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RESEARCH ARTICLE

Generalized Soft Multi Functions

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Abstract

Objectives: To acquaint generalized soft multi functions and graphs. **Methods:** Generalized semi soft multi sets was introduced using semi closure operators and open soft multi sets. As a consequence, this study acquaints generalized semi soft multi continuous functions with the help of generalized semi soft multi sets. Also the concept of generalized soft multi homeomorphisms are introduced via generalized semi soft multi continuous, open and closed mappings and also the thought of closed soft multi graph is introduced through the open soft multi sets and soft multi points. **Findings:** This study reveals that composition of two generalized semi soft multi continuous functions do not necessarily generalized semi soft multi continuous but composition of generalized semi soft multi continuous function with any of its stronger forms be a generalized semi soft multi continuous function and also several interesting results are found. **Novelty:** This study acquaints soft multi graph by using soft multi functions and also acquaint generalized semi closed soft multi graph.

Keywords: Generalized Semi Soft Multi Continuous Functions; Generalized Semi Soft Multi Open Functions; Generalized Semi Soft Multi Closed Functions; Generalized Semi Soft Multi Homeomorphisms And Generalized Semi Closed Soft Multi Graphs

1 Introduction

In 2013, Babitha and John were introduced the concept of soft multi sets as acombination of soft sets and multi sets that can be used to solve real life problems in many fields. The same authors initiated the concept of connectedness in soft multi topological spaces. In 2015 Deniz Tokat et al. introduced the idea of soft multi continuous functions and also soft multi semi continuous functions by using soft multi functions.

Then El- Sheik et al. introduced the generalizations of open soft multi sets and mappings in soft multi topological spaces. In 2015, they have introduced the concept of g-closed soft multi sets in soft multi topological spaces. Muhammad Riazet et al. [1] gave the properties of soft multiset topological spaces and also some real life applications in 2020. Seydakaya Pezuk et al. [2] gave an application of soft multisets to a decision-making problem concerning the side effects of COVID-19 vaccines. V. Inthumathi et al. [3] introduced the concepts of generalized closed soft multisets and its properties. Further they[4,5] introduced the concepts of generalized soft multi operators and generalized soft multi connectedness and compactness.

After the introduction of generalizations of open soft multisets and functions in soft multi topological spaces in 2016, no one has attempted to make further generalizations of open soft multi sets. So as an inquiry, we introduce the concept of generalized semi soft multi continuous functions and also generalized semi soft multi closed and open functions. Next we introduce the concept of generalized semi soft multi homeomorphisms by using generalized semi soft multi continuous functions. Finally, we initiate the concept of soft multi graph in soft multi topological spaces and extend in generalize case such as generalized semi closed soft multi graphs and discuss some related properties in detail.

Preliminaries

Throughout this paper SMTS denotes the soft multi topological spaces, sm denotes soft multi.

Definition1. Let f_E be a smset over X_E.f_E is claimed to be smpoint if there exists $e\in E$ and $n/x\in X, 1\leq n\leq m$ such that

$$f(\varepsilon)=\begin{cases}\{n/x\} \text{ if } \varepsilon=e, 1\leq n\leq m\\ \phi \quad \text{ if } \varepsilon\in E-\{e\}\end{cases}$$

We Symbolize the smpointf_E by $[(n/x)_e]_E$. The family of all smpoints is marked by $P(X,E)$ *or* P.

i.e. $P(X,E)=\left\{\left[(n/x_i)_{e_j}\right]_E : n/x_i\in X, e_j\in E, 1\leq n\leq m\right\}$

Definition 2. A smsetS_E in a SMTS $(X,\ \tau,\ E)$ is claimed to be a generalized semi closed soft multiset(in summary *gscs* mset) if$C_{scl(S)(e)}(x)\leq C_{U(e)}(x)$ whenever $C_{S(e)}(x)\leq C_{U(e)}(x)$and $U_E\in OSM(X_E)$. The complement of *gscs* mset is a *gsos* mset.

2 Methodology

1. Cardinality of smsets are checked.

2. Applied the concept of soft multi functions to define the generalized semi soft multi continuous functions, where domains and codomains need not have the same cardinality.

3. Applied the operators like smclosure and smsemi closure to develop generalized semi closed soft multisets.

4. Semi soft multi homeomorphism concept is applied to obtain generalized semi soft multi homeomorphism.

5. The notion of graph of the functions is used to develop generalized semi closed soft multi graph.

Generalized semi soft multi continuous functions

Remark 2.1.1

The definition of smfunction defined by Ismail Osmanoglu et al.is applicable only when the domain and co-domain elements have same cardinality, that means m is common for all elements in domain and co-domain or the cardinality of an element should be same to its image element. We have given the definition for image of a smfunction in below which is applicable for all types of cardinality of the element in SMTS.

Definition 2.1. 2 Let $u\ :M\longrightarrow N$ and $p\ :L\longrightarrow Q$ be functions. Then a multi function u is defined as, $u(m/v)\ =n/w,\ \forall\ m/v\in\ M$ and $n/w\in N$ and is marked by $u=(m/v,\ n/w)/mn\ : u(m/v)\ =n/w$ and the smfunction $r\ :\ M_L\rightarrow N_Q$, where M_L and N_Q are smclasses, is defined as, for a smset S_I in M_L, $(r(S_I))_J,\ \ J=p(I)\subseteq Q$ is a smset in N_Q given by

$$r(S_I)(q)=\begin{cases}u\left(\cup_{l\in p^{-1}(q)\cap I}S(l)\right), & \text{if } p^{-1}(q)\cap I\neq\phi\\ \phi & \text{Otherwise}\end{cases}$$

for $q\ \in J\ \subseteq Q,\ (r(S_I))_J$ is supposedly as image of a smset S_I.

