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# Separation Axioms through Nano JD open set

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## Abstract

**Objective:** This study aims to introduce NanoJD-*T*ispaces ( $i=0,1,2$ ) using Nano JD open set and investigate their properties. **Methods:** To deal with the separation axioms first we need to find the NanoJD-*T*ispaces ( $i=0,1,2$ ) using Nano JD open set. **Findings:** We can find Nano JD-*T*ispaces ( $i=0,1,2$ ) using Nano JD open sets. **Novelty:** Various characterization of these spaces are discussed. Also, the relationship among themselves and with known separation axioms are to be studied.

**Keywords:** Nano JD Open sets; Separation Axioms; NanoJD T 0 space; Nano JD T 1 space; Nano JD T 2 space

## 1 Introduction

Nano topology, a relatively recent branch of mathematics, unveils a microscopic world where space is intricately woven with indiscernibility relations. In this study, we focus on separation axioms within the context of Nano topological spaces through Nano JD open sets which extends the boundaries of Nano topology by introducing Nano JD-Ti spaces, as the previous researches mainly delve on the foundational aspects of Nano separation axioms. The axioms found in this study, akin to delicate threads, weave a fabric that defines the spatial relationships between points. This embarks an intellectual voyage, as we study the characteristics of these spaces, guided by the principles of nano topology. Further we unravel the delicate threads that bind points together or set them apart in the nano cosmos, where even the tiniest distinctions matter.

## 2 Methodology

Definition 2.1:<sup>(1)</sup>

Let  $U$  be a non-empty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be Indiscernible with one another. The pair  $(U, R)$  is said to be the approximation space. Let  $A \subseteq U$

1. The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can before certain classified as  $X$  with respect to  $R$  and is defined by

$$LR(X) = \cup_{x \in U} \{R(x) : R(x) \subseteq X\}$$

2. The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as with respect to  $R$  and is defined by

$$UR(X) = \cup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$$

3. The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified by

$$BR(X) = UR(X) - LR(X)$$

**Definition 2.2:**<sup>(2)</sup> Let  $U$  be non-empty, finite universe of objects and  $R$  be an equivalence relation on  $U$ . Let  $X \subseteq U$ . Let  $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ . Then  $\tau_R(X)$  a topology on  $U$ , called as the Nanotopology with respect to  $X$ . Elements of the Nano topology are known as the Nano open sets in  $U$  and  $(U, \tau_R(X))$  is called the Nano topological space.  $(\tau_R(X))^c$  is called the Dual Nanotopology on  $\tau_R(X)$ . Elements of  $(\tau_R(X))^c$  are called as Nano closed sets.

**Definition 2.3:**<sup>(3)</sup> Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then  $\tau_R(X)$  satisfies the following axioms.  $U$  and  $\emptyset \in \tau_R(X)$  The union of the elements of any subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$  The intersection of the elements of any finite subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$ . That is  $\tau_R(X)$  forms a topology on  $U$  called as the Nano Topology on  $U$  with respect to  $X$ . We call  $(U, \tau_R(X))$  as the Nano Topological space. The elements of  $\tau_R(X)$  are called Nano open sets. If  $(U, \tau_R(X))$  is a Nano Topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$  then the Nano interior of  $A$  is defined as the union of all Nano open subsets of  $A$  and it is defined as  $Nint(A)$ . Nano interior is the largest open subset of  $A$ .

**Definition 2.4:**<sup>(4)</sup> The Nano Closure of  $A$  is defined as the intersection of all Nano closed sets containing  $A$  and it is denoted by  $Ncl(A)$ . It is the smallest Nano closed set containing  $A$ .

