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On Sequences of Geophine Triples Involving Padovan and Bernstein Polynomial with Propitious Property

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Abstract

Objective: To bring forth a new conception in the time-honoured field of Diophantine triples, namely "Geophine triple". To examine the feasibility of proliferating an unending sequence of Geophine triples from Geophine pairs with the property $D(\Psi)$ comprising Padovan and Bernstein polynomial. **Method:** Established Geophine triples employing Padovan and Bernstein polynomial by the method of polynomial manipulations. **Findings:** An unending sequences of Geophine triples $\{(t^2, t^4, (3t^2+3t)^2),\ (t^2, (3t^2+3t)^2,\ (5t^2+8t)^2), (t^2, (5t^2+8t)^2,\ (7t^2+15t)^2), \ldots...\}$ and $\{(36t^2, (1-t)^4, (3t^2+10t+3)^2),\ (36t^2, (3t^2+10t+3)^2, (5t^2+34t+5)^2),\ (36t^2, (5t^2+34t+5)^2,\ (7t^2+70t+7)^2),\ \ldots\ldots\ldots\}$ with the property $D(t^4+t^3+t^2)$ and $D(t^4+23t^2+1)$ are promulgated from Geophine pairs, precisely involving Padovan and Bernstein polynomials and few numerical representation of the sequences are computed using MATLAB. **Novelty:** This article carries an innovative approach of determining this definite type of triples using Geometric mean and thereby, two infinite sequences of Geophine triples with the property $D(\Psi)$ are ascertained. Also, few numerical representations of the sequences utilizing MATLAB program are figured out, thus broadening the scope of computational Number Theory.

Keywords: Polynomial Diophantine triple; Geophine triple; Bernstein polynomial; Padovan polynomials; Pell's equation; Special Polynomials

1 Introduction

The study of Diophantine tuples, which began around two thousand years ago, has garnered considerable attention over time[1]. Pioneering work in the field of Diophantine n-tuples was carried out by Diophantus of Alexandria, Euler and Fermat who made use of the property $D(1)$. A Diophantine n-tuple with the property $D(\Psi)$ is such that for a set of n distinct integers, product of any two integers added by

Ψ is a square number [2,3]. The authors [4–6] have studied Special Diophantine triples such that the product of two distinct elements of the triple added to their sum and increased by property is a square number. Diophantine triples involving polynomials are emerging area in Number Theory which are examined using special polynomials of various degrees with integer coefficient instead of figurate numbers [7–9]. Here a new formulation in Diophantine triples, Geophine triples is studied as it varies from Diophantine triple where the Geometric mean of any two different polynomials added to given property yields a square of certain polynomial. Also, two unending sequences of Geophine triples with a particular focus on $D\left(t^4+t^3+t^2\right)$ and $D(t^4+23t^2+1)$ involving Padovan and Bernstein polynomials are generated by polynomial manipulation and few numerical representations are presented by means of MATLAB.

2 Methodology

- **Definition 2.1.** Let $\{r_1(t), r_2(t), \ldots\ldots\ldots r_n(t)\}$ be a set of n distinct polynomials with integer coefficient. This set is called polynomial Diophantine n-tuple with the property $D(\Psi)$ if it satisfies the characteristic that $r_i(t)r_j(t)+\Psi$ is a square of some polynomial with integer coefficient for all $1 \le i < j \le n$, Ψ being an integer or a polynomial with integer coefficient. A Polynomial Diophantine triple and Diophantine pair is a particular case for n=3 and n=2.
- **Definition 2.2.** The set $\{r_1(t), r_2(t), r_3(t), \ldots\ldots\ldots r_n(t)\}$ is called Geophine n-tuple with the property $D(\Psi)$ if the Geometric mean of any two elements from the set convened by Ψ is a square of some polynomial, where Ψ being an integer or a polynomial with integer coefficient. A Geophine pair and triple is a particular case for n=2 and n=3.
- **Definition 2.3.** The Bernstein polynomial is defined as

$$\mathfrak{B}_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i},\ i = 0, \ldots .n$$

- **Definition 2.4.** The Padovan polynomial is defined as

$$p_n(t) = \begin{cases} 1 & if\ n=1 \\ 0 & if\ n=2 \\ t & if\ n=3 \\ t\,p_{n-2}(t)+p_{n-3}(t) & if\ n\ge 4 \end{cases}$$

3 Results and Discussion

3.1 On sequence of Geophine triples with D (t^4+t^3+t^2)

Fix $p = p_5(t) = t^2$ and $\mathfrak{B} = \mathfrak{B}_{4,4}(t) = t^4$. To validate (t^2, t^4) as Geophine pair with the property $D\left(t^4+t^3+t^2\right)$, exert definition 2.2 such that, $\sqrt{p\mathfrak{B}}+t^4+t^3+t^2$. The estimation results in $\left(t^2+t\right)^2$ as desired. The succeeding subsections confirm the possibility of protracting this pair to triple consequently forming an unending sequence of Geophine triples.