Note 2.1.1 Hereupon the functions $u,\ \ p$ and r are mapped from mset$M$ to mset N, parameter set L to parameter set Q and smclass M_L to smclass N_Q respectively.

Definition 2.1.3 Let $r\ :\ M_L\rightarrow N_Q$ be a mapping and T_J be a smset in $N_Q,\ \ J\subseteq Q$. Let$u\ :M\longrightarrow N$ and $p\ :L\longrightarrow Q$ be mappings. Then $(r^{-1}(T_J))_I,\ \ I=p^{-1}(J)$, is a smset in M_L, defined as

$$r^{-1}(T_J)(l)=\begin{cases}u^{-1}(T(p(l))), & p(l)\in J\\ \phi & \text{otherwise}\end{cases}$$

for $l\ \in I\ \subseteq L,\ (r^{-1}(T_J))_I$ is claimed to be inverse image of smset T_J.

Example 2.1.4 Let $M=\{1/v_1,\ 2/v_2,\ 1/v_3\},\ \ L=\{l_1,\ l_2\},\ \ N=\{2/w_1,\ 2/w_2,\ 2/w_3\}$ and $Q=\{q_1,\ q_2\}$. Let u and p be defined as $u=\{(1/v_1,\ 2/w_1)/2,\ (2/v_2,\ 2/w_2)/4,\ \ (1/v_3,\ 2/w_3)/2\}, p(l_1)=q_1,\ \ p(l_2)=q_2.$

Let S_L be a smset defined as $S(l_1)=\{2/v_2\},\ S(l_2)=\{2/v_2\}$ and T_Q be a smset defined as $T(q_1)=\{2/w_1,\ 2/w_2\},\ \ T(q_2)=\{2/w_1,\ 2/w_2\}$.

Then image of smsetS_L under r is obtained as

$$r(S_L)(q_1)=\begin{cases} u\left(U_{l\in p^{-1}(q_1)\cap L}S(l)\right), & \text{if } p^{-1}(q_1)\cap L\neq\phi \\ \phi & \text{otherwise}\end{cases}$$

$=(u(S(l_1))\}$

$=\{u(2/v_2)\}$

$=\{2/w_2\}$.

Similarly,

$$r(S_L)(q_2)=\begin{cases} u\left(U_{l\in p^{-1}(q_2)\cap L}S(l)\right), & \text{if } p^{-1}(q_2)\cap L\neq\phi \\ \phi & \text{otherwise}\end{cases}$$

$=(u(S(l_2))\}$

$=\{u(2/v_2)\}$

$=\{2/w_2\}$.

Hence the image of smset$r(S_L)$ is a smset in order that $r(S_L)(q_1)=\ \{2/w_2\}$,

$$r(S_L)(q_2)=\ \{2/w_2\}.$$

Next the inverse image of smsetT_Q under r is obtained as

$$r^{-1}(T_Q)(l_1)=\begin{cases} u^{-1}(T(p(l))), & p(l)\in L \\ \phi & \text{otherwise}\end{cases}$$

$=\left(u^{-1}(T(q_1))\right\}$

$=\{u^{-1}(2/w_1,\ 2/w_2)\}$

$=\{1/v_1,\ 2/v_2\}$.

Similarly,

$$r^{-1}(T_Q)(l_2)=\begin{cases} u^{-1}(T(p(l))), & p(l)\in L \\ \phi & \text{otherwise}\end{cases}$$

$=\left(u^{-1}(T(q_2))\right\}$

$=\{u^{-1}(2/w_1,\ 2/w_2)\}$

$=\{1/v_1,\ 2/v_2\}$.

Hence the inverse image of smset$r^{-1}(T_Q)$ is a smset in order that $r^{-1}(T_Q)(l_1)=\{1/v_1,\ 2/v_2\},r^{-1}(T_Q)(l_2)=\{1/v_1,\ 2/v_2\}$.

Definition 2.1.5. The smfunction$r$ is supposedly as generalized semi soft multi continuous (in summary *gssm*-cts) if $r^{-1}(S_Q)$ is a *gscs* mset over $(M,\tau,\ L)$ for every closed smset S_Q over $(N,\ \sigma,\ Q)$.

Example 2.1.6. Let $M=\{1/v_1,\ 2/v_2,\ 1/v_3\},\ \ L=\{l_1,\ l_2\}$

and $\tau=\{\widetilde{\phi,\ \widetilde{M}},\ (S_L)_1,\ (S_L)_2,\ (S_L)_3,\ (S_L)_4\}$

where

$S_1(l_1)\ =\{1/v_1,\ 1/v_3\},S_1(l_2)\ =\{1/v_1,\ 1/v_3\},S_2(l_1)\ =\{1/v_1\},S_2(l_2)\ =\{1/v_3\},S_3(l_1)\ =\{2/v_2\},S_3(l_2)\ =\{2/v_2\},S_4(l_1)\ =\{1/v_1,\ 2/v_2\},S_4(l_2)\ =\{2/v_2,1/v_3\}$

and

let $N=\{2/w_1,\ 2/w_2,\ 2/w_3\},\ \ Q=\{q_1,\ q_2\}$ and

$\sigma=\{\widetilde{\phi,\ \widetilde{N}},\ (T_Q)_1,\ (T_Q)_2\}$

where $T_1(q_1)\ =\{2/w_1,\ 2/w_2\},T_1(q_2)\ =\{N\},T_2(q_1)\ =\{2/w_2\},T_2(q_2)\ =\{2/w_2\}$.

