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On the Solution of Blasius Boundary Layer Equations of Prandtl-Eyring Fluid Flow Past a Stretching Sheet

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Abstract

Objective: This paper investigates velocity profile for two-dimensional, incompressible, laminar forced convection flow of the fluid model for Prandtl-Eyring fluid past a stretching sheet in the presence of fluid parameters. **Methods:** The governing partial differential equation for the flow was transformed into non-linear ordinary differential equation by using the deductive one parameter group theoretic method and numerical solution of non-linear ordinary differential equation (ODE) is solved by MATLAB bvp4c solver. **Findings:** The solution of velocity profile obtained as a function of parameter α and β. The effect of the fluid parameter was discussed graphically. **Novelty:** The main goal of this article is to analyze boundary layer flow of Prandtl-Eyring fluid over a stretching surface. The conservation equations of mass, momentum are converted into non-linear ordinary differential equations along with boundary conditions using deductive one parameter group theoretic method and solved by MATLAB ODE solver. Comparisons with previously published works are made, and results show a high level of agreement. This type of research is applicable to extrusion, paper production, fiber glass production, hot rolling, condensation process, crystal growing, polymer sheets etc.

Keywords: Boundary layer; laminar flow; Deductive one parameter Group theoretic method; Absolute invariant; Stretching Sheet; Prandtl-Eyring fluid

1 Introduction

Boundary-layer flow on a continuously moving surface through a quiet ambient fluid is a significant topic in engineering because it has several applications in engineering processes. A number of engineering problems have been encountered where the flow is generated by continuous surface movement, which has numerous applications in industries such as rubber sheet manufacture, glass fiber production, paper production, petroleum extraction, polymer processing, filament extrusion continuously from a die and strengthening of copper wires.

The boundary layer flow of viscous incompressible fluid on moving surface with constant velocity was first examined by[(1)]. The boundary layer flow behavior on a continuous solid surface moving on both flat and the cylindrical surface was analyzed by[2,3]. The boundary layer equations of nonNewtonian Sisko fluid flow were analyzed by[4] using Shooting method over stretching sheet. The similarity equations for steady, two dimensional laminar incompressible boundary layer flow past a moving continuous surface in the presence of transverse magnetic field was derived by[5] using deductive group theoretic method. The problem of boundary layer flow for non-Newtonian power law fluid over continuous moving surface was studied by[6]. The invariance analysis of MHD boundary layer flow for non-Newtonian Williamson fluids past a permeable surface was made by[7]. The Powell-Eyring and Prandtl-Eyring models for heat transfer in forced convection boundary layer flow was studied by[8]. Three dimensional Eyring-Powell fluid flows over a stretching sheet with velocity slip and activation energy investigated by[9]. Mixed convection flow of viscoelastic fluid caused by a vertical stretched surface was studied by[10]. The steady flow of CassonWilliamson fluid on stretchable, impenetrable sheet with ohmic dissipation was investigated by[11]. The laminar boundary layer flow of magneto-hydrodynamics model for Power-law fluid over a continuous moving surface was analyzed by[12]. Water-based Cu and $CoFe_2O_4$ hybrid nano liquid flow was analyzed by[13] on a permeable curved sheet under impact of inertial and Lorentz forces. The impact of temperature dependent viscosity and variable thermal conductivity on nano fluid flow in a rotating system investigated by[14]. Analysis of entropy production and thermal feature of the system in the presence of non-linear radiation, cross-diffusion, activation energy, endothermic/exothermic reaction and dissipation effects were done by[15]. Flow of Magnetohydrodynamics (MHD) third grade liquid through Darcy-Forchheimer's porous space with homogeneous-heterogeneous features were studied by[16]. The entropy production of magnetized hybrid nanomaterials flowing through Darcy-Forchheimer space with varying permeability was investigated by[17]. The impact of heat source and dissipation phenomena on the flow of aqueous alumina-titania hybrid nanoparticle across a rotating channel was analysed by[18].

The similarity analysis of MHD boundary layer flow of non-Newtonian Prandtl-Eyring fluid was investigated by[19] using scaling group symmetry method and deductive group method. The Prandtl-Eyring fluid flow over curved geometry was analyzed by[20]. The effect of normally applied magnetic field on non-Newtonian Prandtl-Eyring fluid flow over stretching sheet analyzed by[21]. The MHD Prandtl-Eyring nano fluid over stretching sheet investigated by[22]. Entropy optimization in chemically radiated MHD flow of Prandtl-Eyring nanoliquid with activation energy over stretched surface was discussed by[23]. The MHD Prandtl-Eyring fluid flow over a linearly stretching surface was analyzed by[24]. Two dimensional MHD Prandtl fluid flow over stretching sheet was studied by[25]. The effect of magnetic field of the stagnation point flow of water base nano fluid was discussed by[26]. The MHD Prandtl nano fluid over a stretched sheet was investigated by[27] with convective boundary condition. A Mathematical model for stagnation point flow of MHD Prandtl-Eyring fluid over a stretchable cylinder was developed by[28]. The magneto-hydrodynamics (MHD) boundary layer flow of a Prandtl-Eyring fluid over a stretched sheet was studied by[29] using Keller box method. The nanofluid's cross flow which is caused by a nonlinear stretching sheet within boundary layer investigated by[30]. Entropy generation and heat transport of steady Prandtl-Eyring hybrid nanofluids are explored by[31].

