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Radio Contra Harmonic Mean Number of Graphs

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Abstract

Objectives: To explore the least upper bound of graphs by radio contra harmonic labeling. **Methods**: Contra harmonic mean function $\left\lceil \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rfloor$, radio mean labeling condition $d(u,v)+\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1+diam(G)$ and radio harmonic mean labeling $d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$ are used. **Findings**: Here we introduce radio contra harmonic mean labeling and its least upper bound, designated as radio contra harmonic mean number, by formulating the constraints. **Novelty**: Based on radio mean labeling and contra harmonic mean labeling, the new concept of radio contra harmonic mean labeling was established. The bounds of some special graphs are encountered here. This kind of labeling is employed in secure communication networks and is also applicable in X-rays, crystallography, coding theory, computing etc.

Keywords: Radio contra harmonic mean labeling; rchmn (G); Order; Diameter; Smallest span

1 Introduction

Graph labeling is a recently developed area in graph theory; for a detailed survey, we refer to Gallian [1]. Hale's radio frequency classification problems were implemented in 1980. The idea of contra harmonic mean labeling was developed by S. S. Sandhya, S. Somasundaram, and J. Rajeshni Golda. R. Sridevi and A L. Yakavi studied the contra harmonic mean labeling of derived graphs [2]. Motivated by channel assignment of FM radio stations, in 2001, G. Chartrand described the term radio labeling of graphs. For coding theory, crystallography, X-ray, routing, computer, etc., radio labeling is employed. R. Ponraj presented the radio mean labeling of graph notation and determined the radio mean number of several graphs in the series. K. Palani, S.S. Sabarina Subi, and V. Maheswari described radio mean labeling of path union of graphs [3]. A radio mean labeling is a one-to-one mapping f from $V(G)$ to N satisfying the condition $d(u,v)+\left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1+diam(G)$ for every $u,v \in V(G)$. The span of a labeling f is the maximum integer that f maps to a vertex of G. The radio mean number of G and $rmn(G)$ is the lowest span taken over all radio mean labelings of the graph G.

Also, R. Revathy and Dr. K. Amuthavalli studied the radio harmonic mean labeling of some star related graphs[4]. A radio mean labeling is a one-to-one mapping f from $V(G)$ to N satisfying the condition $d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$ for every $u,v \in V(G)$. The span of a labeling f is the maximum integer that f maps to a vertex of G. The radio mean number of G and $rmn(G)$ is the lowest span taken over all radio mean labelings of the graph G. K. John Bosco and B.S. Vishnupriya introduced the radio Dd-distance in harmonic mean number of some new graphs[5]. These events served as our inspiration for introducing radio contra harmonic mean labeling and radio contra harmonic mean number of graphs. This study examines the radio contra harmonic mean number and labeling of many typical graphs, including[1] $G_n \odot K_1$, $(W_n \odot mK_1) \odot K_1$, $J(m,n)$, and $P_2(B_{m,n})$.

2 Methodology

2.1 Methodology

A simple graph with p vertices and q edges is given by $V=(G,\ E)$. For a connected graph G, the radio contra harmonic mean labeling is defined as an injective function $f:\ V(G) \to \mathbb{Z}^+$ such that for any two distinct vertices $u,\ v$

$$d(u,v)+\left\lceil \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rceil \geq 1+diam(G)\ \forall\, u,v \in V(G) \tag{1}$$

or

$$d(u,v)+\left\lfloor \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rfloor \geq 1+diam(G)\ \forall\, u,v \in V(G) \tag{2}$$

The radio contra harmonic mean number of f is represented as $rchmn(f)$ is the highest integer allocated to any vertex $v \in V(G)$ under the mapping f. Further, the radio contra harmonic mean number of G is presented as $rchmn(G)$, which is the smallest span of $rchmn(f)$ taken across every radio contra harmonic mean labeling of G. It is obvious from the description that $rchmn(G) \geq |V(G)|$. If $rchmn(G)=|V(G)|$, then G is referred as radio contra harmonic mean graceful graph.

3 Results and Discussion

Theorem 3.1. $G_n \odot K_1$ is a radio contra harmonic mean graceful graph for $n \geq 3$.

