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RESEARCH ARTICLE

An Alternative Method for Finding the Critical Path of the Network in Fuzzy Time Cost Trade off Problem

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Abstract

Background: The critical path approach is used to determine the network's longest path, according to historical records. This study examines a different approach to determining the construction network's longest path. **Method:** Here, the network is viewed as a directed acyclic graph, and the critical path of the network is found using the longest path algorithm of the network. To find the best answer for a building project, the longest path that was found was integrated into a linear programming issue. The triangle fuzzy variable defines all of the project's inputs. The 992 square foot building area is incorporated, and three project manager's quotes are used as a triangular fuzzy variable. **Findings:** This work has the options of getting quotation from the project managers, convert them as the fuzzy variables such as triangular fuzzy variable, Trapezoidal fuzzy variable and pentagonal fuzzy variable. After the network is converted into a linear programming problem using the fully fuzzy mathematical model, the best possible solution is found. **Novelty and applications :** Alternative method for critical path of the network has been incorporated. It has been found that the proposed method reduces the time to find the critical path of the larger networks.

Keywords: Directed Acyclic Graph; Longest Path Algorithm; Triangular Fuzzy Variable; Fuzzy Linear Programming Problem; Fully Fuzzy Mathematical Model

1 Introduction

One of the important branches of Operations research is time cot trade off problems simply called networks in project management. Its primary purpose is to shorten the initial project duration—which is established by the critical path analysis—in order to fulfil a deadline while spending the least amount of money. Furthermore, it could be essential to complete the job by a certain date. The idea of fuzzy sets was first presented by Zadeh and today the development of the lowest cost with the best duration is the basis for all research fields. The idea of fuzzy sets and applications of fuzzy sets was introduced by Zimmermann. Critical path scheduling and planning was first introduced by James E. Kelley. Babu A.J.G. & Suresh N. proposed a new method in the project management using time, cost and quality as parameters. To get an ideal duration not translating the duration of the fuzzy activity to a well-defined number and a current procedure to solve integer linear programming problems using triangular fuzzy numbers was introduced by Shakeela sathish. Ghazanfari presented a new optimality approach for the goal programming fuzzy time-cost trade-off problem. Pandian P. & Jayalakshmi M. gives out a new technique called decomposition technique for linear programming problem which is very useful for splitting fuzzy variables into crisp variables in linear programming problem. Evangeline Jebaseeli M. and Paul Dhayabaran D. proposed an algorithm to solve the fully fuzzy mathematical model and proposed a procedure that uses the multi-objective linear programming technique for solving fully fuzzy time cost tradeoff models. Antony Raj M. & Mariappan P give out a new method called aggregated method for the linear programming problem. LINGO solver package system is used to solve the linear programming problems and it is very much useful and time consumption for solving the LPP with large number of constraints.

2 Literature review

For all real-world project decision makers, the trade-off between project cost and completion time as well as environmental uncertainty are important considerations. One of the most crucial areas in business and industry is project management, and one of its unique facets is precisely scheduling time. Many methods have been put forth in the literature over the years to determine the ideal duration at the lowest possible cost. The development of the lowest cost with the best duration is the basis for all research fields(1). Abinaya B., Evangeline Jebaseeli M. & Henry Amirtharaj E.C. proposed a new solving procedure for solving FTCTO problems using decomposition and aggregated techniques(2). Abinaya B. & Henry Amirtharaj E.C. proposed a new way for an application of multiple attribute group decision making to find the best alternative in fuzzy time cost trade off problems(3). Abinaya B., Evangeline Jebaseeli M. & Henry Amirtharaj E.C employed a new methodology for solving fuzzy time cost trade off problems using triangular and trapezoidal fuzzy variables(4). Abinaya B. & Henry Amirtharaj E.C. gives out a comparison between decomposition method and graded mean value method.(5) Abinaya B. & Henry Amirtharaj E.C. employed a methodology for finding optimality of the project network using triangular fuzzy variable(6). Ghazi H. Shakah & Krasnoproshin V.V. introduces a possibility to free a person from time-consuming, routine work and unleash his creative potential(7). Hema R.& Sudharani R. proposed the arithmetic operators, operational laws and also the score function, accuracy and certainty functions for interval valued bipolar triangular neutrosophic sets and discovered the PROMETHEE II technique to an aggregated weighted arithmetic operator to address the multi-criterion decision-making problem(8). Keshavarz, Esmaeil & Shoul, Abbas employed the novel approach to the multi criteria decision making for the project management using time, cost and quality as the parameters in time cost trade off problems(9). Riddhi K. Rekh and Jayesh M. Dhodiya proposed novel procedure project activities for project management using cost, quality, time and risk and found the solution of this multi-criteria project management problem in fuzzy programming technique by decision maker alpha level set and also compared it with closely related fuzzy group multi-criteria decision making related Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method for the critical path selection.(10) Shanmuga Sundari M. proposed another technique to find the fuzzy optimal arrangement for assignment problems with triangular fuzzy numbers and created fuzzy adaptation of Hungarian algorithm for the arrangement of fuzzy assignment problems without changing over them to established proportional.

