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Lict k-Domination in Graphs

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Abstract

Objectives: In this study, we find lict k-domination number ($\gamma_{kn}(G)$) of various types of graphs. **Methods:** Let G be any graph, a set $D \subseteq V[n(G)]$ is said to be an k-dominating set of lict graph $n(G)$ if every vertex $v \in V[n(G)] - D$ is dominated by at least k vertices in D, that is $|N(v) \cap D| \geq k$. The Lict k-domination number $\gamma_{kn}(G)$ is the minimum cardinality of k-dominating set of $n(G)$. **Findings:** This study is centered on the lict k-domination number $\gamma_{kn}(G)$ of the graph G and developed its relationship with other different domination parameters. **Novelty:** This study introduces the concept of "Lict k-Domination in Graphs". It obtains many bounds on $\gamma_{kn}(G)$ in terms of vertices, edges, and other different parameters of G.

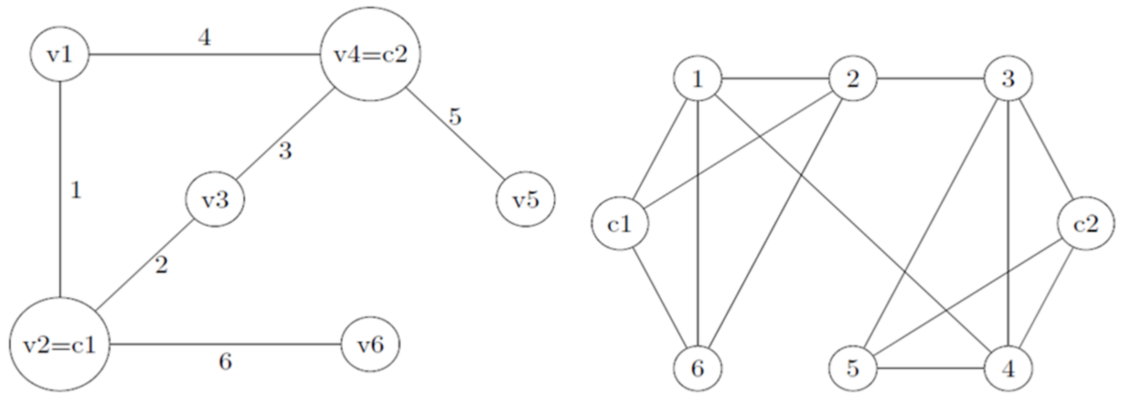
Keywords: Domination number; k-domination number; Lict graph; Lict k-domination number; Total domination number; Independent domination number

1 Introduction

This study considers simple, finite, non-trivial, undirected and connected graph denoted as $G(V,E) = (p,q)$, where V represents the vertex set and E denotes the edge set of the graph. The order and size of graph G is the $|V(G)| = p$ and $|E(G)| = q$. For our notation and corresponding definitions, we follow the F. Harary[1]. A lict graph[2] $n(G)$ associated with graph G is constructed with a vertex set that is the union of the sets of edges and cut vertices of graph G. In this context, two vertices in $n(G)$ are adjacent if and only if their corresponding edges in graph G are adjacent, or the corresponding elements of G are incident (Figure 1).

We begin by recalling some definitions from domination theory.

A set $D \subseteq V(G)$ is a dominating set if for each vertex $u \in D \subseteq V(G)$, $N(D) \bigcap D \neq \phi$. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. If every vertex in $V(G)$ is connected to at least one neighbor in D, then the subset $D \subseteq V(G)$ is defined as a total dominating set. The total domination number $\gamma_t(G)$ is represents the cardinality of a minimum total dominating set[3]. A subset $D \subseteq V(G)$ is an independent dominating set if the induced subgraph $< D >$ contains no edges. The independence domination number $\gamma_i(G)$ is corresponds to the cardinality of a minimum independence dominating set (see[4]).



Graph (G) Lict Graph [n(G)]

2 Related work

Many authors studied different domination in lict graph of graph. In [5] author introduce the new concept weak domination in lict graphs and determine upper and lower bounds for $\gamma_{wn}(G)$ in terms of parameter graph G. In the context of [6] the focus is on set domination in lict graphs and develop the properties of the set-domination. Also, relate $\gamma_s(G)$ with some domination parameter. In [7] studied Split Domination in Lict Subdivision Graph of a Graph and many bounds were obtained in terms of the various element of G. Additionally, they investigated relation between other different domination parameters of graph. Also, in Lict Subdivision Connected Domination in Graphs [8] work on $\gamma_{nsc}(G)$ to relate this with the components of G.