Proof: Let $u_1,\ u_2,\ u_3,\ldots,u_{2n+1}$ be the vertices of the gear graph G_n. Let v_i be the vertices connected to $u_i,\ 1 \leq i \leq 2n+1$. The resultant graph is $G_n \odot K_1$.

Case 1: $n=3,\ 4$

The diameter of the graph $G_n \odot K_1$ is 5 for n=3, 4.

Then the inequality [Equation (1)] reduces to

$$d(u,v)+\left\lceil \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rceil \geq 6 \tag{3}$$

Define a function $f:\ V(G_n \odot K_1) \to \mathbb{Z}^+$ by

$f(v_i)=i,\ 1 \leq i \leq 2n+1$ such that $d(v_1, v_2)=4$

$f(u_i)=2n+i+1,\ 1 \leq i \leq 2n+1$

Clearly, f is an injective function.

Now to show that the graph satisfies the inequality [Equation (3)] by considering the following possibilities:

i) Look at the pair (u_i,u_j) of vertices with $d(u_i,u_j) \geq 1$ where $1 \leq i,\ j \leq 2n+1,\ i \neq j,\ d(u_i,u_j)+\left\lceil \frac{(2n+i+1)^2+(2n+j+1)^2}{4n+i+j+2} \right\rceil \geq 6$

ii) Look at the pair (v_i,v_j) of vertices with $d(v_i,v_j) \geq 3$ where $1 \leq i,\ j \leq 2n+1,\ i \neq j,\ d(v_i,v_j)+\left\lceil \frac{(i)^2+(j)^2}{i+j} \right\rceil \geq 6$

iii) Look at the pair (u_i,v_j) of vertices with $d(u_i,v_j) \geq 1$ where $1 \leq i,\ j \leq 2n+1$,

$d(u_i,v_j)+\left\lceil \frac{(2n+i+1)^2+(j)^2}{2n+i+j+1} \right\rceil \geq 6$

It is obvious from the above possibilities that f holds the inequality [Equation (3)]. The greatest number assigned is $4n+2$ to the vertex u_{2n+1}.

Case 2: $n \geq 5$

For $n \geq 5$ the diameter of the graph $G_n \odot K_1$ is 6.

Then the inequality [Equation (1)] reduces to

$$d(u,v)+\left\lceil \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rceil \geq 7 \tag{4}$$

Define a function $f: V(G_n \odot K_1) \rightarrow Z^+$ by fixing the vertices v_1, v_2 and v_3.

(i) $f(v_i) = i,\ 1 \leq i \leq 2n+1$ such that

(a) $d(v_1, v_2) = 5$

(b) $d(v_1, v_3) \geq d(v_2, v_3) \geq 4$

(ii) $f(u_i) = 2n+i+1,\ 1 \leq i \leq 2n+1$

Clearly, f is an injective function.

Now to show that the graph satisfies the inequality [Equation (4)] by considering the following possibilities:

i) Look at the pair (u_i, u_j) of vertices with $d(u_i, u_j) \geq 1$ where $1 \leq i,\ j \leq 2n+1,\ i \neq j,\ d(u_i,u_j)+\left\lceil \frac{(2n+i+1)^2+(2n+j+1)^2}{4n+i+j+2} \right\rceil \geq 7$

ii) Look at the pair (v_i, v_j) of vertices with $d(v_i, v_j) \geq 3$ where $1 \leq i,\ j \leq 2n+1,\ i \neq j$,

$d(v_i,v_j)+\left\lceil \frac{(i)^2+(j)^2}{i+j} \right\rceil \geq 7$

iii) Look at the pair (u_i, v_j) of vertices with $d(u_i, v_j) \geq 1$where $1 \leq i,\ j \leq 2n+1$,

$d(u_i,v_j)+\left\lceil \frac{(2n+i+1)^2+(j)^2}{2n+i+j+1} \right\rceil \geq 7$

It is obvious from the above labeling pattern that f holds the inequality [Equation (4)].