Define u and p by $u=\{(1/v_1,\ 2/w_1)/2,\ (2/v_2,\ 2/w_2)/4,\ \ (1/v_3,\ 2/w_3)/2\}$ and $p(l_1)=q_1,\ \ p(l_2)=q_2$. Then the mapping r is *gssm*-cts.

Proposition 2.1.7. Every smcontinuous function is *gssm*-cts.

Proof. Presume that r is smcontinuous function and S_Q is a closed smset over (N, σ, Q). Then by assumption $r^{-1}(S_Q)$ is a closed smset over (M, τ, L). Since every closed smset is a *gscs* mset, $r^{-1}(S_Q)$ is a *gscs* mset. Thus r is a *gssm*-cts.

Remark 2.1.8. To the contrary of the above proposition do not necessarily correct as shown from the below example.

Example 2.1.9. In Example 2.1.6, r is *gssm*-ctsbut not smcontinuous.

Remark 2.1.10. Composition of two *gssm*-cts functions do not necessarily *gssm*-cts but the composition of *gssm*-cts function with any of its stronger forms be a *gssm*-cts function.

Example 2.1.11. Let $M = \{1/v_1,\ 2/v_2,\ 1/v_3\},\ \ L = \{l_1,\ l_2\}$ and

$\tau = \{\widetilde{\phi,\ M},\ (S_L)_1,\ (S_L)_2,\ (S_L)_3,\ (S_L)_4\}$

where

$S_1(l_1) = \{1/v_1,\ 1/v_3\}, S_1(l_2) = \{1/v_1,\ 1/v_3\}, S_2(l_1) = \{1/v_1\}, S_2(l_2) = \{1/v_3\}, S_3(l_1) = \{2/v_2\}, S_3(l_2) = \{2/v_2\}, S_4(l_1) = \{1/v_1,\ 2/v_2\}, S_4(l_2) = \{2/v_2, 1/v_3\}$

and

let $N = \{2/w_1,\ 2/w_2,\ 2/w_3\},\ \ Q = \{q_1,\ q_2\}$

and $\sigma = \{\widetilde{\phi,\ N},\ (T_Q)_1,\ (T_Q)_2\}$

where

$T_1(q_1) = \{2/w_1,\ 2/w_2\}, T_1(q_2) = \{N\}, T_2(q_1) = \{2/w_2\}, T_2(q_2) = \{2/w_2\}$

and let $O = \{3/z_1,\ 2/z_2,\ 2/z_3\},\ \ T = \{t_1,\ t_2\}$

and $\eta = \{\widetilde{\phi,\ O},\ (H_T)_1\}$

where $H_1(t_1) = \{3/z_1\}, H_1(t_2) = (\phi\}$.

Define $u : M \longrightarrow N$ and $p : L \longrightarrow Q$ by $u = \{(1/v_1,\ 2/w_1)/2,\ (2/v_2,\ 2/w_2)/4,\ \ (1/v_3,\ 2/w_3)/2\}$

and $p(l_1) = q_1,\ \ p(l_2) = q_2$

and defined $d : N \longrightarrow O$ and $e : Q \longrightarrow T$

by $d = \{(2/w_1,\ 3/z_1)/6,\ (2/w_2,\ 2/z_2)/4,\ \ (2/w_3,\ 2/z_3)/4\}$

and $e(q_1) = t_1,\ \ e(q_2) = t_2$

Then the mapping $r : M_L \to N_Q$and $g : N_Q \to O_T$are*gssm*-cts but their composition $g\ o\ r$ is not *gssm*-cts.

Proposition 2.1.12. Let (M, τ, L), (N, σ, Q) and (O, η, T) be SMTSs and let $u : M \longrightarrow N\ \ d : N \longrightarrow O\ \ p : L \longrightarrow Q$ and $e : Q \longrightarrow T$ be functions. If $r : M_L \to N_Q$ is *gssm*-cts and $g : N_Q \to O_T$ is sm continuous, then their composition $g\ o\ r : M_L \to O_T$ is *gssm*-cts.

Proof. Let $S_T$be any closed smset over (O, η, T).Since g is smcontinuous, $g^{-1}(S_T)$ is closed smset over (N, σ, Q).Since r is *gssm*-cts$C_{r^{-1}\left(g^{-1}(S)\right)(t)}(z) = C_{(gor)^{-1}(S)(t)}(z)$ is *gscs* mset over (M, τ, L) and so $g\ o\ r$ is *gssm*-cts.

Corollary 2.1.13. If$r : M_L \to N_Q$ is smcontinuous and $g : N_Q \to O_T$is smcontinuous, then their composition $g\ o\ r : M_L \to O_T$ is *gssm*-cts.

Proof. It is easy from the result that every closed smset is a *gscs* mset.

Theorem 2.1.14. smfunction$r$ is a *gssm*-cts if and only if for each open smset S_Q over (N, σ, Q), $r^{-1}(S_Q)$ is a *gsos* mset over (M, τ, L).

