma/\underset{-}{x}) = \frac{G^n}{\sigma^{n+1}\Gamma(n)} exp\left(-\frac{G}{\sigma}\right) \tag{1.3}$$

$$\pi_2(\sigma/\underset{-}{x}) = \frac{G^{n+2}}{\sigma^{n+3}\Gamma(n+2)} exp\left(-\frac{G}{\sigma}\right) \tag{1.4}$$

$$\pi_3(\sigma/\underset{-}{x}) = \frac{G^{n-1}}{\sigma^{n}\Gamma(n-1)} exp\left(-\frac{G}{\sigma}\right) \tag{1.5}$$

$$\pi_4(\sigma/\underset{-}{x}) = \frac{M^N}{\sigma^{N+1}\Gamma(N)} exp\left(-\frac{M}{\sigma}\right) \tag{1.6}$$

where $G = \sum_{i=1}^{n} |x_i - \mu|$, $M = a + \sum_{i=1}^{n} |x_i - \mu|$, $N = n+1$.

The loss functions should be analyzed to select the best estimator and also used to show the associated penalty. The statement that Bayesian methods always perform good regardless of the situations, is not always true. In the present paper, we have considered different loss functions such as SELF (Squared Error Loss Function), GELF (General Entropy Loss Function), ELF (Entropy Loss Function), WSELF (Weighted Squared Error Loss Function), KLF (K- Loss Function), M/QSELF (Modified/Quadratic Squared Error Loss Function) and PLF (Precautionary Loss Function).

The Bayesian estimators under SELF, GELF, ELF, WSELF, KLF, M/QSELF and PLF loss functions denoted by $\overset{\wedge}{\sigma}_S, \overset{\wedge}{\sigma}_G, \overset{\wedge}{\sigma}_E, \overset{\wedge}{\sigma}_W, \overset{\wedge}{\sigma}_K, \overset{\wedge}{\sigma}_M$ and $\overset{\wedge}{\sigma}_P$ are as

$$\overset{\wedge}{\sigma}_S = E(\sigma/\underset{-}{x}) \tag{1.7}$$

$$\overset{\wedge}{\sigma}_G = \left[E\left(\sigma^{-c}/\underset{-}{x}\right)\right]^{-1/c} \tag{1.8}$$

$$\overset{\wedge}{\sigma}_E = \left[E\left(\sigma^{-1}/\underset{-}{x}\right)\right]^{-1} \tag{1.9}$$

$$\overset{\wedge}{\sigma}_W = \left[E\left(\sigma^{-1}/\underset{-}{x}\right)\right]^{-1} \tag{1.10}$$

$$\hat{\sigma}_K = \sqrt{\frac{E(\sigma/\underline{x})}{E\left(\sigma^{-1}/\underline{x}\right)}} \tag{1.11}$$

$$\hat{\sigma}_M = \frac{E\left(\sigma^{-1}/\underline{x}\right)}{E\left(\sigma^{-2}/\underline{x}\right)} \tag{1.12}$$

$$\hat{\sigma}_P = \sqrt{E\left(\sigma^{2}/\underline{x}\right)} \tag{1.13}$$

and posterior risk under above mentioned loss functions denoted by $\hat{\sigma}_{PS}, \hat{\sigma}_{PG}, \hat{\sigma}_{PE}, \hat{\sigma}_{PW}, \hat{\sigma}_{PK}, \hat{\sigma}_{PM}$ and $\hat{\sigma}_{PP}$ are as

$$\hat{\sigma}_{PS} = V(\sigma/\underline{x}) \tag{1.14}$$

$$\hat{\sigma}_{PG} = cE(ln\sigma/\underline{x}) + ln\left[E\left(\sigma^{-c}/\underline{x}\right)\right] \tag{1.15}$$

$$\hat{\sigma}_{PE} = E(ln\sigma/\underline{x}) + ln\left[E\left(\sigma^{-1}/\underline{x}\right)\right] \tag{1.16}$$