Therefore, for both cases, the inequalities hold and the graph $G_n \odot K_1$ satisfies the radio contra harmonic mean labeling. The greatest number assigned is $4n+2$ to vertex u_{2n+1}.

Radio contra harmonic mean number, $rchmn(G_n \odot K_1) = 4n+2 = |V(G_n \odot K_1)|$.

Hence, $G_n \odot K_1$ indicates a radio contra harmonic mean graceful graph.

Theorem 3.2. The radio contra harmonic mean number of $(W_n \odot mK_1) \odot K_1$ is its order $\forall\, n \geq 3$.

Proof: Let $v_1, v_2, v_3, \ldots, v_{n+1}$ be the vertices of the wheel graph W_n.

Let $V(mK_1) = \{t_i : 1 \leq i \leq m(n+1)\}$. Now $t_i, t_{i+(n+1)}, \ldots, t_{i+(m-1)(n+1)}$ are the vertices joined to the vertices v_i, $1 \leq i \leq n+1$. The resultant graph is $(W_n \odot mK_1) \odot K_1$.

Now let s_i be the vertices joined to t_i, $1 \leq i \leq m(n+1)$ and u_i be the vertices adjacent to v_i, $1 \leq i \leq n+1$. The resultant graph is $(W_n \odot mK_1) \odot K_1$.

Case (1): $n = 3$

The diameter of the graph $(W_n \odot mK_1) \odot K_1$ is 5 for $n = 3$.

Then the inequality [Equation (1)] reduces to

$$d(u,v)+\left\lceil \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rceil \geq 6 \tag{5}$$

Define a function $f: V((W_n \odot mK_1) \odot K_1) \rightarrow \mathbb{Z}^+$ as follows

$f(s_i) = i,\ 1 \leq i \leq m(n+1)$

$f(t_i) = m(n+1)+i,\ 1 \leq i \leq m(n+1)$

$f(u_i) = 2m(n+1)+i,\ 1 \leq i \leq n+1$

$f(v_i) = (2m+1)(n+1)+i,\ 1 \leq i \leq n+1$.

Clearly f indicates an injective function.

Now to show that the graph satisfies the inequality [Equation (5)] by considering the following possibilities:

i) Look at the pair (s_i, s_j) of vertices with $d(s_i, s_j) \geq 4$ where $1 \leq i,\ j \leq m(n+1),\ i \neq j$

$d(s_i,s_j)+\left\lceil \frac{(i)^2+(j)^2}{i+j} \right\rceil \geq 6$

ii) Look at the pair (u_i, u_j) of vertices with $d(u_i, u_j) = 3$ where $1 \leq i,\ j \leq n+1,\ i \neq j$

$\left\lceil \frac{(2mn+2m+i)^2+(2mn+2m+j)^2}{4mn+4m+i+j} \right\rceil \geq 3$

iii) Look at the pair (s_i, u_j) of vertices with $d(s_i, u_j) \geq 3$ where $1 \leq i \leq m(n+1),\ 1 \leq j \leq n+1$

$d(s_i,u_j)+\left\lceil \frac{(i)^2+(2mn+2m+j)^2}{2mn+2m+i+j} \right\rceil \geq 6$

iv) Look at the pair (t_i, u_j) of vertices with $d(t_i, u_j) \geq 2$ where $1 \leq i \leq m(n+1),\ 1 \leq j \leq n+1$

$d(t_i,u_j)+\left\lceil \frac{(mn+n+i)^2+(2mn+2m+j)^2}{3mn+2m+n+i+j} \right\rceil \geq 6$

v) Look at the pair $(t_i,\ t_j)$ of vertices with $d(t_i,\ t_j) \geq 2$ where $1 \leq\ i,\ j \leq m(n+1),\ i \neq j$

$d(t_i,t_j)+\left\lceil \frac{(mn+n+i)^2+(mn+n+j)^2}{2mn+2n+i+j} \right\rceil \geq 6$

vi) Look at the pair $(v_i,\ v_j)$ of vertices with $d(v_i,\ v_j) = 1$ where $1 \leq\ i,\ j \leq n+1,\ i \neq j$