Proof. Surmise that r is *gssm*-cts and S_Q is a open smset over (N, σ, Q). Then $(S_Q)^c$ is a closed smset in (N, σ, Q) and by hypothesis $r^{-1}((S_Q)^c)$ is *gscs* mset in (M, τ, L). But $C_{r^{-1}((S)^c)(q)}(w) = C_{\left(r^{-1}(S)\right)^c(q)}(w)$ and so $r^{-1}(S_Q)$ is a *gsos* mset over (M, τ, L).

Contrarily, surmise that $r^{-1}(S_Q)$ is a *gsos* mset in (M, τ, L) for each open smset S_Q over (N, σ, Q). Let T_Q be a closed smset in (N, σ, Q). Then $(T_Q)^c$ is a open smset in (N, σ, Q) and by assumption,$r^{-1}((T_Q)^c)$ is a *gsos* mset in (M, τ, L). Since $C_{r^{-1}((T)^c)(q)}(w) = C_{(r^{-1}(T))^c(q)}(w)$, we have $r^{-1}(T_Q)$ is a *gscs* mset in (M, τ, L) and so r is *gssm*-cts.

Proposition 2.1.15. If r is *gssm*-cts then for each smpoint$[(n/v)_l]_L$ in M_L for every open smset T_Q containing $r([(n/v)_l]_L)$ there exists a *gsos* msetS_L containing $[(n/v)_l]_L$ such that $C_{r(S)(l)}(v) \leq C_{T(q)}(w)$.

Proof. Let T_Q be any open smset over (N, σ, Q) with the property that $r([(n/v)_l]_L) \in T_Q$. Then $[(n/v)_l]_L \in r^{-1}(T_Q)$. By hypothesis $r^{-1}(T_Q)$ is a *gsos* mset in (M, τ, L). If we take $C_{S(l)}(v) \leq C_{r^{-1}(T)(q)}(w)$, then $S_L$is a *gsos* mset in (M, τ, L) containing $[(n/v)_l]_L$ such that $C_{r(S)(l)}(v) \leq C_{r\left(r^{-1}(T)\right)(q)}(w) \leq C_{T(q)}(w)$.

Proposition 2.1.16. If r is *gssm*-cts then for every sub smsetS_L of (M, τ, L), $C_{r(gssm-cl(S))(l)}(v) \leq C_{cl(r(S))(l)}(v)$.

Proof. Presume that r is *gssm*-cts and S_L is a smset over (M, τ, L). Then $cl(r(S_L))$ is a closed smset in (N, σ, Q) and so $r^{-1}(cl(r(S_L)))$ is a *gscs* mset in (M, τ, L).

Therefore $C_{gssm-cl\left(r^{-1}(cl(r(S)))\right)(l)}(v) = C_{r^{-1}(cl(r(S)))(l)}(v)$. Now $C_{S(l)}(v) \leq C_{r^{-1}(r(S))(l)}(v) \leq C_{r^{-1}(cl(r(S)))(l)}(v)$.Thus $C_{gssm-cl(S)(l)}(v) \leq C_{gssm-cl\left(r^{-1}(cl(r(S)))\right)(l)}(v) = C_{r^{-1}(cl(r(S)))(l)}(v)$. Hence $C_{r(gssm-cl(S))(l)}(v) \leq C_{cl(r(S))(l)}(v)$.

Proposition 2.1.17. If r is $gssm$-cts then for every sub smsetT_Q over (N, σ, Q), $C_{gssm-cl\left(r^{-1}(T)\right)(q)}(w) \leq C_{r^{-1}(cl(T))(q)}(w)$.

Proof. Surmise that r is $gssm$-cts and T_Q is any sub smset over (N, σ, Q). Then $cl(T_Q)$ being closed smset, by assumption, $r^{-1}(cl(T_Q))$ is a $gscs$ mset. Now, $C_{r^{-1}(T)(q)}(w) \leq C_{r^{-1}(cl(T))(q)}(w)$ and so $C_{gssm-cl\left(r^{-1}(T)\right)(q)}(w) \leq C_{gssm-cl\left(r^{-1}(cl(T))\right)(q)}(w) = C_{r^{-1}(cl(T))(q)}(w)$.

Proposition 2.1.18. If r is $gssm$-cts then $C_{r^{-1}(int(S))(l)}(v) \leq C_{gssm-int\left(r^{-1}(S)\right)(l)}(v)$ for every sub smset $S_L$over (M, τ, L).

Proof. Surmise that S_L is a sub smset over (M, τ, L). Then $int(S_L)$ being open smset, by hypothesis, $r^{-1}(int(S_L))$ is $gsos$ mset. Since $C_{int(S)(l)}(v) \leq C_{S(l)}(v)$, we have $C_{r^{-1}(int(S))(l)}(v) \leq C_{r^{-1}(S)(l)(v)}$ and so $C_{gssm-int\left(r^{-1}(int(S))\right)(l)}(v) \leq C_{gssm-int\left(r^{-1}(S)\right)(l)}(v)$. Thus $C_{r^{-1}(int(S))(l)}(v) \leq C_{gssm-int\left(r^{-1}(S)\right)(l)}(v)$ for every sub smset $S_L$over (M, τ, L).