$$\hat{\sigma}_{PW} = E(\sigma/\underline{x}) - \left[E\left(\sigma^{-1}/\underline{x}\right)\right]^{-1} \tag{1.17}$$

$$\hat{\sigma}_{PK} = 2\left[E(\sigma/\underline{x})\left[E\left(\sigma^{-1}/\underline{x}\right)\right] - 1\right] \tag{1.18}$$

$$\hat{\sigma}_{PM} = 1 - \frac{\left[E\left(\sigma^{-1}/\underline{x}\right)\right]^2}{E\left(\sigma^{-2}/\underline{x}\right)} \tag{1.19}$$

$$\hat{\sigma}_{PP} = 2\left[\sqrt{E\left(\sigma^{2}/\underline{x}\right)} - E(\sigma/\underline{x})\right] \tag{1.20}$$

3 Results and Discussion

3.1 Proposed Bayesian Estimators

The novelty of the paper is to propose the Bayesian estimators and posterior risks of dynamic weighted cumulative residual entropy for Laplace distribution. In the following theorems, Bayesian estimators and posterior risks of DWCRE using different loss functions and priors are proposed, as discussed in section 2.

Theorem 3.1 : The Bayesian estimators of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Jeffrey's prior (Equation (1.3)) under different loss functions, using Equations (1.7), (1.8), (1.9), (1.10), (1.11), (1.12) and (1.13), are given by,

$$\overset{\wedge}{\varepsilon}^{w}(t)_{JS} = \tfrac{(t+2)G}{n-1}, \overset{\wedge}{\varepsilon}^{w}(t)_{JE} = \tfrac{(t+2)G}{n}, \overset{\wedge}{\varepsilon}^{w}(t)_{JW} = \tfrac{(t+2)G}{n}, \overset{\wedge}{\varepsilon}^{w}(t)_{JK} = \tfrac{(t+2)G}{\sqrt{n(n-1)}}$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{JG} = (t+2)G\left[\tfrac{\Gamma(n)}{\Gamma(n+c)}\right]^{\frac{1}{c}}, \overset{\wedge}{\varepsilon}^{w}(t)_{JM} = \tfrac{(t+2)G}{n+1}, \overset{\wedge}{\varepsilon}^{w}(t)_{JP} = \tfrac{(t+2)G}{\sqrt{(n-1)(n-2)}}$$

Proof: For the Laplace distribution (Equation (1.1)) and Jeffrey's prior (Equation (1.3)), we have

$$E\left[\varepsilon^{w}(t)/\underset{-}{x}\right] = \int_0^{\infty} \frac{\sigma(t+2)G^{n}}{\sigma^{n+1}\Gamma(n)} exp\left(-\frac{G}{\sigma}\right) d\sigma = \frac{G(t+2)}{n-1} \tag{1.21}$$

$$\Rightarrow \overset{\wedge}{\varepsilon}^{w}(t)_{JS} = E\left[\varepsilon^{w}(t)/\underset{-}{x}\right] = \frac{(t+2)G}{n-1}$$

$$E\left[\varepsilon^{w}(t)^2/\underset{-}{x}\right] = \int_0^{\infty} \frac{\sigma^2(t+2)^2G^{n}}{\sigma^{n+1}\Gamma(n)} exp\left(-\frac{G}{\sigma}\right) d\sigma = \frac{G^2(t+2)^2}{(n-1)(n-2)},$$

$$\Rightarrow \overset{\wedge}{\varepsilon}^{w}(t)_{JP} = \sqrt{E\left[\varepsilon^{w}(t)^2/\underset{-}{x}\right]} = \frac{(t+2)G}{\sqrt{(n-1)(n-2)}}.$$