$\left\lceil \frac{(2mn+2m+n+i+1)^2+(2mn+2m+n+j+1)^2}{4mn+4m+2n+i+j+2} \right\rceil \geq 5$

vii) Look at the pair $(s_i,\ v_j)$ of vertices with $d(s_i,\ v_j) \geq 2$ where $1 \leq\ i \leq m(n+1),\ 1 \leq\ j \leq n+1$

$d(s_i,v_j)+\left\lceil \frac{(i)^2+(2mn+2m+n+j+1)^2}{2mn+2m+n+i+j+1} \right\rceil \geq 6$

viii) Look at the pair $(t_i,\ v_j)$ of vertices with $d(t_i,\ v_j) \geq 1$ where $1 \leq\ i \leq m(n+1),\ 1 \leq\ j \leq n+1$

$d(t_i,v_j)+\left\lceil \frac{(mn+n+i)^2+(2mn+2m+n+j+1)^2}{3mn+2m+2n+i+j+1} \right\rceil \geq 6$

ix) Look at the pair $(s_i,\ t_j)$ of vertices with $d(s_i,\ t_j) \geq 1$ where $1 \leq\ i,\ j \leq\ m(n+1)$

$d(s_i,t_j)+\left\lceil \frac{(i)^2+(mn+n+j)^2}{mn+n+i+j} \right\rceil \geq 6$

x) Look at the pair $(u_i,\ v_j)$ of vertices with $d(u_i,\ v_j) \geq 1$ where $1 \leq i,\ \ j \leq n+1$

$d(u_i,v_j)+\left\lceil \frac{(2mn+2m+i)^2+(2mn+2m+n+j+1)^2}{4mn+4m+n+i+j+1} \right\rceil \geq 6$

Case (2): $n \geq 4$

The diameter of the graph $(W_n \odot mK_1) \odot K_1$ is 6 for $n \geq 4$.

Then the inequality [Equation (1)] reduces to

$$d(u,v)+\left\lceil \frac{(f(u))^2+(f(v))^2}{f(u)+f(v)} \right\rceil \geq 7 \tag{6}$$

Label the vertices as in case (1) under the defined function f.

i) Look at the pair $(s_i,\ s_j)$ of vertices with $d(s_i,\ s_j) \geq\ 4$ where $1 \leq\ i,\ j \leq m(n+1),\ i \neq j$

$d(s_i,s_j)+\left\lceil \frac{(i)^2+(j)^2}{i+j} \right\rceil \geq 7$

ii) Look at the pair $(u_i,\ u_j)$ of vertices with $d(u_i,\ u_j) \geq 3$ where $1 \leq\ i,\ j \leq n+1,\ i \neq j$

$d(u_i,\ u_j)+\left\lceil \frac{(2mn+2m+i)^2+(2mn+2m+j)^2}{4mn+4m+i+j} \right\rceil \geq 7$

iii) Look at the pair $(s_i,\ u_j)$ of vertices with $d(s_i,\ u_j) \geq 3$ where $1 \leq\ i \leq m(n+1),\ 1 \leq\ j \leq n+1$

$d(s_i,u_j)+\left\lceil \frac{(i)^2+(2mn+2m+j)^2}{2mn+2m+i+j} \right\rceil \geq 7$

iv) Look at the pair $(t_i,\ u_j)$ of vertices with $d(t_i,\ u_j) \geq 2$ where $1 \leq\ i \leq m(n+1),\ 1 \leq\ j \leq n+1$

$d(t_i,u_j)+\left\lceil \frac{(mn+n+i)^2+(2mn+2m+j)^2}{3mn+2m+n+i+j} \right\rceil \geq 7$

v) Look at the pair $(t_i,\ t_j)$ of vertices with $d(t_i,\ t_j) \geq 2$ where $1 \leq\ i,\ j \leq m(n+1),\ i \neq j$

$d(t_i,t_j)+\left\lceil \frac{(mn+n+i)^2+(mn+n+j)^2}{2mn+2n+i+j} \right\rceil \geq 7$

vi) Look at the pair $(v_i,\ v_j)$ of vertices with $d(v_i,\ v_j) \geq 1$ where $1 \leq\ i,\ j \leq n+1,\ i \neq j$