The longest path problem in graph theory and theoretical computer science is the task of determining a simple path with the maximum length in a given graph. If a path has no repeated vertices, it is considered simple. The path's length can be determined by adding up all of its edge weights or by counting the number of edges. The longest path problem is NP hard. The decision version of the problem asks whether a path exits of at least a given length is NP complete. In contrast, the shortest path problem can be solved in polynomial time in graphs without negative-weight cycles. This indicates that unless P=NP, the decision problem cannot be solved in polynomial time for any graph. Stronger hardness results are also known, indicating that approximation is challenging. On the other hand, it provides a linear time solution for directed acyclic networks, which is useful for determining the critical path in scheduling issues.

Research Statement: This paper gives out the alternative method of finding critical path of the network using longest path algorithm in the directed acyclic graph which is very effective and time consuming for the project managers.

3 The preliminary

Definition 3.1

The characteristic function μ_B in a certain (crisp) set B$\subseteq$T allots a value which is 0 or 1 for every member in T. The feature is expanded to a function $\mu_{\tilde{B}}$ so that the element's assigned value of S falls inside a given range i.e. $\mu_B : \mathrm{T} \to [0,1]$. The allocated values $\mu_{\tilde{B}}(T)$ for eacht $\in$ T indicate the element's membership grade inside the set. B. The set $B = \{\mathrm{B}, \mu_B(y) : \mathrm{y} \in Y\}$is called

Fuzzy Set.

Definition 3.2

As demonstrated in the example below, a triangular fuzzy number is one that has three points:

$B = (j_1, j_2, j_3)$ Membership functions are the interpretation of this representation:

We use "F(R)" to represent a collection of fuzzy triangular integers.

$$\mu_B(x) = \begin{cases} 0 & \text{if } y < j_1 \text{ and } j_3 > y \\ \frac{y-j_1}{j_2-j_1} & \text{if } j_1 \leq y < j_2 \\ \frac{j_3-y}{j_3-j_2} & \text{if } j_2 \leq y \leq j_3 \end{cases}$$

Definition 3.3

Let (j_1, j_2, j_3) and (k_1, k_2, k_3) are two triangular fuzzy numbers.

Then

$(j_1, j_2, j_3) \oplus (k_1, k_2, k_3) = (j_1 + k_3, j_2 + k_2, j_3 + k_1)$

$(j_1, j_2, j_3) - (k_1, k_2, k_3) = (j_1 - k_3, j_2 - k_2, j_3 - k_1)$

$(j_1, j_2, j_3) = (cj_1, cj_2, cj_3)$, for $c \geq 0$.

$c(j_1, j_2, j_3) = (cj_3, cj_2, cj_1)$, for $c < 0$.

$\frac{(j_1, j_2, j_3)}{(k_1, k_2, k_3)} = \left(\frac{j_1}{k_3}, \frac{j_2}{k_2}, \frac{j_3}{k_1}\right)$

Definition 3.4

"Linear programming is one of the most widely used operations research approaches, assuming that all variables and parameters are real values. In practice, we don't have enough reliable data. Thus, fuzzy variables and fuzzy integers are used in a linear programming issue. The fully fuzzy linear programming problems in standard form are as follows with n fuzzy variables and m fuzzy constants:

Maximimize or (Minimize) $\left(B^T \otimes \tilde{Y}\right)$

Subject to $E\tilde{Y} = \tilde{d}$

$\tilde{Y} isanon - negative fuzzyvariable$

$B^T = b_{j_{1\times n}} \tilde{Y} = y_{i_{nx1}}, E = [e_{ij}]_{mxn}, \tilde{d} = [d_i]_{m\times 1}$ and $Where$

$b_j, y_j, e_{ij},\ d_i \in F(R)$

$i = 1, 2, \ldots m \& j = 1, 2, \ldots n$

Definition 3.5

An activity-on-activity arc network can be used to define a fuzzy project network. P=(N,L) where N={1,2,...,m} is the set of nodes(points) and A is the set of arcs(oriented lines) represents the activities. In the fuzzy project network, node 1 and n denotes the initial and terminal of the project respectively. The following is the full fuzzy mathematical model for fully fuzzy time cost trade-off problems:

Min $\tilde{Z} = \sum_k \sum_l A_{kl}$

subject to

$\tilde{D}_1 = 0, \tilde{D}_l - \tilde{D}_k - \tilde{y}_{kl} \geq 0, \tilde{D}_m \leq \tilde{D}; \tilde{a}_k = \tilde{s}^* \left(N\tilde{D}_{kl} - \tilde{y}_{kl}\right), A\tilde{D}_{kl} \leq \tilde{y}_k \leq N\tilde{D}_k$

$\forall (k,l) \in P, \tilde{A}_k = \sum_k \sum_l \tilde{a}_{kl} + \tilde{I}^* \left(\tilde{D}_m - \tilde{D}_1\right) + \sum m\tilde{K}_m;$ Wherea $= (1, 2, \ldots m)$ and $b = (1, 2, \ldots m)$.

Theorem 3.1(Decomposition Theorem)

A triangular fuzzy number $\tilde{y} = (y_1, y_2, y_3)$ is an optimal result of the problem (P) if and only if y_1, y_2 and $y_3$are optimal results of the prescribed crisp linear programming problems (P2), (P1) and (P3) respectively where:

$(P) Maximize \tilde{Z} = Ay \quad Subjectto \quad \tilde{B}y \leq \tilde{d}, y \geq 0 (P2) Maximize Z_2 = Ay_2 Subjectto By_2 \leq d_2, y_2 \geq 0 (P1) Maximize Z_1 = Ay_1 Subjectto By_1 \leq d_1, y_1 \geq 0, y_1 \leq y_2 (P3) Maximize Z_3 = Ay_3 Subjectto By_3 \leq d_3, y_3 \geq 0, y_3 \geq y_2$

4 Algorithm of longest path in directed a cyclic graph

Every directed acyclic graph has a topological sorting

Suppose we have labelled the vertices such as (g,h) is a directed edge only if g<h

Let OPT(h=length of the longest path ending at h.

Suppose that OPT(h) is (g_1,g_2) , (g_2,g_3) , …(g_{k-1},g_k) , (g_k,h) then;

Obs1 ; $g_1 \leq g_2 \leq \ldots \leq g_k \leq h$

Obs2 : (g_1,g_2) , (g_2,g_3) , …(g_{k-1},g_k) is the longest path ending at g_k

OPT(h)=1+OPT(g_k)

Suppose we have labelled the vertices such that (g,h) is a directed edge only if g<h

Let OPT(h)=length of the longest path ending at h.

$$OPT(h) = \left(\begin{array}{cc} 0 & if\ j\ is\ a\ source \\ 1 + max_{g:(g,h) an\ edge} OPT(g) & otherwise \end{array} \right\}$$

5 Procedure to solve

Step 1 Obtain the critical path of the network using longest path algorithm in directed a cyclic graph

Step 2 Obtain the direct cost and the cost slope of the fuzzy time cost trade-off problem using triangular fuzzy variable

Step 3 Using fully fuzzy mathematical structures; convert the fuzzy project network into a fuzzy linear programming problem.

Step 4 Decomposition technique is used to divide fuzzy linear programming problems into crisp linear programming problems.

Step 5 The corresponding variables contain the best solution for the crash duration and cost for each activity.

6 Illustration

The list of the activities involved in building a 992 square foot house, along with the pertinent data. Table 1 elaborates on the project's specifics. Three builders have provided quotes for the project, which are regarded as triangular fuzzy variables. Indirect cost of the project per day is (250,250,250). The project manager hopes to finish in (138,140,142) days (i.e.) around 4 to 5 months.

Table 1. Description of the project network

Activities	**Description**
1-2(α)	Preparation of site
1-3(β)	Basic levelling work
2-4(γ)	PPC& excavation
3-4(δ)	Barpending work
4-5(ε)	Plinth beam foundation work
5-6(ζ)	Super structure and column construction
6-7(η)	Brick work
7-8(θ)	Lintel over door window gaps
8-9(ϑ)	Roof construction
9-10(ι)	Plumbing and electrical work
9-11(κ)	Ceiling work
10-12(λ)	Tiles Pasting
11-12(μ)	Modular kitchen
12-13(ν)	Door windows framing
12-14(ξ)	Painting