$$E\left[\frac{1}{\varepsilon^{w}(t)^{c}}/\underset{-}{x}\right] = \int_0^{\infty} \frac{G^{n}}{\sigma^{n+c+1}(t+2)^{c}\Gamma(n)} exp\left(-\frac{G}{\sigma}\right) d\sigma = \frac{\Gamma(n+c)}{G^{c}(t+2)^{c}\Gamma(n)}. \tag{1.22}$$

$$\Rightarrow \overset{\wedge}{\varepsilon}^{w}(t)_{JG} = \left[E\left[\varepsilon^{w}(t)^{-c}/\underset{-}{x}\right]\right]^{\frac{-1}{c}} = (t+2)G\left[\frac{\Gamma(n)}{\Gamma(n+c)}\right]^{\frac{1}{c}}.$$

Put c = 1 in Equation (1.22), we get

$$E\left[\frac{1}{\varepsilon^{w}(t)}/\underset{-}{x}\right] = \frac{\Gamma(n+1)}{G(t+2)\Gamma(n)} = \frac{n}{(t+2)G}, \tag{1.23}$$

$$\Rightarrow \overset{\wedge}{\varepsilon}^{w}(t)_{JE} = \left[E\left[\varepsilon^{w}(t)^{-1}/\underset{-}{x}\right]\right]^{-1} = \frac{(t+2)G}{n}$$

Similarly,

$$\overset{\wedge}{\varepsilon}^{w}(t)_{JW} = \left[E\left[\varepsilon^{w}(t)^{-1}/\underset{-}{x}\right]\right]^{-1} = \frac{(t+2)G}{n}$$

Put c = 2 in Equation (1.22), we get

$$E\left[\frac{1}{\varepsilon^{w}(t)^2}/\underset{-}{x}\right] = \frac{\Gamma(n+2)}{G^2(t+2)^2\Gamma(n)} = \frac{n(n+1)}{(t+2)^2G^2} \tag{1.24}$$

Using Equations (1.21) and (1.23), we get

$$\overset{\wedge}{\varepsilon}^{w}(t)_{JK} = \sqrt{\frac{E\left[\varepsilon^{w}(t)/\underset{-}{x}\right]}{E\left[\varepsilon^{w}(t)^{-1}/\underset{-}{x}\right]}} = \frac{(t+2)G}{\sqrt{n(n-1)}}.$$

Using Equations (1.23) and (1.24), we get

$$\overset{\wedge}{\varepsilon}^{w}(t)_{JM} = \frac{E\left[\varepsilon^{w}(t)^{-1}/\underset{-}{x}\right]}{E\left[\varepsilon^{w}(t)^{-2}/\underset{-}{x}\right]} = \frac{(t+2)G}{n+1}.$$

This proves the theorem.

Theorem 3.2: The posterior risks of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Jeffrey's prior (Equation (1.3)) under different loss functions, using equations Equations (1.14), (1.15), (1.16), (1.17), (1.18), (1.19) and (1.20), are given by,

$$\overset{\wedge}{\varepsilon}^{w}(t)_{JPS} = \tfrac{(t+2)^2G^2}{(n-1)^2(n-2)}, \overset{\wedge}{\varepsilon}^{w}(t)_{JPE} = lnn - \tfrac{\Gamma'(n)}{\Gamma(n)}, \overset{\wedge}{\varepsilon}^{w}(t)_{JPW} = \tfrac{(t+2)G}{n(n-1)}, \overset{\wedge}{\varepsilon}^{w}(t)_{JPK} = \tfrac{2}{n-1},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{JPG} = ln\tfrac{\Gamma(n+c)}{\Gamma(n)} - c\tfrac{\Gamma'(n)}{\Gamma(n)}, \overset{\wedge}{\varepsilon}^{w}(t)_{JPM} = \tfrac{1}{n+1}, \overset{\wedge}{\varepsilon}^{w}(t)_{JPP} = 2\left[\tfrac{G(t+2)}{\sqrt{(n-1)(n-2)}} - \tfrac{G(t+2)}{n-1}\right].$$