$d(v_i,v_j)+\left\lceil \frac{(2mn+2m+n+i+1)^2+(2mn+2m+n+j+1)^2}{4mn+4m+2n+i+j+2} \right\rceil \geq 7$

vii) Look at the pair $(s_i,\ v_j)$ of vertices with $d(s_i,\ v_j) \geq 2$ where $1 \leq\ i \leq m(n+1),\ 1 \leq\ j \leq n+1$

$d(s_i,v_j)+\left\lceil \frac{(i)^2+(2mn+2m+n+j+1)^2}{2mn+2m+n+i+j+1} \right\rceil \geq 7$

viii) Look at the pair $(t_i,\ v_j)$ of vertices with $d(t_i,\ v_j) \geq 1$ where $1 \leq\ i \leq m(n+1),\ 1 \leq\ j \leq n+1$

$d(t_i,v_j)+\left\lceil \frac{(mn+n+i)^2+(2mn+2m+n+j+1)^2}{3mn+2m+2n+i+j+1} \right\rceil \geq 7$

ix) Look at the pair $(s_i,\ t_j)$ of vertices with $d(s_i,\ t_j) \geq 1$ where $1 \leq\ i,\ j \leq\ m(n+1)$

$d(s_i,t_j)+\left\lceil \frac{(i)^2+(mn+n+j)^2}{mn+n+i+j} \right\rceil \geq 7$

x) Look at the pair $(u_i,\ v_j)$ of vertices with $d(u_i,\ v_j) \geq 1$ where $1 \leq i,\ \ j \leq n+1$

$d(u_i,v_j)+\left\lceil \frac{(2mn+2m+i)^2+(2mn+2m+n+j+1)^2}{4mn+4m+n+i+j+1} \right\rceil \geq 7$

Therefore, the above cases hold for both inequalities [Equations (5) and (6)]. In both cases, the greatest number assigned is $(2m+1)(n+1)$ to vertex v_{n+1}. Hence, the graph $(W_n \odot mK_1) \odot K_1$ satisfies the radio contra harmonic mean labeling.

Hence, radio contra harmonic mean number, $rchmn((W_n \odot mK_1) \odot K_1) = 2(n+1)(m+1)$ which is the order of the graph $(W_n \odot mK_1) \odot K_1$.

Theorem 3.3. Radio contra harmonic mean number of subdivision of jellyfish, $rchmn(S(J(m,n))) = 2m + 2n + 10\ for\ n,\ m \geq 2$.

Proof: Let $J(m,n)$ be a jellyfish formed by combining u_1 and u_3 with an edge to a 4-cycle $u_1u_2u_3u_4$ by making $v_1,\ v_2,\ v_3, \ldots, v_m$ and $w_1,\ w_2,\ w_3, \ldots, w_n$ pendant vertices next to u_2 and u_4 respectively.

Let x_i represent the vertices that subdivide the edges $v_iu_2,\ 1 \leq i \leq m,\ y_j$ considered to be the vertices that subdivide the edges $w_ju_4, 1 \leq j \leq n\ and\ z_k,\ 1 \leq k \leq 5$ represent the vertices that subdivide the edges $u_1u_2, u_2u_3,\ u_3u_4, u_4u_1$ and u_1u_3.

The resultant graph is $S(J(m,n))$.

The diameter of $S(J(m,n))$ is 8.

Then the inequality [Equation (1)] reduces to

$$d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u) + f(v)} \right\rceil \geq 9 \tag{7}$$

Let f be a function defined on the vertex set of $S(J(m,n))$ to $\mathbb{Z}^+$.

If the vertex v_1 of G is allocated 1 under f, then the vertex assigned 2 must be located 7 distances away from v_1.

The vertices v_1 and w_1 should be given the numbers 1 and 2, respectively, without losing generality. Now to assign the label 3, α (say) be any vertex the label 3 is to be assigned.