3.1.1 Protraction to triples

Let $p = p_5(t) = t^2$ and $\mathfrak{B} = \mathfrak{B}_{4,4}(t) = t^4$. Define $\mathfrak{C}$, $\mathfrak{C} \in \mathbb{Z}[t]$. Then,

$$\sqrt{p\mathfrak{C}}+t^4+t^3+t^2 = \theta^2 \tag{1}$$

$$\sqrt{\mathfrak{B}\mathfrak{C}}+t^4+t^3+t^2 = \tau^2 \tag{2}$$

Detecting the transformations $\theta = \alpha + \sqrt{p}\ \beta$, $\tau = \alpha + \sqrt{\mathfrak{B}}\ \beta$. Eliminating $\mathfrak{C}$ from above equations and incorporating transformations lead to prominent Pell's equation $\alpha^2 = \ell\beta^2+t^4+t^3+t^2$, $\ell = t^3$ resulting $\theta = t^2+2t$. Substituting the values of p, θ in Equation (1) the third tuple $\mathfrak{C}$ is determined as

$\mathfrak{C} = \left(3t^2+3t\right)^2$

Thus, $\left(t^2, t^4, \left(3t^2+3t\right)^2\right)$ is Geophine triple with $D\left(t^4+t^3+t^2\right)$ formed from (t^2, t^4).

3.1.2 Proliferation of an unending sequence of Geophine triple with D ($t^4+t^3+t^2$)

Commence with a Diophantine pair $\left(t^2,\left(3t^2+3t\right)^2\right)$ from the aforementioned triples. Set $\mathfrak{D}$, $\mathfrak{D}\in\mathbb{Z}[t]$. Then,

$$\sqrt{\wp\mathfrak{D}+t^4+t^3+t^2}=\theta^2 \tag{3}$$

$$\sqrt{\mathfrak{C}\mathfrak{D}+t^4+t^3+t^2}=\tau^2 \tag{4}$$

Determining the transformations $\theta=\alpha+\sqrt{\wp}\,\beta$, $\tau=\alpha+\sqrt{\mathfrak{C}}\,\beta$. Dropping of $\mathfrak{D}$ from Equations (3) and (4) and applying transformations lead to prominent Pell's equation $\alpha^2=\mathscr{b}\beta^2+t^4+t^3+t^2$, $\mathscr{b}=3t^3+3t^2$ resulting $\theta=t^2+3t$. Applying $\wp,\theta$ in Equation (3)$\mathfrak{D}$is determined as

$\mathfrak{D}=\left(5t^2+8t\right)^2$

Thus, $\left(t^2,\left(3t^2+3t\right)^2,\left(5t^2+8t\right)^2\right)$ is Geophine triple with $D\left(t^4+t^3+t^2\right)$ formed from $\left(t^2,\left(3t^2+3t\right)^2\right)$.

Analogously, for the Diophantine pair $\left(t^2,\left(5t^2+8t\right)^2\right)$. Set $\mathfrak{E}$, $\mathfrak{E}\in\mathbb{Z}[t]$. Progressing with the above mentioned steps yields a triple $\left(t^2,\left(5t^2+8t\right)^2,\left(7t^2+15t\right)^2\right)$. Continuing the same procedure leads to the formation of an unending chain of triples $\left\{\left(t^2,t^4,\left(3t^2+3t\right)^2\right),\left(t^2,\left(3t^2+3t\right)^2,\left(5t^2+8t\right)^2\right),\left(t^2,\left(5t^2+8t\right)^2,\left(7t^2+15t\right)^2\right),\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\right\}$.

3.2 On sequence of Geophine triples with D (t^4+23t^2+1)

Let $\wp=12\wp_8(t)=36t^2$ and $\mathfrak{B}=\mathfrak{B}_{0,4}(t)=(1-t)^4$ be Padovan and Bernstein polynomial. The geometric mean of these two polynomials convened by t^4+23t^2+1 gives square of some other polynomial, resulting in a Geophine pair $\left(36t^2,(1-t)^4\right)$ with property $D(t^4+23t^2+1)$. Progressing the procedure outlined in 3.1.1 this pair is protracted to triple $\left(36t^2,(1-t)^4,\left(3t^2+10t+3\right)^2\right)$. Contemplating the pair $\left(36t^2,\left(3t^2+10t+3\right)^2\right)$. In accordance with the process detailed in 3.1.2 an unending sequence of Geophine triples $\left\{\left(36t^2,(1-t)^4,\left(3t^2+10t+3\right)^2\right),\left(36t^2,\left(3t^2+10t+3\right)^2,\left(5t^2+34t+5\right)^2\right),\left(36t^2,\left(5t^2+34t+5\right)^2,\left(7t^2+70t+7\right)^2\right),\ldots.\right\}$ with the property $D(t^4+23t^2+1)$ is proliferated.