Theorem 3.3: The Bayesian estimators of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Hartigan prior (Equation (1.4)) under different loss functions, using Equations (1.7), (1.8), (1.9), (1.10), (1.11), (1.12) and (1.13), are given by

$$\overset{\wedge}{\varepsilon}^{w}(t)_{HS} = \tfrac{(t+2)G}{n+1}, \overset{\wedge}{\varepsilon}^{w}(t)_{HE} = \tfrac{(t+2)G}{n+2}, \overset{\wedge}{\varepsilon}^{w}(t)_{HW} = \tfrac{(t+2)G}{n+2}, \overset{\wedge}{\varepsilon}^{w}(t)_{HK} = \tfrac{(t+2)G}{\sqrt{(n+1)(n+2)}},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{HG} = (t+2)G\left[\tfrac{\Gamma(n+2)}{\Gamma(n+c+2)}\right]^{\frac{1}{c}}, \overset{\wedge}{\varepsilon}^{w}(t)_{HM} = \tfrac{(t+2)G}{n+3}, \overset{\wedge}{\varepsilon}^{w}(t)_{HP} = \tfrac{(t+2)G}{\sqrt{n(n+1)}}.$$

Theorem 3.4: The posterior risks of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Hartigan prior (Equation (1.4)) under different loss functions, using Equations (1.14), (1.15), (1.16), (1.17), (1.18), (1.19) and (1.20), are given by

$$\overset{\wedge}{\varepsilon}^{w}(t)_{HPS} = \tfrac{(t+2)^2G^2}{n(n+1)^2}, \overset{\wedge}{\varepsilon}^{w}(t)_{HPE} = ln(n+2) - \tfrac{\Gamma'(n+2)}{\Gamma(n+2)}, \overset{\wedge}{\varepsilon}^{w}(t)_{HPW} = \tfrac{(t+2)G}{(n+1)(n+2)}, \overset{\wedge}{\varepsilon}^{w}(t)_{HPK} = \tfrac{2}{n+1},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{HPG} = ln\tfrac{\Gamma(n+c+2)}{\Gamma(n+2)} - c\tfrac{\Gamma'(n+2)}{\Gamma(n+2)}, \overset{\wedge}{\varepsilon}^{w}(t)_{HPM} = \tfrac{1}{n+3}, \overset{\wedge}{\varepsilon}^{w}(t)_{HPP} = 2\left[\tfrac{G(t+2)}{\sqrt{n(n+1)}} - \tfrac{G(t+2)}{n+1}\right].$$

Theorem 3.5: The Bayesian estimators of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Uniform prior (Equation (1.5)) under different loss functions, using Equations (1.7), (1.8), (1.9), (1.10), (1.11), (1.12) and (1.13), are given by

$$\overset{\wedge}{\varepsilon}^{w}(t)_{US} = \tfrac{(t+2)G}{n-2}, \overset{\wedge}{\varepsilon}^{w}(t)_{UE} = \tfrac{(t+2)G}{n-1}, \overset{\wedge}{\varepsilon}^{w}(t)_{UW} = \tfrac{(t+2)G}{n-1}, \overset{\wedge}{\varepsilon}^{w}(t)_{UK} = \tfrac{(t+2)G}{\sqrt{(n-1)(n-2)}},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{UG} = (t+2)G\left[\tfrac{\Gamma(n-1)}{\Gamma(n+c-1)}\right]^{\frac{1}{c}}, \overset{\wedge}{\varepsilon}^{w}(t)_{UM} = \tfrac{(t+2)G}{n}, \overset{\wedge}{\varepsilon}^{w}(t)_{UP} = \tfrac{(t+2)G}{\sqrt{(n-2)(n-3)}}.$$