The Concept of k-domination and its generalization as total k-domination number $\gamma_{kt}(G)$ used in [9] strong product graph to obtain several bounds for the $\gamma_{kt}(G)$ of the strong product of two graphs. As in 2020, Ekinci G.B., Bujtas C. characterized bipartite graphs satisfying the equality for $k \geq 3$ and presented a necessary and sufficient condition for a bipartite graph to satisfy the equality hereditarily if $k = 3$ [10]. In 2022 [11], author studied relationships between the k-tuple domination number and parameters of graph and different dominations. Also, [12] studied the total k-domination number of Cartesian product of two complete graphs and obtained some lower and upper bounds for the total k-domination number of Cartesian product of two complete graphs.

This work uses the concept of k-domination within the lict graph of graph G. The lict k-domination number denoted as $\gamma_{kn}(G)$, represents the minimum cardinality among all k-dominating set of lict graph $n(G)$. Our investigation yields numerous bounds on $\gamma_{kn}(G)$ in terms various parameters such as vertices, edges, and other distinct parameters of G. Additionally, this study establishes relationships between $\gamma_{kn}(G)$ other different domination parameters of G.

3 Methodology

Theorem 2.1. [13,14] A subset $D \subseteq V(G)$ is k-dominating set if every vertex of $D \subseteq V(G)$ is adjacent to at least k vertices of D. The k-domination number $\gamma_k(G)$ represents the minimum cardinality of k-dominating set of G.

Theorem 2.2. [15] For any Tree T, $\gamma_{se}(n(T)) \leq c$, where c is the number of cut-vertices of T.

Theorem 2.3. [16] For any connected graph G, $\gamma_c(G) \geq \gamma(G)$.

Theorem 2.4. [16] For any connected graph G with $p \geq 2$ *vertices*, $\gamma_{ct}(G) \geq \gamma_t(G)$.

Theorem 2.5. [15] For any connected graph G, $\gamma_{se}(n(G)) \geq \gamma'(G)$.

Theorem 2.6. [17] For any graph G, $\gamma_t(n(G)) \geq \gamma'(G)$.

4 Results and Discussion

Theorem 3.1: If $G = (p,q)$ be any graph, then $\gamma_{kn}(G) \geq \frac{q+c}{\triangle'(n(G))+1}$.

Proof. Let $V(n(G))$ be the set $\{u_1, u_2, \ldots, u_{q+c}\}$, representing the vertex set of the lict graph of graph G. There exists a minimal set $A_1 = \{u_1, u_2, \ldots, u_{q+c}\} \subseteq V(n(G))$, such that every vertex in $V(n(G)) - A_1$ is adjacent to at least k vertices of A_1. This establishes A_1 as the minimal k-dominating set of $n(G)$. Clearly, $|A_1| = \gamma_{kn}(G)$.

It gives that each vertex in $V\left(n(G)\right)-\ A_1$ contributes at least one to the sum of degree of vertices of A_1. This implies that the size of set $(V\left(n(G)\right)-\ A_1$ is less than or equal to the sum of the degrees of vertices in A_1, given by:

$$|V\left(n\left(G\right)\right)-\ A_1|\le\sum_{u_i\in A_1}deg(u_i)$$

In the lict graph $n(G)$, there exists at least one vertex $u_j\in V(n(G))$ such that $deg(u_i)=\ \triangle^{'}\left(n\left(G\right)\right)$.

We can conclude that,

$$q+c-\ \gamma_{kn}\left(G\right)=|V\left(n\left(G\right)\right)-\ A_1|\le\sum_{u_i\in A_1}deg(u_i)$$

$$\le\ \gamma_{kn}\left(G\right)*\triangle^{'}\left(n\left(G\right)\right).$$

Therefore, $q+c-\ \gamma_{kn}\left(G\right)\le\ \ \gamma_{kn}\left(G\right)*\triangle^{'}\left(n\left(G\right)\right)$.

Thus, $\gamma_{kn}(G)\ge\frac{q+c}{\triangle^{'}(n(G))+1}$.

Theorem 3.2: If $G=(p,q)$ be any graph with $2\le k\le\ \triangle(G)$, then $\gamma_{kn}\left(G\right)\ge\ \gamma(G)+k-2$.