In view of aforementioned literature survey, it is concluded that Prandtl-Eyring fluid flow on stretching sheet is discussed using deductive group theoretic method first time. In the paper, the problem of Blasius boundary layer for Prandtl-Eyring fluid the governing partial differential equations past a stretching sheet is discussed. The governing equations of the flow problem are non-linear partial differential equations. The similarity of these equations is obtained using one parameter deductive group theoretic method, the most powerful similarity method. The resulting higher order non-linear ODE are then solved using MATLAB bvp4c solver and presented graphically. We also carried out the analysis of different parameters α and β and presented graphically and discussed their effect on the velocity profile. A similar study was also carried out by[21] using shooting method.

2 Mathematical Formulation

The two-dimensional laminar boundary layer flows pasts a stretching sheet is considered. The governing equation of continuity and momentum equation of Blasius boundary layer flow of Prandtl-Eyring fluid past a stretching sheet are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial \tau_{yx}}{\partial y} \tag{2}$$

With boundary conditions

$$\left.\begin{array}{l} u=U(x),\ y=0 \\ v=0,\ y=0 \end{array}\right\} \tag{3}$$

$$u(x,y) \rightarrow 0 \text{ as } y \rightarrow \infty \tag{4}$$

$$\tau_{yx} = B \sinh^{-1}\left(\frac{1}{C}\frac{\partial^2 \psi}{\partial y^2}\right) \tag{5}$$

Using stream function $\psi(x,y)$, to reduce into one dependent variable which satisfies the Equation (1)

$$u = \frac{\partial \psi}{\partial y} \;\&\; v = -\frac{\partial \psi}{\partial x} \tag{6}$$

The dimensionless basic partial differential equations for forced convection flow of nonNewtonian fluid with stream function $\psi(x,y)$ can be derived as follows:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial \tau_{yx}}{\partial y} \tag{7}$$

Subject to boundary conditions:

$$y = 0 \Rightarrow \frac{\partial \psi}{\partial y} = U(x), -\frac{\partial \psi}{\partial x} = 0 \tag{8}$$

$$y \rightarrow \infty \Rightarrow \frac{\partial \psi}{\partial y} = 0 \tag{9}$$

With stress-strain relation

$$\tau_{yx} = B \sinh^{-1}\left(\frac{1}{C}\frac{\partial^2 \psi}{\partial y^2}\right) \tag{10}$$

Where, B and C are fluid parameters.

2.1 Deductive one Parameter Group Theoretic Method

In this paper, the deductive one parameter group theoretic method is used. Under this group transformation, two independent variables will be reduced by one, and boundary value type partial differential Equations (6), (7), (8), (9) and (10) which have two independent variables and transform into boundary value type ordinary differential equations in one independent variable, which is called similarity equation.

Introduced a one-parameter group transformation of the form

$$G: \begin{cases} \bar{x} = (h^x)(a)x + k^x(a) \\ \bar{y} = h^y(a)y + k^y(a) \\ \bar{\psi} = h^\psi(a)\psi + k^\psi(a) \\ \bar{U} = h^U(a)U + k^U(a) \end{cases} \tag{11}$$

Where ' a ' is the parameter of the transformation h^s and k^s are real valued and at least differentiable in their real argument ' a '. Equation (7) remains invariant under the group of transformations defined by G in Equation (11).

$$\frac{\partial \bar{\psi}}{\partial \bar{y}}\frac{\partial^2 \bar{\psi}}{\partial \bar{x}\partial \bar{y}} - \frac{\partial \bar{\psi}}{\partial \bar{x}}\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} - \frac{\partial \overline{\tau_{yx}}}{\partial \bar{y}} = \lambda(a)\left[\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \tau_{yx}}{\partial y}\right]$$