**Definition 2.5:**<sup>(5)</sup> If  $(U, \tau_R(X))$  is a Nano Topological space with respect to  $X$  and  $A \subseteq U$ . Then  $A$  is said to be i) Nano semi-open if  $A \subseteq Ncl(Nint(A))$ . ii) Nano pre-open if  $A \subseteq Nint(Ncl(A))$  iii) Nano  $\alpha$ -open if  $A \subseteq Nint(Ncl(Nint(A)))$ . iv) Nano  $\beta$ -open if  $A \subseteq Ncl(Nint(Ncl(A)))$  v) Nano regular-open if  $A = Nint(Ncl(A))$ .  $NSO(U, \tau_R(X))$ ,  $NPO(U, X)$ ,  $N\alpha O(U, X)$ ,  $N\beta O(U, X)$ , and  $NRO(U, X)$  respectively denote the families of all Nano-semiopen, Nano pre-open, Nano  $\alpha$ -open, Nano  $\beta$ -open, Nano regular open, and Nano  $b$ -open subsets of  $U$ .

**Definition 2.6:**<sup>(5)</sup> Let  $(U, \tau_R(X))$  be a Nano Topological space and  $A \subseteq U$ .  $A$  is said to be Nano semi-closed, Nano pre-closed, Nano  $\alpha$ -closed, Nano  $\beta$ -closed, and Nano regular closed if its complement is Nano semi-open, Nano pre-open, Nano open, Nano semi-pre-open, and Nano regular open respectively.

## 3 Results and Discussion

### 3.1 Nano JD $T_0$ space

Here, we define Nano  $T_0$  space and study some of its characterizations in Nano topological spaces.

**Definition 3.1.1:** A Nano topological space  $(M, \tau(P))$  is said to be Nano JD- $T_0$  space if for each pair of distinct points  $k, l$  of  $M$ , there exist an Nano JD open set containing one point and not the other.

**Theorem 3.1.2:** Every Nano  $T_0$  space is a Nano JD  $T_0$  space.

**Proof:** Let  $M$  be a Nano  $T_0$  space. Let  $k, l$  be two distinct points in  $M$ . Since  $M$  is a Nano  $T_0$  space, there exists an Nano open set  $G$  in  $M$  such that  $k \in G, l \notin G$ . Since every Nano open set is Nano JD open,  $G$  is Nano JD open in  $M$ . Thus, for any two distinct points  $k, l$  in  $M$  there exists an Nano JD open set  $G$  such that  $k \in G, l \notin G$ . Hence  $M$  is a Nano JD  $T_0$  space.

**Remark 3.1.3:** Reverse implications need not hold true as evidenced by the illustration below.

**Example 3.1.4:** Let  $U = \{l, m, n, o\}$ ;  $U \setminus R = \{\{l\}, \{n\}, \{m, o\}\}$   $X = \{l, m\}$ .  $\tau_R(U, \tau_R(X)) = \{U, \emptyset, \{l\}, \{l, m, o\}, \{m, o\}\}$   $NJDO(U, \tau_R(X)) = \{U, \emptyset, \{l\}, \{m\}, \{o\}, \{l, m\}, \{m, n\}, \{n, o\}, \{l, o\}, \{m, o\}, \{l, n\}, \{l, m, n\}, \{l, m, o\}\}$  Clearly  $U$  is Nano JD  $T_0$  space but not Nano JD  $T_0$  space.

**Theorem 3.1.5:** Every Nano  $\alpha T_0$  space is a Nano JD  $T_0$  space.

**Proof:** Let  $M$  be a Nano  $\alpha T_0$  space. Let  $k, l$  be two distinct points in  $M$ . Since  $M$  is Nano  $\alpha T_0$  space, there exists a Nano  $\alpha$  open set  $G$  in  $M$  such that  $k \in G, l \notin G$  Since every Nano  $\alpha$  open set is Nano JD open,  $G$  is Nano JD open in  $M$ . Thus, for any two distinct points  $k, l$  in  $M$  there exists a Nano JD open set  $G$  such that  $k \in G, l \notin G$ . Hence  $M$  is a Nano JD  $T_0$  space.