Proof. Let $V(G)=\left\{u_1,u_2,\ldots,u_p\right\}$ represent the vertex set a graph G. Then, there is subset D of $V(G)$ that covers all the vertices of G. This shows that D is dominating set of G. Consider $V\left(n\left(G\right)\right)=\left\{w_1,w_2,\ldots,w_{q+c}\right\}$ as the vertex set of lict graph of graph G, and let $S\subseteq V(n(G))$ be a minimum k-dominating set of $n(G)$.

Now, take $w\in V\left(n\left(G\right)\right)-S$, and let $w_1,w_2,\ldots,w_k$ be distinct vertices in S which dominate w. Since S is a k-dominating set, each vertex in $V\left(n\left(G\right)\right)-S$ is dominated by at least one vertex in $S-\{w_2,w_3,\ldots,w_k\}$. Therefore, w dominates each vertex in $\{w_2,w_3,\ldots,w_k\}$.

Which implies,

$$|D|\le|S-\{w_2,w_3,\ldots,w_k\}\cup\{w\}|$$

$\gamma(G)\le\gamma_{kn}\left(G\right)-(k-1)+1$

$=\gamma_{kn}\left(G\right)-k+2$

Thus, $\gamma_{kn}\left(G\right)\ge\ \gamma(G)+k-2$.

Theorem 3.3: For any graph $G=(p,q)$ and a positive integer $k\ge2$, then $\gamma_{kn}\left(G\right)\ge c$. Where c is number of cut vertices of G.

Proof. Case-I: If the graph has no cut vertex, then result holds trivially.

Case-II: If the graph has cut vertices.

Assume $A_1=\{m_1,m_2,\ldots,m_c\}$ be subset of a vertex set of $n(G)$ contains cut vertices of G. Let $V\left(n(G)\right)=\left\{w_1,w_2,\ldots,w_q,m_1,m_2,\ldots,m_c\right\}$ be a vertex set of $n(G)$, where $w_1,w_2,\ldots,w_q$ corresponds to edges incident on vertices of graph G.

Consider a subset D_1 of $V\left(n\left(G\right)\right)$, which contains vertices of $n(G)$ which corresponding to edges incident with cut vertices of graph G. Then, D_1 forms a minimal k-dominating set of lict graph of G. Otherwise, we can add cut vertices in D_1 to get a k-dominating set of $V\left(n\left(G\right)\right)$. A cardinality of such a set is greater or equal to the number of cut vertices of G.

Therefore, $\gamma_{kn}\left(G\right)\ge c$.

Corollary 3.4: For any tree T and a positive integer $k\ge2$ then $\gamma_{kn}\left(T\right)\ge\gamma_{se}\left(n(T)\right)$.

Proof. From Theorem 2.2, we have

$$\gamma_{se}\left(n(T)\right)\le c \tag{1}$$

From Theorem 3.3, we get

$$\gamma_{kn}\left(G\right)\ge c \tag{2}$$

From Equations (1) and (2), we Conclude that, this is required result.

Theorem 3.5: If $G=(p,q)$ is any graph and k is a positive integer with $k\ge2$, then $\gamma_{kn}\left(G\right)\ge\gamma_i\left(G\right)$.

Proof. Let $A=\{u_1,u_2,\ldots,u_s\}$ be a dominating set of G. If the induces subgraph $<A>$ contains vertices of degree zero, then A itself is an independent dominating set. Otherwise, let $S=A_1\cup A_2$, where $A_1\subseteq A$ and $A_2\subseteq V\left(G\right)-A$ forming a minimal independent dominating set of G such that $|S|=\gamma_i\left(G\right)$.

Now, consider $D=\{e_1,e_2,\ldots,e_s,m_1,m_2,\ldots,m_t\}$, a subset of vertex set of $V\left(n\left(G\right)\right)$, where $e_i^{'}$s are edges incident on vertices of set S and $m_i^{'}$s are the cut vertices of G. If every vertex of $V\left(n\left(G\right)\right)-D$ is adjacent to at least k vertices of D then D itself is a k-dominating set of $n(G)$. Otherwise, let $S_1 = D_1 \cup D_2$, where $D_1 \subseteq D$ and $D_2 \subseteq V(n(G)) - D$, forms minimal k-dominating set of $n(G)$, and it gives $|S| \leq |S_1|$.

Thus, $\gamma_{kn}(G) \geq \gamma_i(G)$.

Theorem 3.6: If $G = (p,q)$ is any graph and k is a positive integer with $k \leq \triangle(G)$, then $\gamma_{kn}(G) + \gamma(n(G)) \geq q + c$.