Therefore

$$\frac{\partial \bar{\psi}}{\partial \bar{y}}\frac{\partial^2 \bar{\psi}}{\partial \bar{x}\partial \bar{y}} - \frac{\partial \bar{\psi}}{\partial \bar{x}}\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} - \frac{1}{\sqrt{\frac{C^2}{B^2} + \frac{1}{B^2}\left(\frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2}\right)^2}}\frac{\partial^3 \bar{\psi}}{\partial \bar{y}^3}$$

$$= \lambda(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{1}{\sqrt{\frac{C^2}{B^2} + \frac{1}{B^2}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2}}\frac{\partial^3\psi}{\partial y^3}\right] \quad (12)$$

The invariance of Equation (12) gives,

$$\frac{\left(h^{2\psi}\right)}{\left(h^{4y}\right)} = 1$$

Therefore

$$h^{\psi} = h^{2y} = \left(h^{y}\right)^2 \quad (13)$$

and

$$\frac{\left(h^{2\psi}\right)}{h^{x}\left(h^{2y}\right)} = \frac{h^{\psi}}{h^{3y}} = \lambda(a)$$

Therefore,

$$h^{x} = h^{3y} = \left(h^{y}\right)^3 \quad (14)$$

The invariance of boundary condition gives:

$$k^{y} = 0,\ k^{U} = 0,\ h^{U} = h^{y} \quad (15)$$

Combining the results Equations (13), (14) and (15) into Equation (11) the group reduce in the following form

$$G: \begin{cases} \bar{x} = \left(h^{y}\right)^3(a)x + k^{x}(a) \\ \bar{y} = h^{y}(a)y \\ \overline{\psi} = \left(h^{y}\right)^2(a)\psi + k^{\psi}(a) \\ \overline{U} = h^{y}(a)U \end{cases} \quad (16)$$

Our aim to represent the problem in the form of ordinary differential equation. Now we have proceeded in our analysis to obtain a complete set of absolute invariants. If $\eta = \eta(x,y)$ is the absolute invariant of independent variables then,

$$g_j(x,y,\psi,U) = f_j(\eta)\ where,\ j = 1,2$$

is the absolute invariant for dependent variables ψ and U. Using these new variables original problem will convert into an ordinary differential equation in new similarity variable using group theoretic method. The application of a basic theory,[32,33] states that: A function $g(x,y,\psi,U)$ is an absolute invariant of a one-parameter group if it satisfies the following first-order linear differential equation,

$$\Sigma_{i=1}^{4}(\alpha_i S_i + \beta_i)\frac{\partial g}{\partial S_i} = 0 \quad (17)$$

Where, $S_i = x, y, \psi, U$

$$\alpha_i = \frac{\partial h^{S_i}}{\partial a}|_{a_0},\ \beta_i = \frac{\partial k^{S_i}}{\partial a}|_{a_0},\ Where\ i = 1,2,3,4 \quad (18)$$

and a_0 denotes the value of a which yields the identity element of the group G.

Equation (16) becomes,

$$(\alpha_1 x + \beta_1)\frac{\partial g}{\partial x} + (\alpha_2 y + \beta_2)\frac{\partial g}{\partial y} + (\alpha_3\psi + \beta_3)\frac{\partial g}{\partial\psi} + (\alpha_4 U + \beta_4)\frac{\partial g}{\partial U} = 0 \quad (19)$$

Using definition of $\alpha_i, \beta_i (i = 1,2,3,4)$ from Equations (13), (14) and (15) we obtain the relation between α_i 's and β_i 's as follows.

$$\alpha_1 = 3\alpha_2,\ \alpha_3 = 2\alpha_2,\ \alpha_4 = \alpha_2,\ \beta_2 = 0\ and\ \beta_4 = 0 \tag{20}$$

The absolute invariant of independent variable owing the Equation (19) is $\eta = \eta(x,y)$ if it satisfies the first order linear partial differential equation.

$$(\alpha_1 x + \beta_1)\frac{\partial \eta}{\partial x} + (\alpha_2 y + \beta_2)\frac{\partial \eta}{\partial y} = 0$$

Applying the variable separable method we get,

$$\eta = (x+\lambda)^{-\frac{1}{3}} y,\ where,\ \lambda = \frac{\beta_1}{\alpha_1} \tag{21}$$

Similarly, the absolute invariants for dependent variables owing (Equation (19)), one can derive, are:

$$\psi = (x+\lambda)^{\frac{2}{3}} f(\eta) - \frac{\beta_3}{\alpha_3} \tag{22}$$

$$U = c(x+\lambda)^{\frac{1}{3}} \tag{23}$$

2.2 Reduction to an ordinary differential equation

Independent and dependent absolute invariants are used to convert Equations (7), (8), (9) and (10) into the following nonlinear ordinary differential equations

$$\frac{1}{3\alpha}\sqrt{\beta + (f''(\eta))^2}((f'(\eta))^2 - 2f(\eta)f''(\eta)) = f'''(\eta) \tag{24}$$

Where, $\alpha = B$ and $\beta = C^2$,

Subject to the boundary conditions

$$\eta = 0 \Rightarrow f(0) = 0\ and\ f'(0) = 1 \tag{25}$$

$$\eta \to \infty \Rightarrow f'(\eta) = 0 \tag{26}$$

Finally, we have third order ordinary non-linear differential Equation (24) with boundary conditions (25) and (26) (Equations (25) and (26)) which is solved by using MATLAB software in the form of graph (Figures 1 and 2).