2.2 Generalized semi soft multi homeomorphism

Definition 2.2 .1. The bijective smfunction$r$ is claimed to be generalized semi soft multi homeomorphism (in summary $gssm$-h) if r and r^{-1} are $gssm$-cts.

Example 2.2.2. Let $M = \{2/v_1, 2/v_2\}, \ L = \{l_1, l_2\}$
and $\tau = \{\widetilde{\phi}, \widetilde{M}, (S_L)_1, (S_L)_2\}$
where $S_1(l_1) = \{2/v_1\}, S_1(l_2) = \{2/v_1\}, S_2(l_1) = \{2/v_2\}, S_2(l_2) = \{2/v_2\}$
and
let $N = \{3/w_1, 3/w_2\}, \ Q = \{q_1, q_2\}$
and $\sigma = \{\widetilde{\phi}, \widetilde{N}, (T_Q)_1\}$
where $T_1(q_1) = \{3/w_1\}, T_1(q_2) = \{3/w_2\}$.
Define u and p by $u = \{(2/v_1, 2/w_1)/4, (2/v_2, 2/w_2)/4\}$ and $p(l_1) = q_1, \ p(l_2) = q_{2.}$
Then the mapping r is $gssm$-h.

Definition 2.2.3. The smfunction$r$ is claimed to be generalized semi soft multi closed function (in summary $gssm$-cf) if the image of each closed smset over (M, τ, L) is a $gscs$ mset over (N, σ, Q).

Example 2.2. 4. Let $M = \{2/v_1, 2/v_2, 1/v_3\}, \ L = \{l_1, l_2\}$
and $\tau = \{\widetilde{\phi}, \widetilde{M}, (S_L)_1, (S_L)_2\}$
where $S_1(l_1) = \{2/v_1, 2/v_2\}, S_1(l_2) = \{M\}, S_2(l_1) = \{2/v_2\}, S_2(l_2) = \{2/v_2\}$
and
let $N = \{1/w_1, 2/w_2, 1/w_3\}, \ Q = \{q_1, q_2\}$
and $\sigma = \{\widetilde{\phi}, \widetilde{N}, (T_Q)_1, (T_Q)_2, (T_Q)_3, (T_Q)_4\}$
where $T_1(q_1) = \{1/w_1, 1/w_3\}, T_1(q_2) = \{1/w_1, 1/w_3\}, T_2(q_1) = \{1/w_1\}, T_2(q_2) = \{1/w_3\}, T_3(q_1) = \{2/w_2\}, T_3(q_2) = \{2/w_2\}$,
$T_4(q_1) = \{1/w_1, 2/w_2\}, T_4(q_2) = \{2/w_2, 1/w_3\}$.
Define u and p by $u = \{(2/v_1, 1/w_1)/2, (2/v_2, 2/w_2)/4, (1/v_3, 1/w_3)/1\}$ and $p(l_1) = q_1, \ p(l_2) = q_{2.}$ Then r is $gssm$-cf.

Definition 2.2.5. The smfunction$r$ is allegedly as generalized semi soft multi open function (in summary $gssm$-of) if the image of each open smset over (M, τ, L) is a $gsos$mset over (N, σ, Q).

Proposition 2.2.6. If r is a $gssm$-of then$C_{r(int(S))(l)}(v) \leq C_{gssm-int(r(S))(l)}(v)$ for every S_L in (M, τ, L).

Proof. Presume that r is a $gssm$-of and $S_L$is any sub smset of (M, τ, L). Since $int(S_L)$ is open smset in (M, τ, L), $r(int(S_L))$ is a $gssm$-of in (N, σ, Q). Consequently, $C_{gssm-int(r(int(S)))(l)}(v) = C_{r(int(S))(l)}(v)$ and $C_{r(int(S))(l)}(v) \leq C_{r(S)(l)}(v)$. Thus $C_{r(int(S))(l)}(v) \leq C_{gssm-int(r(S))(l)}(v)$.

Proposition 2.2.7. If r is a $gssm$-cf then $C_{gssm-cl(r(S))(l)}(v) \leq C_{r(cl(S))(l)}(v)$ for every S_L in (M, τ, L).

Proof. Surmise that r is a $gssm$-cf. Since $cl(S_L)$ is a closed smset, $r(cl(S_L))$ is a $gssm$-cf. Therefore $C_{gssm-cl(r(cl(S)))(l)}(v) = C_{r(cl(S))(l)}(v)$. We know that $C_{S(l)}(v) \leq C_{cl(S)(l)}(v)$ and so $C_{r(S)(l)}(v) \leq C_{r(cl(S))(l)}(v)$. Therefore $C_{gssm-cl(r(S))(l)}(v) \leq C_{gssm-cl(r(cl(S)))(l)}(v) = C_{r(cl(S))(l)}(v)$.

Proposition 2.2.8. If $C_{r(int(S))(l)}(v) \leq C_{gssm-int(r(S))(l)}(v)$ for every $S_L$in (M, τ, L), then the image of the open smset $S_L$in (M, τ, L) is equal to its $gssm$-interior.

Proof. Presume that $C_{r(int(S))(l)}(v) \leq C_{gssm-int(r(S))(l)}(v)$ for every S_L in (M, τ, L). Let S_L be a open smset in (M, τ, L). Then we have $C_{r(S)(l)}(v) = C_{r(int(S))(l)}(v) \leq C_{gssm-int(r(S))(l)}(v)$. But $C_{gssm-int(r(S))(l)}(v) \leq C_{r(S)(l)}(v)$ always. Therefore $C_{gssm-int(r(S))(l)}(v) = C_{r(S)(l)}(v)$. Hence the image of the open smset $S_L$in $(M, \tau, L)$is equal to its $gssm$-interior.