**Remark 3.1.6:** Reverse implications need not hold true as evidenced by the illustration below.

**Example 3.1.7:**

Let  $M = \{l, r, v, w\}$   $M \setminus R = \{\{l\}, \{v\}, \{r, v\}\}$   $P = \{l, r\}$   $\tau(P) = \{M, \emptyset, \{l\}, \{l, r, w\}, \{r, w\}\}$  Nano JD open =  $\{M, \emptyset, \{l\}, \{r\}, \{w\}, \{l, r\}, \{r, v\}, \{v, w\}, \{l, w\}, \{r, w\}, \{l, v\}, \{l, r, v\}, \{l, r, w\}, \{r, v, w\}, \{l, v, w\}\}$  clearly  $M$  is a Nano JD  $T_0$  space but not Nano  $\alpha$  space.

**Theorem 3.1.8:** A Nano topological space  $(M, \tau(P))$  is a Nano JD  $T_0$  space if and only if Nano JD closures of distinct points are distinct.

**Proof:** Let  $k, l \in M$  and with  $k \neq l$  and  $M$  be a Nano JD  $T_0$  space. we shall show that  $NJDcl(k) \neq NJDcl(l)$ . Since,  $M$  is Nano JD- $T_0$  space, there exists a Nano JD open set  $G$  such that  $k \in G, l \notin G$ . Also,  $k \notin M - G, l \in M - G$ , where  $M - G$  is Nano JD closed set in  $M$ . Since  $NnJDcl(\{l\})$  is the intersection of all Nano JD closed sets which contain  $l$ . Hence  $l \in NnJDcl(\{l\})$  but  $k \notin NJDcl(\{l\})$  as  $k \notin M - G$ . Therefore,  $NJDcl(k) \neq NJDcl(l)$ . Conversely, suppose that for any pair of distinct points  $k, l \in M, NJDcl(k) = NJDcl(l)$ . Then there exist at least one point  $z \in G$  such that  $z \in NJDcl(\{k\})$  but  $z \notin NJDcl(\{l\})$ . we claim that  $k \notin NJDcl(l)$ . If  $k \in NJDcl(\{l\})$  then  $NJDcl(k) \subset NJDcl(l)$ . So  $z \in NJDcl(k)$  which is a contradiction. Now,  $k \notin NJDcl(l)$  implies  $k \in M - NJDcl(l)$  which is a Nano JD open set in  $M$  containing  $k$  but not  $l$ . Hence,  $M$  is Nano JD- $T_0$  space.

**Theorem 3.1.9:** Let  $h : (M, \tau(P)) \rightarrow (N, \tau(Q))$  be a bijection, Nano JD open map and  $M$  is Nano JD- $T_0$  space, then  $N$  is also Nano JD- $T_0$  space.

**Proof:** Let  $l_1, l_2 \in N$  with  $l_1 \neq l_2$ . Since  $h$  is a bijection, there exist  $k_1, k_2 \in M$  with  $h(k_1) = l_1$  and  $h(k_2) = l_2$ . Since,  $M$  is Nano JD- $T_0$  space, there exists a Nano JD open set  $G$  in  $M$  such that  $k_1 \in G, k_2 \notin G$ . Since  $h$  is Nano JD open map,  $h(G)$  is a Nano JD open set in  $N$ . Now, we have  $k_1 \in G \Rightarrow h(k_1) \in h(G) \Rightarrow l_1 \in h(G)$  and  $k_2 \notin G \Rightarrow h(k_2) \notin h(G)$ . Hence for any two distinct points  $l_1, l_2 \in N$ , there exists a Nano JD open set  $h(G)$  in  $N$  such that  $l_1 \in h(G)$  and  $l_2 \notin h(G)$ . Hence  $N$  is a Nano JD- $T_0$  space.

**Theorem 3.1.10:** Let  $h : (M, \tau(P)) \rightarrow (N, \tau(Q))$  be a bijection, Nano JD irresolute and  $N$  is Nano JD- $T_0$  space, then  $M$  is also Nano JD- $T_0$  space.