Proof. For $k \leq \triangle(G)$, let $S = \{w_1, w_2, \ldots, w_r\} \subseteq V(n(G))$ be minimal k-dominating set of $n(G)$. Assume that there exists vertex $w \in S$ that is not adjacent to any vertex in $V(n(G)) - S$. This implies that w is adjacent to at least k vertices of set S. This Shows that the set $S - \{w\}$ is minimal k-dominating set of $n(G)$. Which contradicts the set S is minimal k-dominating set. Therefore, every vertex in S is adjacent to at least one vertex of $V(n(G)) - S$. This implies $V(n(G)) - S$ is dominating set of $n(G)$. Clearly, there exists set $D \subseteq [V(n(G)) - S]$ is minimal dominating set of $n(G)$ such that $|D| + |S| \leq q + c$.

Thus, $\gamma_{kn}(G) + \gamma(n(G)) \geq q + c$.

Theorem 3.7: If $G = (p,q)$ is any graph. Then, $\gamma'(G) \leq \gamma(n(G)) \leq \gamma_{kn}(G)$.

Proof. Case-I: If the graph has no cut vertex, the result holds obviously.

Case-II: If the graph has cut vertices.

Let $E(G) = \{e_1, e_2, \ldots, e_q\}$ be edge set of G, and $C(G) = \{m_1, m_2, \ldots, m_c\}$ be set of cut vertices of G. The vertex set of lict graph is $V(n(G)) = E(G) \cup C(G)$. Let $E_1 = \{e_1, e_2, \ldots, e_s\} \subseteq E(G)$ be minimal edge dominating set of G such that $|E| = \gamma'(G)$. Since $E(G) \subseteq V(n(G))$, if set E_1 forms a dominating set of $n(G)$ then result holds. Otherwise, let $A = V(n(G)) - E_1$, then $A_1 \cup E_2$, where $A_1 \subseteq A$ and $E_2 \subseteq E_1$ covers all vertices of $n(G)$, and which is minimal dominating set of $n(G)$.

Clearly, $|E_1| \leq |A_1 \cup E_2|$.

Therefore, $\gamma'(G) \leq \gamma(n(G))$. Also, we get required result.

Theorem 3.8: If $G = (p,q)$ is any graph. Then, $\gamma_{kn}(G) \leq q + \alpha_0(G)$.

Proof. Case-I: If the graph has no cut vertex, the result holds obviously.

Case-II: If the graph has cut vertices.

As $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ contains q edges and c cut vertices. Let $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ Subset $V(n(G))$ corresponding to edges of graph G. A set $A = \{u_1, u_2, \ldots, u_s\}$ be the vertex cover of G, which also contain a cut vertices of graph G. Consider $E_1 \subseteq E$ and $A_1 \subseteq A$, then $A_1 \cup E_1$ forms minimal a k-dominating set of $n(G)$ such that $|A_1 \cup E_1| \leq |E| + |A|$.

We get, $\gamma_{kn}(G) \leq q + \alpha_0(G)$.

Theorem 3.9: Let $G = (p,q)$ be any graph. Then, $\gamma_{kn}(G) \leq p + \alpha_1(G)$.

Proof. Let $A = \{u_1, u_2, \ldots, u_p\}$ be vertex set of graph G. A set $B = \{e_1, e_2, \ldots, e_s\}$ is edge cover of graph G such that $|B| = \alpha_1(G)$.

As $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ is the vertex set of $n(G)$, which is a collection of edges and cut vertices of G. We take a set $D_1 = \{w_1, w_2, \ldots, w_q\}$ be the edges incident on vertices of G, and $C_1 = \{w_{q+1}, w_{q+2}, \ldots, w_{q+i}\}$ are the cut vertices adjacent to element of edge cover of G. Consider $D_2 \subseteq D_1$ and $C_2 \subseteq C_1$, then $D_2 \cup C_2$ forms minimal a k-dominating set of $n(G)$.

Clearly, $|D_2 \cup C_2| \leq |A| + |B|$.

Thus, $\gamma_{kn}(G) \leq p + \alpha_1(G)$.

Theorem 3.10: Let $G = (p,q)$ be any graph. Then, $\gamma_{kn}(G) < q + diam(G)$.

Proof. Case-I: If the graph has no cut vertex, the vertex set of lict graph contains vertices, which corresponds to edges of G, such that $|V(n(G))| = q$. This follows the result.

Case-II: If the graph has cut vertices.