Consider $f(v_1) = 1$ and $f(\alpha) = 3$, then

$d(v_1,\alpha) + \left\lceil \frac{(f(v_1))^2 + (f(\alpha))^2}{f(v_1)+f(\alpha)} \right\rceil \geq 9$

$d(v_1,\alpha) + \left\lceil \frac{1^2+3^2}{1+3} \right\rceil \geq 9$

$d(v_1,\alpha) + \lceil 2.5 \rceil \geq 9$

$d(v_1,\alpha) + 3 \geq 9$

$d(v_1,\alpha) \geq 6$

Therefore, from the vertex v_1, the vertex α must be at least 6 distances apart. For v_1, the possibilities to fix α are chosen from any one of the vertices,

i) u_4

ii)y_j , $1 \leq j \leq n$

iii)w_j, $2 \leq j \leq n$

Now, for $f(w_1) = 2$ and $f(\alpha) = 3$, then

$d(w_1,\alpha) + \left\lceil \frac{(f(w_1))^2 + (f(\alpha))^2}{f(w_1)+f(\alpha)} \right\rceil \geq 9$

$d(w_1,\alpha) + \left\lceil \frac{2^2+3^2}{2+3} \right\rceil \geq 9$

$d(w_1,\alpha) + \lceil 2.6 \rceil \geq 9$

$d(w_1,\alpha) + 3 \geq 9$

$d(w_1,\alpha) \geq 6$

Thus, with respect to w_1, α is to be fixed for any one of the vertices,

i) u_2

ii)x_i , $1 \leq i \leq m$

iii)v_i, $2 \leq i \leq m$

which is impossible, since α is to be assigned at least 6 distances apart from v_1.

So next we assign 2 to the vertex v_1 and 3 to the vertex w_1 and $f(\alpha) = 4$ in such a manner that we get $d(v_1,\alpha) \geq 5,\ d(w_1,\alpha) \geq 5$. Now label the vertex z_5 (which is α) as 4.

$f(v_i) = 3 + i,\ 2 \leq\ i \leq m$

$f(w_j) = m + 2 + j,\ 2 \leq\ j \leq n$

$f(x_i) = m + n + 2 + i,\ 1 \leq\ i \leq m$

$f(y_j) = 2m + n + 2 + j,\ 1 \leq\ j \leq n$

$f(u_p) = 2m + 2n + 2 + p,\ 1 \leq\ p \leq 4$

$f(z_q) = 2m + 2n + 6 + q,\ 1 \leq\ q \leq 4$

Thus, f is an injective function and for every distinct pair of vertices, the inequality [Equation (7)] holds.

Additionally, f reaches its maximum at z_4 and is $2(m+n+5)$ for $n,\ m\ \geq 2$ in this labeling.

Similarly, even if we acquire any radio contra harmonic mean labeling, the span will undoubtedly be bigger if we apply labels to the vertices using integers greater than 3. When compared to the radio contra harmonic mean labeling presented above, the smallest span is $2(m+n+5)$.

Hence, the graph $S(J(m,n))$ satisfies the radio contra harmonic mean labeling and $rchmn(S(J(m,n))) = 2m+2n+10$.

Theorem 3 .4. For $n,\ m\ \geq 2$, the radio contra harmonic mean number of path union for two copies of the Bistar graph is $2(m+n+3)$.

Proof: Let $P_2(B_{m,n})$ be path union for two copies of the bistar graph.

Let $u,\ v$ be the vertices of the path $P_2$and $V(B_{m,n}) = \left\{u_i, v_j, w_1, w_2, u_i^{'}, v_j^{'}, w_1^{'}, w_2^{'} : 1 \leq i \leq m,\ 1 \leq j\ \leq n\right\}$, $E(B_{m,n}) = \{w_1u_i, v_jw_2, w_1w_2, w_1^{'}\ u_i^{'}, v_j^{'}w_2^{'}\ , w_1^{'}w_2^{'}\ /1 \leq i \leq m, 1 \leq j \leq n\}$. Now fuse the vertex u_1 to the vertex u and $u_1^{'}$ to the vertex v.