Theorem 3.6: The posterior risks of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Uniform prior (Equation (1.5)) under different loss functions, using Equations (1.14), (1.15), (1.16), (1.17), (1.18), (1.19) and (1.20), are given by

$$\overset{\wedge}{\varepsilon}^{w}(t)_{UPS} = \tfrac{(t+2)^2G^2}{(n-2)^2(n-3)}, \overset{\wedge}{\varepsilon}^{w}(t)_{UPE} = ln(n-1) - \tfrac{\Gamma'(n-1)}{\Gamma(n-1)}, \overset{\wedge}{\varepsilon}^{w}(t)_{UPW} = \tfrac{(t+2)G}{(n-1)(n-2)}, \overset{\wedge}{\varepsilon}^{w}(t)_{UPK} = \tfrac{2}{n-2},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{UPG} = ln\tfrac{\Gamma(n+c-1)}{\Gamma(n-1)} - c\tfrac{\Gamma'(n-1)}{\Gamma(n-1)}, \overset{\wedge}{\varepsilon}^{w}(t)_{UPM} = \tfrac{1}{n}, \overset{\wedge}{\varepsilon}^{w}(t)_{UPP} = 2\left[\tfrac{G(t+2)}{\sqrt{(n-2)(n-3)}} - \tfrac{G(t+2)}{n-2}\right].$$

Theorem 3.7: The Bayesian estimators of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Gumbel Type II prior (Equation (1.6)) under different loss functions, using Equations (1.7), (1.8), (1.9), (1.10), (1.11), (1.12) and (1.13), are given by

$$\overset{\wedge}{\varepsilon}^{w}(t)_{GS} = \tfrac{(t+2)M}{N-1}, \overset{\wedge}{\varepsilon}^{w}(t)_{GE} = \tfrac{(t+2)M}{N}, \overset{\wedge}{\varepsilon}^{w}(t)_{GW} = \tfrac{(t+2)M}{N}, \overset{\wedge}{\varepsilon}^{w}(t)_{GK} = \tfrac{(t+2)M}{\sqrt{N(N-1)}},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{GG} = (t+2)M\left[\tfrac{\Gamma(N)}{\Gamma(N+c)}\right]^{\frac{1}{c}}, \overset{\wedge}{\varepsilon}^{w}(t)_{GM} = \tfrac{(t+2)M}{N+1}, \overset{\wedge}{\varepsilon}^{w}(t)_{GP} = \tfrac{(t+2)M}{\sqrt{(N-1)(N-2)}}.$$

Theorem 3.8: The posterior risks of DWCRE for Laplace distribution (Equation (1.1)) with posterior distribution of Gumbel Type II prior (Equation (1.6)) under different loss functions, using Equations (1.14), (1.15), (1.16), (1.17), (1.18), (1.19) and (1.20), are given by

$$\overset{\wedge}{\varepsilon}^{w}(t)_{GPS} = \frac{(t+2)^2M^2}{(N-1)^2(N-2)}, \overset{\wedge}{\varepsilon}^{w}(t)_{GPE} = lnN - \frac{\Gamma'(N)}{\Gamma(N)}, \overset{\wedge}{\varepsilon}^{w}(t)_{GPW} = \frac{(t+2)M}{N(N-1)}, \overset{\wedge}{\varepsilon}^{w}(t)_{GPK} = \frac{2}{N-1},$$
$$\overset{\wedge}{\varepsilon}^{w}(t)_{GPG} = ln\frac{\Gamma(N+c)}{\Gamma(N)} - c\frac{\Gamma'(N)}{\Gamma(N)}, \overset{\wedge}{\varepsilon}^{w}(t)_{GPM} = \frac{1}{N+1}, \overset{\wedge}{\varepsilon}^{w}(t)_{GPP} = 2\left[\frac{M(t+2)}{\sqrt{(N-1)(N-2)}} - \frac{M(t+2)}{N-1}\right].$$