As vertex set of $n(G)$ is $V(n(G)) = E(G) \cup C(G)$, where $E(G)$ is the edge set of G and $C(G)$ is set of cut vertices of G. Let $E = \{w_1, w_2, \ldots, w_q\}$ subset $V(n(G))$ corresponding to the edges of graph G. A set $D = \{w_{q+1}, w_{q+2}, \ldots, w_{q+c}\}$ be cut vertices of G, which are lie on the longest path between two vertices u_1 and u_2 of G such that $dist(u_1, u_2) = diam(G)$. Consider $E_1 \subseteq E$ and $D_1 \subset D$, then $E_1 \cup D_1$ forms minimal a k-dominating set of $n(G)$.

It follows that, $|E_1 \cup D_1| < |E| + dist(u_1, u_2)$.

We get, $\gamma_{kn}(G) < q + diam(G)$.

Theorem 3.11: If $G = (p,q)$ is any graph. Then, $\gamma_{kn}(G) < q + \beta_0(G)$.

Proof. Case-I: If the graph has no cut vertex, the result holds.

Case-II: If the graph has cut vertices.

A vertex set of $n(G)$ is a collection those vertices which corresponds to edges and cut vertices of G. Let $E = \{w_1, w_2, \ldots, w_q\}$ subset $V(n(G))$ corresponding to edges of graph G, and set $B = \{u_1, u_2, \ldots, u_s\}$ be the maximum independent set of G such that $|B| = \beta_0(G)$. Consider set $C = \{w_{q+1}, w_{q+2}, \ldots, w_{q+c}\}$ contain cut vertices of G which are either member of B or adjacent to elements of B such that $|C| < |B|$.

Let $E_1 \subseteq E$ and $C_1 \subset C$, then $E_1 \cup C_1$ forms minimal a k-dominating set of $n(G)$.

Clearly, $|E_1 \cup C_1| < |E| + |B|$.

Thus, $\gamma_{kn}(G) < q + \beta_0(G)$.

Theorem 3.12: If $G = (p,q)$ is any graph. Then, $\gamma_{kn}(G) \leq q + \gamma_t(G)$.

Proof. Case-I: If the graph has no cut vertex, then result holds.

Case-II: If the graph has cut vertices.

Let $D = \{u_1, u_2, \ldots, u_s\}$ be a dominating set of G and $D_1 = V(G) - D$ be a set such that $H \subseteq D_1$ with a minimum set of vertices. If $< D >$ does not contain an isolated vertex, then D itself total dominating set of G. Otherwise, some $u_i \in H$ such that $\forall\, u_j \in D$, $(u_i, u_j) \in E(G)$

and $< D_1 > = < D \cup \{u_i\} >$ has no isolated vertex. Then $D_1 = D \cup \{u_i\}$ is a total dominating set of G such that $D_1 = \gamma_t(G)$.

Let $E = \{w_1, w_2, \ldots, w_q\}$ subset $V(n(G))$ corresponding to edges of graph G. A set $D_1 = \{u_1, u_2, \ldots, u_t\}$ be a minimal total dominating set, and the cut vertices are also member of such set. Let $E_1 \subseteq E$ and $D_2 \subseteq D_1$, then $E_1 \cup D_2$ forms minimal a k-dominating set of $n(G)$. It gives, $|E_1 \cup D_2| \leq |E| + |D_1|$.

Hence, the result.

Theorem 3.13: If $G = (p,q)$ is any graph and k is a positive integer with $k \geq 2$, then $\gamma_{kn}(G) \geq \frac{p}{\triangle'(G)}$

Proof. Let $V(G) = \{u_1, u_2, \ldots, u_p\}$ be vertex set of G, and $E(G) = \{e_1, e_2, \ldots, e_q\}$ are edges incident on vertices of G. Which are members of vertex set of $n(G)$, and set $E(G) = \{e_1, e_2, \ldots, e_q\}$ cut vertices of G.

For $k \geq 2, V(n(G)) = E(G) \cup C(G)$ be a vertex set of lict graph of G. Then, there exists a set B such that every vertex of $V(n(G)) - B$ adjacent to at least k vertices of B. This implies that B forms minimal k-dominating set of $n(G)$. for every graph, there exists an edge $e \in E(G)$ of maximum degree, i.e., $deg(e) = \triangle'(G)$ Such that, $\triangle'(G) * |B| \geq |V(G)|$.

$\therefore\ \triangle'(G) * \gamma_{kn}(G) \geq p$

Thus, $\gamma_{kn}(G) \geq \frac{p}{\triangle'(G)}$.

Theorem 3.14: If $G = (p,q)$ is any graph, then $\gamma_{kn}(G) + \gamma_t(G) \leq q + p$.