3 Result and Discussion

Numerical calculations are performed to analyze the behavior of the Blasius boundary layer model for Prandtl-Eyring fluid flow on stretching sheet. Differential Equation (24) subjected to the boundary conditions (25) & (26) (Equations (25) and (26)) are solved by MATLAB. The main reason behind solving the present problem is to determine the impact of parameter on velocity profile when fluid flow on a moving surface.

The following two cases depict the graphical representations of the numerical results for different governing parameters influencing the proposed model's flow behavior.

1. Figure 1 represents the graph of the velocity profile for different values of α. In Figure 1, it is observed that the value of parameter α increases then velocity increases.

2. Figure 2 represents the graph of the velocity profile for different values of β. In Figure 2, it is observed that the value of parameter β increases then velocity decreases.

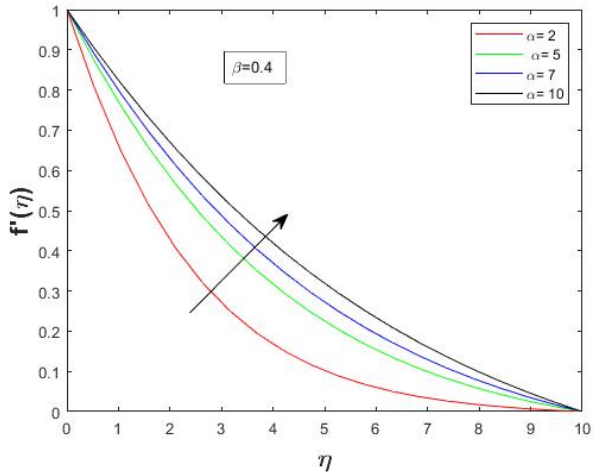

Fig 1. $f^{'}(\eta)$ **versus** η **for different values of** α

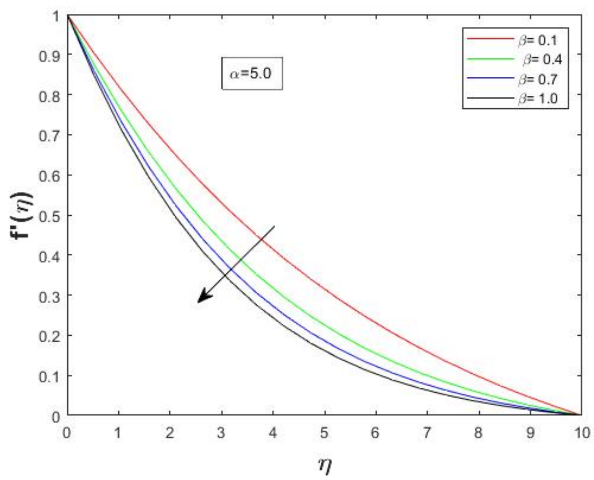

Fig 2. $f^{'}(\eta)$ **versus** η **for different values of** β

4 Conclusion

The present study shows the flow behavior of the Blasius boundary layer model for PrandtlEyring fluid flow on stretching sheet. The key findings of the current analysis are summarized as

- The parameter α is proportional to velocity.
- The parameter β is transversely proportional to velocity.

In the present work, we consider $\tau = Bsinh^{-1}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right)$ and in the article[21], consider $\tau = \frac{B}{C}\frac{\partial^2 u}{\partial y^2} - \frac{B}{6}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right)^3$ so, in the article[21] considered as approximation of $\tau = Bsinh^{-1}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right)$ so, as compare to[21] we have more accuracy in the result.

This helps in regulating the rate of fluid velocity in manufacturing processes and hence industrial applications to produce product of the appropriate quality.

5 Nomenclature

u Velocity in $x - direction\ \left(ms^{-1}\right)$; v Velocity in y-direction (ms^{-1}); ψ Stream function; $U(x)$ Velocity of main stream in x direction; G Group; B, C Material fluid parameter; η Independent Similarity variable; τ_{yx} Shear stress in the direction of x, perpendicular to y ; f Similarity function;α, β and $\alpha_1, \ldots \alpha_4$ and $\beta_1, \ldots \beta_4$ Real constants; λ Invariant conditional function; x,y Dimensional space coordinates (m) ; superscript ' differentiation with respect to η

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