3.2 Numerical Computation and Graphical Analysis

To analyze the efficiency of the estimators, several computations and graphical analyses have been done along with a simulation study. The sample size is taken as $5,10,25,50,100, \sigma = 0.1,\ 0.3, 0.5, 0.7, 0.9, c = 0.9$ and $\mu = 2$ for the purpose. The computations and graphical interpretations are also done for $n = 5,10,25,50,100$ and $\sigma = 0.1, 0.3, 0.5, 0.7$ and 0.9. The Bayesian estimators and posterior risks (in the parenthesis) of DWCRE for $n = 100$ are given in the following table:

Table 1. Bayesian estimators and posterior risks (in the parenthesis) of DWCRE for$n = 100$

σ	Loss function	Jeffrey	Hartigan	Uniform	Gumble Type II
	SELF	0.363(0.001)	0.355(0.001)	0.366(0.001)	0.382(0.001)
	GELF	0.359(0.004)	0.352(0.004)	0.363(0.004)	0.378(0.004)
	ELF	0.359(0.005)	0.352(0.005)	0.363(0.005)	0.378(0.005)
0.1	WSELF	0.359(0.004)	0.352(0.003)	0.363(0.004)	0.378(0.004)
	KLF	0.361(0.02)	0.354(0.02)	0.365(0.02)	0.38(0.02)
	M/QSELF	0.355(0.01)	0.349(0.01)	0.359(0.01)	0.374(0.01)
	PLF	0.365(0.004)	0.357(0.004)	0.368(0.004)	0.383(0.004)
	SELF	1.088(0.012)	1.066(0.011)	1.099(0.012)	1.01(0.012)
	GELF	1.078(0.004)	1.056(0.004)	1.089(0.004)	1.089(0.004)
	ELF	1.077(0.005)	1.056(0.005)	1.088(0.005)	1.089(0.005)
0.3	WSELF	1.077(0.011)	1.056(0.01)	1.088(0.011)	1.089(0.011)
	KLF	1.083(0.02)	1.061(0.02)	1.094(0.02)	1.094(0.02)
	M/QSELF	1.066(0.01)	1.046(0.01)	1.077(0.01)	1.078(0.01)
	PLF	1.094(0.011)	1.072(0.011)	1.105(0.011)	1.105(0.01)
	SELF	1.813(0.034)	1.778(0.032)	1.832(0.035)	1.818(0.033)
	GELF	1.796(0.004)	1.761(0.004)	1.814(0.004)	1.8010.004)
	ELF	1.795(0.005)	1.76(0.005)	1.813(0.005)	1.8(0.005)
0.5	WSELF	1.795(0.018)	1.76(0.017)	1.813(0.019)	1.8(0.018)
	KLF	1.804(0.02)	1.769(0.02)	1.823(0.02)	1.809(0.02)
	M/QSELF	1.777(0.01)	1.743(0.01)	1.795(0.01)	1.782(0.01)
	PLF	1.823(0.018)	1.786(0.018)	1.841(0.019)	1.827(0.018)
	SELF	2.539(0.066)	2.488(0.062)	2.564(0.068)	2.536(0.065)
	GELF	2.514(0.004)	2.465(0.004)	2.54(0.004)	2.512(0.004)
	ELF	2.513(0.005)	2.464(0.005)	2.539(0.005)	2.511(0.005)
0.7	WSELF	2.513(0.025)	2.464(0.024)	2.539(0.026)	2.511(0.025)
	KLF	2.526(0.02)	2.476(0.02)	2.552(0.02)	2.523(0.02)
	M/QSELF	2.488(0.01)	2.440(0.01)	2.513(0.01)	2.486(0.01)
	PLF	2.552(0.026)	2.501(0.025)	2.578(0.026)	2.548(0.026)
	SELF	3.264(0.109)	3.199(0.102)	3.297(0.112)	3.254(0.107)
	GELF	3.233(0.004)	3.169(0.004)	3.266(0.005)	3.223(0.004)
	ELF	3.231(0.005)	3.168(0.005)	3.264(0.005)	3.222(0.005)
0.9	WSELF	3.231(0.033)	3.168(0.031)	3.264(0.033)	3.222(0.032)
	KLF	3.248(0.02)	3.184(0.02)	3.281(0.02)	3.238(0.02)
	M/QSELF	3.199(0.01)	3.137(0.01)	3.231(0.01)	3.19(0.01)
	PLF	3.281(0.033)	3.215(0.032)	3.314(0.034)	3.27(0.033)