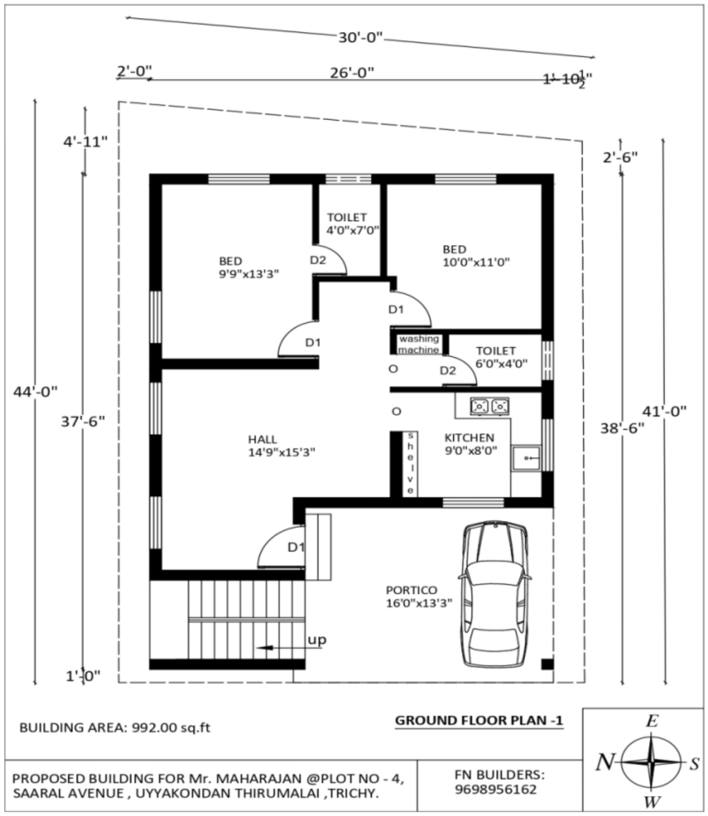

Fig 1. Blueprint of 992 square foot building area

Table 2. Quotations from three project managers are given below

Description	Quotation 1 Sakthi constructors	Quotation 2 Ananya Builders	Quotation 3 Bala Enterprises
House Area	2182400	2083200	1884800
Sump or Septic tank	50000	45000	50000
Head room	90000	100000	110000
Electricity board	7500	1000	7500
Motor and accessories	34000	38000	34000
Elevation and gate	95000	10000	150000
False Ceiling(Hall only)	65000	70000	75000
Modern kitchen	65000	70000	75000
PVC cupboard	30000	40000	25000
3 cupboard door	40000	40000	50000
Total	2593900	2596200	2386300

By using the above quotations cost, duration and cost slope for each activity is given as below.

Critical path of the network is obtained using longest path algorithm of directed a cyclic graph.

Table 3. Cost and duration of each activity

Activities	Cost	Duration	Cost Slope($\frac{\triangle c}{\triangle t}$)
1-2(α)	(26600,28000,30000)	(2,2,2)	-
1-3(β)	(18000,19000,20000)	(3,3,3)	-
2-4(γ)	(20000,25000,30000)	(4,4,4)	(5000,10000,15000)
3-4(δ)	(18000,19000,20000)	(3,3,3)	-
4-5(ε)	(200000,220000,300000)	(25,26,30)	(0,9000,12500)
5-6(ζ)	(300000,350000,400000)	(18,21,25)	(15000,16667,50000)
6-7(η)	(550000,570000,600000)	(19,20,25)	(0,6000,12500)
7-8(θ)	(450000,490000,500000)	(18,19,20)	(3333,10000,20000)
8-9(ϑ)	(270000,290000,300000)	(18,19,20)	(0,6667,6667)
9-10(ι)	(48000,49000,50000)	(2,2,2)	(0,20000,30000)
9-11(κ)	(36000,38000,40000)	(2,2,2)	-
10-12(λ)	(90000,95000,100000)	(3,4,5)	-
11-12(μ)	(180000,190000,200000)	(8,9,10)	(0,10000,13333)
12-13(ν)	(150000,155000,200000)	(8,9,10)	(0,75000,95000)
12-14(ξ)	(80000,85000,100000)	(10,11,12)	(5000,25000,40000)

Project total durations is (132,142,160)

Project Direct cost is (2262600, 2441000 and 2650000)

Project Total cost is (2295600, 2476500 and 2690000)

Network total duration, total cost and directs cost is obtained as crisp variables using decomposition method.

7 Conclusion

By use of the longest path algorithm in the directed a cyclic graph, critical path of the project network is obtained. This alternative procedure consume the duration of calculating the critical path for wide project network. For project managers handling large-scale projects like apartments, government initiatives, schools, etc., this approach is quite helpful.

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