Proposition 2.2.9. If $C_{gssm-cl(r(S))(l)}(v) \leq C_{r(cl(S))(l)}(v)$ for every S_L in M_L,then the image of the closed smsetS_L in (M, τ, L) is equal to its $gssm$-closure.

Proof. Let S_L be a closed smset in (M, τ, L). Then we have $C_{S(l)}(v) = C_{cl(S)(l)}(v)$ and so $C_{gssm-cl(r(S))(l)}(v) \leq C_{r(S)(l)}(v)$. Therefore $C_{gssm-cl(r(S))(l)}(v) = C_{r(S)(l)}(v)$. Since $C_{r(S)(l)}(v) \leq C_{gssm-cl(r(S))(l)}(v)$ always. Hence the image of the closed smset $S_L$in (M, τ, L) is equal to its *gssm*-closure.

Theorem 2.2.10. smfunction$r$ is a *gssm*-cf if and only if for each sub smset C_Q of (N, σ, Q) and each open smset S_L of (M, τ, L) containing $r^{-1}(C_Q)$, there is a *gsos* mset T_Q of (N, σ, Q) containing C_Q in order that $C_{r^{-1}(T)(q)}(w) \leq C_{S(l)}(v)$.

Proof. Presume that r is a *gssm*-cf and C_Q is a sub smset of (N, σ, Q) in order that $C_{r^{-1}(C)(q)}(w) \leq C_{S(l)}(v)$ where S_L is a open smset of (M, τ, L). Since r is a *gssm*-cf, $r((S_L)^c)$ is a *gssm*-cf in (N, σ, Q). Thus $C_{T(q)}(w) = C_{(r(S^c)^c)(l)}(v)$ is a *gssm*-of in (N, σ, Q).But we have $C_{C(q)}(w) \leq C_{T(q)}(w)$, since $C_{r^{-1}(C)(q)}(w) \leq C_{S(l)}(v)$ and so $C_{r^{-1}(T)(q)}(w) \leq C_{r^{-1}(r(S)^c)^c(l)}(v) \leq C_{((S)^c)^c(l)}(v) = C_{S(l)}(v)$. This implies $C_{r^{-1}(T)(q)}(w) \leq C_{S(l)}(v)$.

Contrarily, Presume that S_L be a closed smset of (M, τ, L) and $C_{U(q)}(w) = C_{(r(S))^c(l)}(v)$. Then $C_{r^{-1}(U)(q)}(w) = C_{r^{-1}((r(S))^c)(l)}(v) \leq C_{(S)^c(l)}(v)$ and $(S_L)^c$ is a open smset. By assumption, there exists a *gsos* mset T_Q in (N, σ, Q) such that $C_{U(q)}(w) \leq C_{T(q)}(w)$ and $C_{r^{-1}(T)(q)}(w) \leq C_{(S)^c(l)}(v)$ and so $C_{S(l)}(v) \leq C_{r^{-1}((T)^c)(q)}(w)$. Hence $C_{(T)^c(q)}(w) \leq C_{r(S)(l)}(v) \leq C_{r(r^{-1}((S)^c))(l)}(v)) \leq C_{(T)^c(q)}(w)$ which implies $r(S_L)$is *gscs* mset. Hence r is a *gssm*-cf.

Remark 2.2.11. In the following example, the composition of *gssm*-cf do not necessarily *gssm*-cf.

Example 2.2.12.

Let $M = \{2/v_1, 2/v_2, 1/v_3\}$, $L = \{l_1, l_2\}$

and $\tau = \{\widetilde{\phi}, \widetilde{M}, (S_L)_1, (S_L)_2\}$

where $S_1(l_1) = \{2/v_1, 2/v_2\}, S_1(l_2) = \{M\}, S_2(l_1) = \{2/v_2\}, S_2(l_2) = \{2/v_2\}$

and

let $N = \{1/w_1, 2/w_2, 1/w_3\}$, $Q = \{q_1, q_2\}$

and $\sigma = \{\widetilde{\phi}, \widetilde{N}, (T_Q)_1, (T_Q)_2, (T_Q)_3, (T_Q)_4\}$

where

$T_1(q_1) = \{1/w_1, 1/w_3\}, T_1(q_2) = \{1/w_1, 1/w_3\}, T_2(q_1) = \{1/w_1\}, T_2(q_2) = \{1/w_3\}$, $T_3(q_1) = \{2/w_2\}, T_3(q_2) = \{2/w_2\}, T_4(q_1) = \{1/w_1, 2/w_2\}, T_4(q_2) = \{2/w_2, 1/w_3\}$and

let $O = \{3/z_1, 2/z_2, 2/z_3\}$, $T = \{t_1, t_2\}$ and $\eta = \{\widetilde{\phi}, \widetilde{O}, (H_T)_1\}$ where $H_1(t_1) = \{3/z_1, 2/z_3\}, H_1(t_2) = (3/z_1, 2/z_3\}$.