Proof. Case-I: If the graph has no cut vertex, then $\gamma_{kn}(G) \leq q$ and $\gamma_t(G) \leq p$. We get required result.

Case-II: If the graph has cut vertices.

Let $V(G) = \{u_1, u_2, \ldots, u_p\}$ be vertex set of G then there exists a set A= $\{u_1, u_2, \ldots, u_s\} \subseteq V(G)$ such that $< A >$ has a no isolated vertex. This shows A is total dominating set of G and $|A| = \gamma_t(G)$.

Let set $C = \{m_1, m_2, \ldots, m_c\}$ be the cut vertices of G and $E = \{e_1, e_2, \ldots, e_q\}$ be edge set of G. Which are the members of vertex set of $n(G)$. Consider $E_1 \subseteq E$ and $C_1 \subseteq C$, then $E_1 \cup C_1$ forms minimal a k-dominating set of $n(G)$ such that $|E_1 \cup C_1| + |A| \leq |E(G)| + |V(G)|$.

Thus, $\gamma_{kn}(G) + \gamma_t(G) \leq q + p$.

Theorem 3.15: If $G = (p,q)$ is any graph, then $\gamma_{kn}(G) + \gamma(G) \geq \gamma_t(G)$.

Proof. Case-I: If $\gamma(G) = \gamma_t(G)$, then the result holds clearly.

Case-II: If $\gamma(G) \neq \gamma_t(G)$.

Let $D = \{u_1, u_2, \ldots, u_p\}$ be a dominating set of G and $< D >$ contain isolated vertex. Then, there exists $u_i \in V(G) - D$ such that $\forall\, u_j \in D$, $(u_i, u_j) \in E(G)$ and $< D_1 > = < D \cup \{u_i\} >$ has no isolated vertex. Then, $D_1 = D \cup \{u_i\}$ is total dominating set of G. Which contains cut vertices of G and $V(n(G))$ be vertex set of $n(G)$ contains vertices which correspond to edges and cut vertices of G. So, there is minimal set $A \subseteq V(n(G))$ which is k-dominating set of $n(G)$ such that, $|A| + |D| \geq |D_1|$.

We get, $\gamma_{kn}(G) + \gamma(G) \geq \gamma_t(G)$.

Corollary 3.16: If $G = (p,q)$ is any graph, then $\gamma_{kn}(G) + \gamma_c(G) . \geq \gamma_t(G)$.

Proof. Using Theorem 2.2 and Theorem 3.16, we get required result.

Theorem 3.17: If $G = (p,q)$ is any graph, then $\gamma_{kn}(G) + \gamma_i(G) \leq q + p$.

Proof. Case-I: If the graph has no cut vertex, then $\gamma_{kn}(G) \leq q$ and $\gamma_i(G) \leq p$. We get required result.

Case-II: If the graph has cut vertices.

Let $A = \{u_1, u_2, \ldots, u_s\}$ be dominating set of G. If induced subgraph $< A >$ contains vertices of degree zero, then set A itself minimal independent dominating set of G. Otherwise, $S = A_1 \cup A_2$, where $A_1 \subseteq A$ and $A_2 \subseteq V(G) - A$ forms minimal independent dominating set of G such that $|S| = \gamma_i(G)$.

A set $E = \{e_1, e_2, \ldots, e_q\}$ be edge set of G and $E = \{e_1, e_2, \ldots, e_q\}$ be set of cut vertices of G. The vertex set of lict graph is $V(n(G)) = E(G) \cup C(G)$. Consider $E_1 \cup C_1$, where $E_1 \subseteq E$ and $C_1 \subseteq C$ forms minimal a k-dominating set of $n(G)$ such that $|E_1 \cup C_1| + |S| \leq |V(G)| + |E(G)|$.

Thus, $\gamma_{kn}(G) + \gamma_i(G) \leq q + p$.

Theorem 3.18: If $G = (p,q)$ is any graph, then $\gamma_{kn}(G) \leq diam(G) + \beta_0(G) + \triangle'(G)$.

Proof. Case-I: If the graph has no cut vertex.