3.3 MATLAB demonstration

The findings of section 3.1 and 3.2 are implemented in MATLAB and few numerical representations are ascertained.

```
close all;
clc;
t = 2:6;
P = t.^2;
B = t.^4;
C = (3.*t.^2 + 3.*t).^2;
D = (5.*t.^2 + 8.*t).^2;
E = (7.*t.^2 + 15.*t).^2;
disp('output for 3.1:')
disp('((P,B,C),(P,C,D),(P,D,E))')
disp('t=1 value does not hold')
disp('for t=2:')
for i= 1
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P
(i),D(i),E(i));
end
disp('for t=3:')
for i= 2
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P
(i),D(i),E(i));
end
disp('for t=4:')
for i= 3
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P
(i),D(i),E(i));
end
disp('for t=5:')
for i= 4
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P
(i),D(i),E(i));
end
disp('for t=6:')
for i= 5
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P
(i),D(i),E(i));
end
```

Fig 1. Program for section 3.1

```
output for 3.1:
((P,B,C),(P,C,D),(P,D,E))
t=1 value does not hold
for t=2:
4   16  324 4   324 1296    4   1296    3364
for t=3:
9   81  1296    9   1296    4761    9   4761    11664
for t=4:
16  256 3600    16  3600    12544   16  12544   29584
for t=5:
25  625 8100    25  8100    27225   25  27225   62500
for t=6:
36  1296    15876   36  15876   51984   36  51984   116964
>>
```

Fig 2. Output for section 3.1

```
close all;
clc;
t = 2:6;
P = 36.*t.^2;
B = (1-t).^4;
C = (3.*t.^2 + 10.*t + 3).^2;
D = (5.*t.^2 + 34.*t + 5).^2;
E = (7.*t.^2 + 70.*t + 7).^2;
disp('output for 3.2:')
disp('((P,B,C),(P,C,D),(P,D,E))')
disp('t=1 value does not hold')
disp('for t=2:')
for i= 1
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P↙
(i),D(i),E(i));
end
disp('for t=3:')
for i= 2
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P↙
(i),D(i),E(i));
end
disp('for t=4:')
for i= 3
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P↙
(i),D(i),E(i));
end
disp('for t=5:')
for i= 4
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P↙
(i),D(i),E(i));
end
disp('for t=6:')
for i= 5
    fprintf('%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\t%d\n',P(i),B(i),C(i),P(i),C(i),D(i),P↙
(i),D(i),E(i));
end
```

Fig 3. Program for section 3.2

```
output for 3.2:
((P,B,C),(P,C,D),(P,D,E))
t=1 value does not hold
for t=2:
144 1   1225    144 1225    8649    144 8649    30625
for t=3:
324 16  3600    324 3600    23104   324 23104   78400
for t=4:
576 81  8281    576 8281    48841   576 48841   159201
for t=5:
900 256 16384   900 16384   90000   900 90000   283024
for t=6:
1296    625 29241   1296    29241   151321  1296    151321  461041
>>
```

Fig 4. Output for section 3.2

4 Conclusion

In this manuscript two unending sequences,

$$\left\{\left(t^2,t^4,\left(3t^2+3t\right)^2\right),\ \left(t^2,\left(3t^2+3t\right)^2,\left(5t^2+8t\right)^2\right),\left(t^2,\left(5t^2+8t\right)^2,\left(7t^2+15t\right)^2\right),\ldots\ldots\ldots\right\} \text{ and}$$

$$\left\{\left(36t^2,(1-t)^4,\left(3t^2+10t+3\right)^2\right),\left(36t^2,\left(3t^2+10t+3\right)^2,\left(5t^2+34t+5\right)^2\right),(36t^2,\left(5t^2+34t+5\right)^2,\left(7t^2+70t+7\right)^2),\ldots\right\}$$

of Geophine triples with the property $D\left(t^4+t^3+t^2\right)$ and $D(t^4+23t^2+1)$ are determined involving Padovan and Bernstein polynomials and some numerical representations are displayed utilizing MATLAB. Accordingly, one may look for other special polynomials in preference to Padovan and Bernstein polynomials that demonstrates propitious characteristics.

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