Define $u : M \longrightarrow N$ and $p : L \longrightarrow Q$ by $u = \{(2/v_1, 1/w_1)/2, (2/v_2, 2/w_2)/4, (1/v_3, 1/w_3)/1\}$ and $p(l_1) = q_1$, $p(l_2) = q_2$ and define $d : N \longrightarrow O$ and $e : Q \longrightarrow T$ by $d = \{(1/w_1, 3/z_1)/3, (2/w_2, 2/z_2)/4, (1/w_3, 2/z_3)/2\}$

and $e(q_1) = t_1$, $e(q_2) = t_2$.Then the mapping $r : M_L \to N_Q$ and $g : N_Q \to O_T$are *gssm*-cf but their composition $g \ o \ r$ is not *gssm*-cf.

Proposition 2.2.13. Let (M, τ, L), (N, σ, Q) and (O, η, T) be SMTSs and let $u : M \longrightarrow N$ $d : N \longrightarrow O$ $p : L \longrightarrow Q$ and $e : Q \longrightarrow T$ be functions. If $r : M_L \to N_Q$issmclosed and $g : N_Q \to O_T$is smclosed, then their composition $g \ o \ r : M_L \to O_T$ is *gssm*-cf.

Proof. Proof is obvious from the definition.

Proposition 2.2.14. Every smopen (resp.smclosed) function is a *gssm*-of (resp. *gssm*-cf)

Proof. obvious.

Remark 2.2.15. Reverse implication of the above proposition do not necessarily correct as shown from the below example.

Example 2.2.16. In example 2.2.4 the function r is a *gssm*-of (resp. *gssm*-cf) but not a smopen (resp. smclosed) function.

Proposition 2.2.17. For a smfunction$r$, the following are coequal.

1. r^{-1} is a *gssm*-cts.
2. r is a *gssm*-of.
3. r is a *gssm*-cf.

Proof.1 $\Rightarrow$ 2Let S_L be an open smset in (M, τ, L). By hypothesis $C_{(r^{-1})^{-1}(S)(l)}(v) = C_{r(S)(l)}(v)$ is a *gssm*-of in (N, σ, Q). Thus r is a *gssm*-of.

2 $\Rightarrow$ 3 Let T_L be a closed smset in (M, τ, L). Then $(T_L)^c$ is an open smset in (M, τ, L). By hypothesis, $C_{r((T)^c)(l)}(v) = C_{(r(T))^c(l)}(v)$ is a *gssm*-of in (N, σ, Q) and therefore $r(T_L)$ is a *gssm*-cf in (N, σ, Q). Hence r is a *gssm*-cf.

3 $\Rightarrow$ 1 Let S_L be a closed smset in (M, τ, L). By assumption $r(S_L)$ is a *gssm*-cf in (N, σ, Q). But $C_{r(S)(l)}(v) = C_{(r^{-1})^{-1}(S)(l)}(v)$ and hence r^{-1} is a *gssm*-cts.

Generalized semi closed soft multi graph

Definition 2.3 .1. The smgraph of r is a smset $G(r)_{LQ}$, where $G(r)_{LQ}\ : L\ \ Q\ \longrightarrow P^{*}(M\ \ N)$ is defined by

$$G(r)(l,q)=\begin{cases}G(r) & \text{if } p(l)=q\\ 0 & \text{if } p(l)\neq q\end{cases}$$

where $G(r)$ is usual graph of the function r.

Definition 2.3.2. If $G(r)_{LQ}$ is closed smset in soft multi product topological space $(M\,X\,N,\ \tau\,X\,\sigma,\ L\,X\,Q)$, then r is smfunction with closed smgraph.

Definition 2.3.3. A smfunction$r$ has generalized semi closed soft multi graph (in short $gscs$-mg) if for each $([(n/v)_l]_L,\ [(n/w)_q]_Q)\ \in\ M_L X\ N_Q\ - G(r)_{LQ}$, there exists a $gsos$ mset S_L and an open smset T_Q containing $[(n/v)_l]_L$ and $[(n/w)_q]_Q$ respectively in order that $S_L\ X\ cl(T_Q)$ and $G(r)_{LQ}$ have no common ordered pairs.

Example 2.3.4. Let $M=\{2/v_1,\ 2/v_2\},\ L=\{l_1,\ l_2\}$ and $\tau=\{\widetilde{\phi,\ M},\ (S_L)_1,\ (S_L)_2,(S_L)_3\}$ where $S_1\,(l_1)\ =\{2/v_1\},S_1\,(l_2)\ = \{\phi\},S_2\,(l_1)\ =\{2/v_2\},S_2\,(l_2)\ =\{\phi\}\ ,S_3\,(l_1)\ =\{M\},S_2\,(l_2)\ =\{\phi\}$and let $N=\{2/w_1,\ 2/w_2\},\ Q=\{q_1,\ q_2\}$ and σ be the discrete SMTS over N_Q. Define u and p by $u=\{(2/v_1,\ 2/w_1)/4,\ (2/v_2,\ 2/w_2)/4\}$ and $p\,(l_1)\ =q_1,\ \ p\,(l_2)\ =q_2.$ Then the function$r$ is $gscs$-mg.

Proposition 2.3.5. A smfunction$r$ has a $gscs$-mg if and only if for every $([(n/v)_l]_L,\ [(n/w)_q]_Q)\ \in\ M_L X\ N_Q$ in order that $r([(n/v)_l]_L)\neq[(n/w)_q]_Q$, there exists a $gsos$ mset S_L and an open smset T_Q containing $[(n/v)_l]_L$ and $[(n/w)_q]_Q)$ respectively such that $r(S_L)$ and $cl(T_Q)$ have no common elements.