The graphs in Figures 1, 2 and 3 represent posterior risks of DWCRE for n=5 with different values of σ for different loss functions. In Figures 1, 2 and 3, posterior risks are higher for Uniform prior and smaller for Hartigan prior. The posterior risks increase as σ increases. Also, the posterior risks are invariant for all values of σ for other loss functions (ELF, GELF, M/QSELF and KLF). Hartigan prior is better than all other prior as the posterior risks are smaller for this prior.

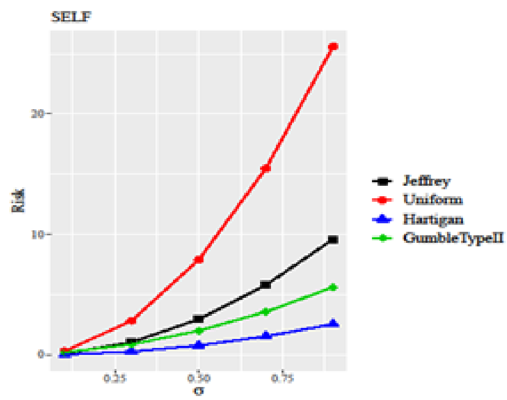

Fig 1. P.R. of DWCRE w.r.t. σ for SELF for $n = 5$

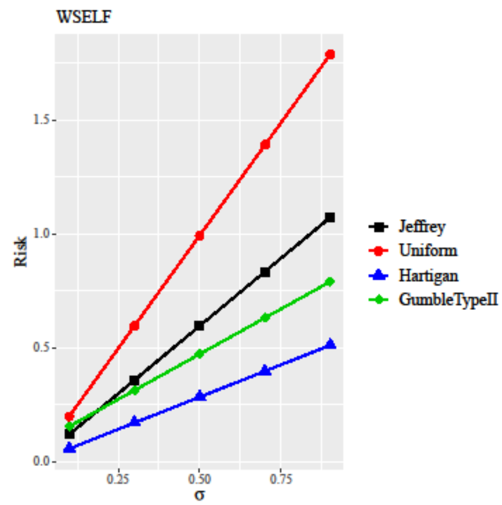

Fig 2. P.R. of DWCRE w.r.t. σ for WSELF for $n = 5$

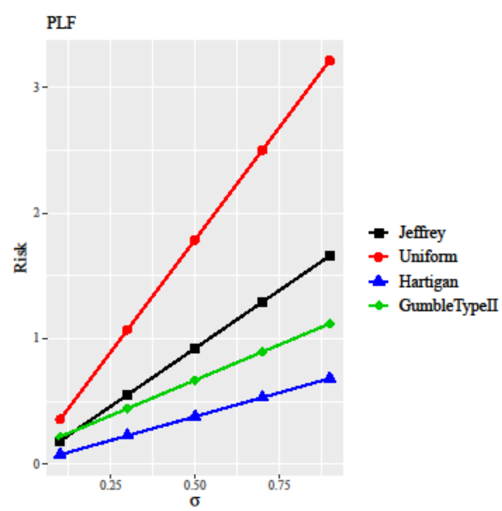

Fig 3. P.R. of DWCRE w.r.t. σ for PLF for $n = 5$

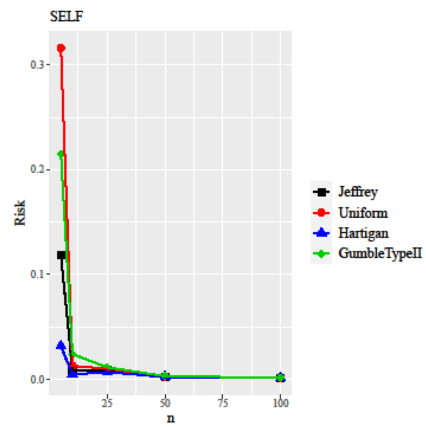

Fig 4. P.R. of DWCRE w.r.t. n **for SELF for** $\sigma = 0.1$

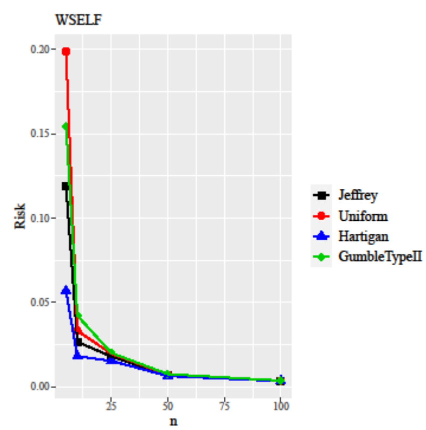

Fig 5. P.R. of DWCRE w.r.t. n **for WSELF for** $\sigma = 0.1$

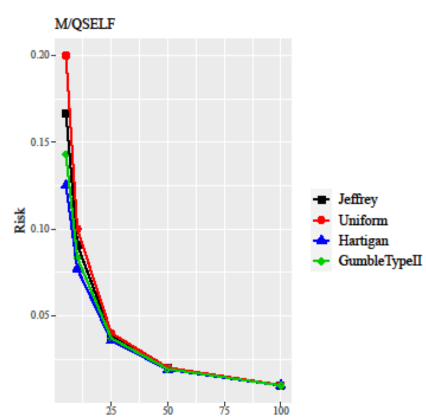

Fig 6. P.R. of DWCRE w.r.t. n **for** $M/QSELF$ **for** $\sigma = 0.1$

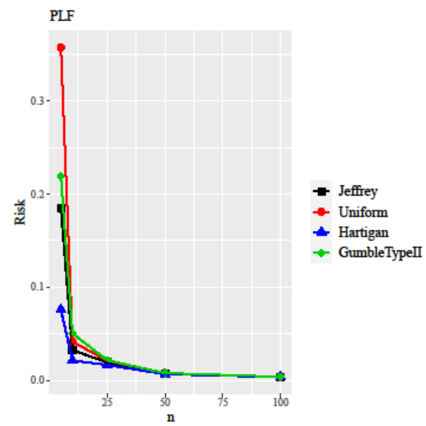

Fig 7. P.R. of DWCRE w.r.t. n **for PLF for** $\sigma = 0.1$

The graphs in Figures 4, 5, 6 and 7 represent posterior risks of DWCRE for $\sigma = 0.1$ with different loss functions for different values of n. In Figures 4, 5, 6 and 7, posterior risks decrease as n increases for all prior and for n=50 onward posterior risks are almost same. Also, the posterior risks decrease as n increases for other loss functions (ELF, GELF and KLF).

4 Conclusion

In the paper, Bayesian estimators and posterior risks of dynamic weighted cumulative residual entropy for the Laplace distribution using different priors under several loss functions have been obtained. It has been concluded that for all the priors, posterior risks of DWCRE for the distribution increases as σ increases for SELF, WSELF and PLF, however for other loss functions, these remain invariant. The posterior risks, in cases of all the loss functions, decrease as n increases for all values of σ. In these studies, Hartigan prior is found to be better in comparison to other priors as it has smaller risk and thereafter, Jeffrey and Gumble Type II priors can be given preference. Further the Uniform prior has always higher risk in comparison to other priors under different loss functions taken in the paper.

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