The resultant graph is a path union of two copies of the bistar graph $P_2(B_{m,n})$ and the diameter of this graph is 7. The inequality [Equation (1)] reduces to

$$d(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq 8 \tag{8}$$

Let f be a function defined on the vertex set of $P_2(B_{m,n})$ to $\mathbb{Z}^+$

If, under f, the vertex v_1 of $P_2(B_{m,n})$ is assigned the value 1, then the vertex assigned the value 2 must be at least six distances away from v_1.

Assign 1 and 2 to the vertices v_1 and $v_1^{'}$ respectively, without losing generality.

For $f(v_1) = 1$ and $f(\alpha) = 3$, we get $d(v_1, \alpha) \geq 5$. So, with respect to the vertex v_1, α is to be fixed to any one of the vertices $w_1^{'}, w_2^{'}, v_j^{'}$, $2 \leq j \leq n$. For $f\left(v_1^{'}\right) = 2$ and $f(\alpha) = 3$, we get $d\left(v_1^{'}, \alpha\right) \geq 5$. So, with respect to the vertex $v_1^{'}$, α is to be fixed to any one of the vertices $w_1, w_2,\ v_j$, $2 \leq j \leq n$, which is impossible.

The vertex allocated 3 must be located 5 distances away from v_1 if we assign 2 to the vertex v_1 of $P_2(B_{m,n})$. Now assign 2 and 3 to the vertices v_1 and $v_1^{'}$ respectively. Next, we have to assign the label 4, β (say) be any vertex the label 4 is to be assigned. For $f(v_1) = 2$ and $f(\beta) = 4$, we get $d(v_1, \beta) \geq 4$. So, with respect to the vertex v_1 , β is to be fixed to any one of the vertices $w_1^{'},\ w_2^{'},\ \ v_j^{'}$ where $2 \leq j \leq n$, $u_i^{'}$ where $1 \leq i \leq m$. For $f(v_1^{'}) = 3$ and $f(\beta) = 4$, we get $d\left(v_1^{'}, \beta\right) \geq 4$. So, with respect to the vertex $v_1^{'}$, β is to be fixed to any one of the vertices $w_1,\ \ w_2, v_j$ where $2 \leq j \leq n$,u_i where $1 \leq i \leq m$ which is also not possible.

Hence, we assign the label to the vertices as follows:

$f(v_1) = 3;\ f(v_1^{'}) = 4\ ; f(u_1) = 5; f(u_1^{'}) = 6$

$f(w_i) = 6+i,\ 1 \leq i \leq 2$

$f(w_i^{'}) = 8+i,\ 1 \leq i \leq 2$

$f(u_i) = 9+i,\ 2 \leq\ i \leq m$

$f(u_i^{'}) = m+8+j,\ 2 \leq\ i \leq m$

$f(v_j) = 2m+7+j,\ 2 \leq\ j \leq n$

$f(v_j^{'}) = 2m+n+6+j,\ 2 \leq\ j \leq n.$

Clearly, f is an injective function and holds the inequality [Equation (8)].

Furthermore, f reaches its maximum corresponding to $v_n^{'}$ and is $2(m+n+3)$ for $n,\ m\ \geq 2$ in this labeling. Similarly, even if radio mean labeling occurs, the span will undoubtedly be greater if we label the vertices using integers $\geq$ 4 instead. The smallest span, when compared to the radio contra harmonic mean labeled above, is $2(m+n+3)$.

Hence, $rchmn(P_2(B_{m,n})) = 2(m+n+3)$.

3.1 Open Problem

1. Find $rchmn(P_n(B_{m,n}))$
2. Find $rchmn(P_n(J(m,n)))$

Thus, for any connected graph G, the radio contra harmonic mean number is less than or equal to radio mean number i.e., $rchmn(G) \leq rmn(G)$. Similarly, the radio contra harmonic mean number is less than or equal to radio harmonic mean number i.e., $rchmn(G) \leq rhmn(G)$. Therefore, radio contra harmonic mean labeling is more effective than radio mean labeling and radio harmonic mean labeling.

4 Conclusion

The radio contra harmonic mean labeling, radio contra harmonic mean graceful graph, and radio contra harmonic mean number of various graphs with diameters of 5, 6, 7, and 8 have been covered in this work. In the next study pertaining to this idea, several graph features will be investigated.

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