In this scenario, $V(n(G)) = \{w_1, w_2, \ldots, w_q\}$ is the vertex set of $n(G)$ with vertices corresponding to edges incident on vertices of G. Let $D = \{e_i \mid 1 \le i \le s\}$ be the edges lying on the longest path between two vertices u and v of G. Additionally, let $A = \{u_1, u_2, \ldots, u_t\}$ be maximum independent set of G and $B = \{e_j \mid 1 \le j \le m\}$ be edges such that e_j incident on u_j, for all $u_j \in A$. Set $C = \{e_r \mid 1 \le r \le l\}$ consist of edges adjacent to an edge of maximum degree other than the element of D and B such that $(C| \le \triangle'(G)$. Consider $D_1 \cup B_1 \cup C_1$, where $D_1 \subseteq D, C_1 \subseteq C$ and $B_1 \subseteq B$ forms minimal a k-dominating set of $n(G)$, such that $|D_1 \cup B_1 \cup C_1| \le |D| + |B| + |C|$.

This implies that, $\gamma_{kn}(G) \le diam(G) + \beta_0(G) + \triangle'(G)$.

Case-II: If the graph has cut vertices.

In this case, $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ be vertex set of $n(G)$ with vertices of lict graph corresponding to edges and cut vertices of G. Let $U = \{u_1, u_2, \ldots, u_s\}$ contain vertices of lict graph corresponding to all cut vertices lying on the longest path between two vertices u and v of G such that |U| = diam(G). Additionally, let $T = \{e_r \mid 1 \le r \le l\}$ be edges adjacent to an edge of maximum degree such that $|T| = \triangle'(G)$, and let $S = \{e_i \mid 1 \le i \le t\}$ contains edges such that e_i incident on u_i, for all $u_i \in A$, where $A = \{u_1, u_2, \ldots, u_m\}$ is maximum independent set. Consider $U_1 \cup T_1 \cup S_1$, where $U_1 \subseteq U$, $T_1 \subseteq T$ and $S_1 \subseteq S$ forms minimal a k-dominating set of $n(G)$, such that $|U_1 \cup T_1 \cup S_1| \le |U| + |T| + |S|$.

This implies that, $\gamma_{kn}(G) \le diam(G) + \beta_0(G) + \triangle'(G)$.

Thus, it follows the result.

Theorem 3.19: If $G = (p,q)$ is any graph and k is a positive integer with $k \ge 2$, then $\gamma_{kn}(G) \ge \beta_1(G)$.

Proof. Let $A = \{e_1, e_2, \ldots, e_s\}$ be maximum subset of the edge set of G. Where no two edges from set A are adjacent. Therefore, $|A| = \beta_1(G)$. A set $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ be vertex set of $n(G)$ with vertices correspond to edges and cut vertices of G.

For $k \ge 2$, if every vertex of $V(n(G)) - A$ is adjacent to at least k vertices of A, then A is minimal k-dominating set of $n(G)$. Otherwise, by adding vertices from $V(n(G)) - A$ to set A, we can obtain a minimal k-dominating set of $n(G)$.

Clearly, $\gamma_{kn}(G) \ge \beta_1(G)$.

This completes the proof.

Theorem 3.20: If $G = (p,q)$ be any graph, then $\gamma_{kn}(G) \le \alpha_1(G) + \alpha_0(G) + \gamma_t(G)$.

Proof. Let $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ be vertex set of $n(G)$ and members of $V(n(G))$ correspond to edges and cut vertices of G. Let $A_1 = \{e_i \mid 1 \le i \le s\}$ be minimal edge cover set of G such that $|A_1| = \alpha_1(G)$. Also, let $A_2 = \{e_j \mid 1 \le j \le t\}$ contains edges of G incident on vertices of minimal vertex cover set of G such that $|A_2| = \alpha_0(G)$. Additionally, let $T = \{u_1, u_2, \ldots, u_m\}$ be a minimal total dominating set of G, where cut vertices of G are members of T.

Consider, $A \subseteq A_1$, $A_3 \subseteq A_2$ and $T_1 \subseteq T$, then $A \cup A_3 \cup T_1$ forms minimal a T k-dominating set of $n(G)$ such that: $|A \cup A_3 \cup T_1| \le |A_1| + |A_2| + |T|$.

Therefore, $\gamma_{kn}(G) \le \alpha_1(G) + \alpha_0(G) + \gamma_t(G)$.

Theorem 3.21: For any graph $G = (p,q)$ and a positive integer k. If $k = \triangle(n(G))$ then $\gamma_{kn}(G) \ge q + c - s$, where s is number of vertices of degree $\triangle(n(G))$.

Proof. The lict graph has $q + c$ vertices, consisting q edges and c cut vertices. Let $S = \{w_1, w_2, \ldots, w_s\}$ contains a vertices of degree $\triangle(n(G))$. The proof is divided into two cases.