Proof. Suppose that for every $([(n/v)_l]_L,\ [(n/w)_q]_Q)\ \in\ M_L X\ N_Q$ such that $r([(n/v)_l]_L)\neq[(n/w)_q]_Q)$. Then there exists a $gsos$ mset S_L and an open smset T_Q containing $[(n/v)_l]_L$ and $[(n/w)_q]_Q)$ respectively such that $(S_L\ X\ cl\,(T_Q))\cap G\,(r)_{LQ}=\phi$, since r has a $gscs$-mg. Hence for each $[(n/v)_l]_L\in S_L$ and $[(n/w)_q]_Q)\in cl(T_Q)$ with $r([(n/v)_l]_L)\neq[(n/w)_q]_Q$,$r(S_L)$ and $cl(T_Q)$ have no common elements.

Contrarily, let $([(n/v)_l]_L,\ [(n/w)_q]_Q)\notin G\,(r)_{LQ}$. Then $r([(n/v)_l]_L)\neq[(n/w)_q]_Q$ and so there exists an $gsos$ mset $S_L$and an open smset T_Q containing $[(n/v)_l]_L$ and $[(n/w)_q]_Q$ respectively such that $r(S_L)$ and $cl(T_Q)$ have no common elements. This implies for every $[(n/v)_l]_L\in S_L$ and $[(n/w)_q]_Q)\in cl(T_Q)$ with $r([(n/v)_l]_L)\neq[(n/w)_q]_Q$. Therefore $(S_L\ X\ cl(T_Q))$ and $G\,(r)_{LQ}$ have no common ordered pairs. Hence r has a $gscs$-mg.

Proposition 2.3.6. If r is $gssm$-cts from a SMTS $(M,\tau,\ L)$ into a smHausdroff$(N,\ \sigma,\ Q)$, then r has a $gscs$-mg.

Proof. Let $([(n/v)_l]_L,\ [(n/w)_q]_Q)\notin G\,(r)_{LQ}$. Then $r([(n/v)_l]_L)\neq[(n/w)_q]_Q$. Since $(N,\ \sigma,\ Q)$ is sm Hausdroff space, there exist two disjoint open smsetT_Q and R_Q in order that $r([(n/v)_l]_L)\in R_Q$ and $[(n/w)_q]_Q\in T_Q$. Since r is a $gssm$-cts. there exists a $gsos$ mset S_L in order that $[(n/v)_l]_L\in S_L$ and $r(S_L)\subseteq R_Q$ by Proposition 4.15. Thus$r\,(S_L)\subseteq N_Q-cl(R_Q)$. Therefore $r(S_L)$ and $cl(R_Q)$ have no common elements and so r has a $gscs$-mg.

Proposition 2.3.7. If r is surjective smfunction with $gscs$-mg from a SMTS $(M,\tau,\ L)$ onto a SMTS $(N,\ \sigma,\ Q)$, then $(N,\ \sigma,\ Q)$ is smHausdroff.

Proof. Let $[(n/w_1)_q]_Q$ and $[(n/w_2)_q]_Q$ be two distinct smpoints in N_Q. Then there exists a smpoint$[(n/v_1)_l]_L\in M_L$ such that $r([(n/v_1)_l]_L)=[(n/w_1)_q]_Q\neq[(n/w_2)_q]_Q$. Thus $([(n/v_1)_l]_L,[(n/w_1)_q]_Q)\ \notin\ G\,(r)_{LQ}$. Since r has $gscs$-mg, there exist a $gsos$ mset S_L and an open smset T_Q containing $[(n/v_1)_l]_L$ and $[(n/w_2)_q]_Q$, respectively, such that $r(S_L)$ and $cl(T_Q)$ have no common elements and so $r([(n/v_1)_l]_L)\notin cl(T_Q)$. But $N_Q-cl(R_Q)$ is an open smset containing $([(n/v_1)_l]_L)=[(n/w_1)_q]_Q$. Thus T_Q and $N_Q-cl(T_Q)$ have no common elements. Hence $(N,\ \sigma,\ Q)$ is smHausdroff.

Proposition 2.3.8. The SMTS $(M,\tau,\ L)$ is Hausdroff if and only if the smidentity function $r: M_L\longrightarrow M_L$ has a $gscs$-mg.

Proof. It is obvious from above Propositions.

3 Result and Discussion

The following results are obtained:

1. Every smcontinuous function is $gssm$-cts.
2. Composition of $gssm$-cts functions and smcontinuous functions is $gssm$-cts functions.
3. Composition of two smcontinuous functions is $gssm$-cts.
4. smfunction is $gssm$-cts if and only if the inverse image of each open smset is $gsos$ mset.
5. For any $gssm$-cts, the cardinality of the image smset is less than the cardinality of the closed smset.

6. Every *gssm*-cts from a SMTS into smHausdroff is a *gscs*-mg.
Examples were provided then and there to confirm the contrary results.

4 Conclusion

This study has acquainted itself with generalized semi soft multi continuous mappings, generalized semi soft multi open mappings, generalized semi soft multi closed mappings, and also generalized semi soft multi homeomorphisms. Moreover, it has also established a generalized semi closed soft multi graph by using generalized semi closed soft multisets. In the future,the concepts of generalized semi soft multi connectedness and compactness can be extended.

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