Case-I:$|S| = 1$.

In this case, the result hold trivially.

Case-II:$|S| > 1$,

Subcase-I. For $k = \triangle(n(G))$, if the members of S are adjacent to each other:

In this scenario, a k-dominating set contain $|S| - 1$ vertices from S. We get required result.

Subcase-II. For $k = \triangle(n(G))$, if the members of S are not adjacent to each other:

In this case, there exists k members from $V(n(G))$ other than member of S adjacent to every vertex of S. we get, $\gamma_{kn}(G) \ge q + c - s$.

Therefore, the theorem holds true.

Theorem 3.22: If $G = (p,q)$ be any graph, then $\gamma_{kn}(G) \le \gamma_{kl}(G) + c$, where c is number of cut vertices of G.

Proof. Case-I: If the graph has no cut vertex:

In this case, the result holds because the lict graph coincides with line graph.

Case-II: If the graph has cut vertices:

Let D be k dominating set of $L(G)$ and S be set of cut vertices of G, which are not incident with any edge of D. For every cut vertex $u \in S$, consider exactly one edge in E_1, where $E_1 = \{e_j \in E(G) \mid e_j \text{ incident with } u \text{ and } e_j \in N(D)\}$ such that, $|E_1| \le |S|$. This implies that $D \cup E_1$ forms minimal k-dominating set of lict graph of G. It follows that $\gamma_{kn}(G) \le \gamma_{kl}(G) + |S|$.

Therefore, $\gamma_{kn}(G) \le \gamma_{kl}(G) + c$.

Theorem 3.23: If $G = (p,q)$ be any graph and a positive integer k with $k \leq \delta(G)$, then $\gamma_{kn}(G) \leq p$

Proof. Let $V(G) = \{u_1, u_2, \ldots, u_p\}$ be vertex set of G and $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ be the vertex set of lict graph of G. We consider subset $D_1 = \{w_1, w_2, \ldots, w_{q+c}\}$ of $V(n(G))$, which corresponds to edges incident on vertices of G and cut vertices of G.

For $k \leq \delta(G)$, there exists $D_2 \subseteq D_1$ is minimal k-dominating set of lict graph of G such that $|D_2| \leq |V(G)| = p$.

Therefore, $\gamma_{kn}(G) \leq p$, and the proof is complete.

Theorem 3.24: If $G = (p,q)$ be any graph and a positive integer k with $k \leq \delta(G)$, then $\gamma_{kn}(G) \leq q$.

Proof. Let $V(n(G)) = \{w_1, w_2, \ldots, w_{q+c}\}$ be the vertex set of lict graph of G. Consider the subset $E = \{w_1, w_2, \ldots, w_q\}$ of $V(n(G))$ with vertices of E corresponding to edges incident on vertices of G such that $|E| = q$.

For $k \leq \delta(G)$, then there exists a possible $E_1 \subseteq E$ that is minimal k-dominating set of lict graph of G such that $|E_1| \leq |E|$.

Therefore, $\gamma_{kn}(G) \leq q$.

Theorem 3.25: If $G = (p,q)$ be any graph and a positive integer k with $k \geq \triangle(G)$, then $\gamma_{kn}(G) \geq \triangle(G)$.

Proof. Since for every k, and $k \geq \triangle(n(G))$, it follows that $\gamma_{kn}(G) \geq \triangle(n(G))$.

For any graph $\triangle(n(G)) \geq \triangle(G)$.

Therefore, $\gamma_{kn}(G) \geq \triangle(G)$.

Theorem 3.26: If $G = (p,q)$ be any graph with $p \geq 2$, then $\gamma_{kn}(G) + \gamma'(G) \leq q + \gamma_{ct}(G) + \gamma_{se}(n(G))$.

Proof. Using Theorem 2.3, Theorem 2.5 and Theorem 3.12, we get the required result.

Theorem 3.27: If $G = (p,q)$ be any graph with $p \geq 2$, then $\gamma_{kn}(G) + \gamma'(G) \leq q + \gamma_{ct}(G) + \gamma_t(n(G))$.

Proof. Using Theorem 2.3, Theorem 2.6 and Theorem 3.12, we get the required result.

5 Conclusion

This study has computed the results regarding k-domination in lict graph. The study involves deriving theorems specifically related to lict k-domination and relationships with various other domination parameters. Furthermore, this investigation derivation of lict k-domination number, denoted as $\gamma_{kn}(G)$, considering factors such as vertices, edges, and various other parameters of graph G.

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