**Proof:** Let  $l_1, l_2 \in N$  and  $l_1 \neq l_2$ . since  $h$  is a bijection, there exist  $k_1, k_2 \in M$  with  $h(k_1) = l_1$  and  $h(k_2) = l_2 \Rightarrow k_1 = h^{-1}(l_1)$  and  $k_2 = h^{-1}(l_2)$ . Since,  $N$  is Nano JD  $T_0$ , there exists a Nano JD open set  $G$  in  $N$  such that  $l_1 \in G$ . Since  $h$  is Nano JD irresolute map,  $h^{-1}(G)$  is a Nano JD open set in  $M$ . Now, we have  $l_1 \in G \Rightarrow h^{-1}(l_1) \in h^{-1}(G) \Rightarrow k_1 \in h^{-1}(G)$  and  $l_2 \notin G \Rightarrow h^{-1}(l_2) \notin h^{-1}(G) \Rightarrow k_2 \notin h^{-1}(G)$ . Hence for any two distinct points  $k_1, k_2 \in M$ , there exists Nano JD open set  $h^{-1}(G)$  in  $M$  such that  $k_1 \in h^{-1}(G)$  and  $k_2 \notin h^{-1}(G)$ . Hence  $M$  is a Nano JD- $T_0$  space.

**Theorem 3.1.11:** Let  $h : (M, \tau(P)) \rightarrow (N, \tau(Q))$  be a bijection, Nano JD continuous and  $N$  is Nano  $T_0$  space, then  $M$  is also Nano JD  $T_0$  space.

**Proof:** Let  $h : (M, \tau(P)) \rightarrow (N, \tau(Q))$  be a Nano JD continuous map and  $N$  is Nano  $T_0$  space. Let  $k_1, k_2 \in M$  with  $k_1 \neq k_2$ . Let  $l_1 = h(k_1)$  and  $l_2 = h(k_2)$ . Since  $h$  is one-one,  $l_1 \neq l_2$ . Since  $N$  is  $T_0$ , there is Nano open set  $G$  in  $N$  containing  $l_1$  or  $l_2$  but not the other. Thus  $M$  is Nano JD  $T_0$  space.

### 3.2 NANO JD-T<sub>1</sub> SPACE

**Definition 3.2 .1:** A Nano topological space  $M$  is said to be Nano JD  $T_1$  space if for each pair of distinct points  $k, l$  of  $M$ , there exists a pair of Nano JD open sets, one containing  $k$  but not  $l$  and the other containing  $l$  but not  $k$ .

**Remark 3.2 .2:** Every Nano JD  $T_1$  space is a Nano JD  $T_0$  space but the converse is not true.

**Theorem 3.2 .3:** For a Nano topological space  $(M, \tau(P))$  the following are equivalent.

- (i)  $M$  is Nano JD  $T_1$  space.
- (ii) For every  $k \in M, \{k\}$  is a Nano JD closed in  $M$ .
- (iii) Each subset of  $M$  is the intersection of Nano JD open sets containing it.
- (iv) The intersection of all Nano JD open sets in  $M$  containing the points  $k$  is  $\{k\}$ .

**Proof:**

(i)  $\Rightarrow$  (ii) Suppose that  $M$  is a Nano JD  $T_1$  space. Let  $k \in M, \{k\}$ . Then for every  $l \neq k$ , there exists a Nano JD open sets  $G$  in  $M$  containing  $l$  but not  $k$ .  $M \cap \{k\} \neq \emptyset$ . Therefore,  $k \in G$  a contradiction. Thus,  $\{k\}$  is Nano JD closed.

(ii)  $\Rightarrow$  (iii) Let  $J \subset M$ . Then for each  $k \in M \setminus J, \{k\}$  is Nano JD closed in  $M$  and hence,  $M \setminus \{k\}$  is Nano JD open. Clearly,  $J \subset M \setminus \{k\}$  for each  $k \in M \setminus J$ . Therefore,  $J \subset \cap \{M \setminus \{k\} : k \in M \setminus J\}$ . On the other hand, if  $l \notin J$ , then  $l \in M \setminus J$  and  $l \notin M \setminus \{l\}$ . This implies,  $l \notin \cap \{M \setminus \{k\} : k \in M \setminus J\}$ . Hence,  $\cap \{M \setminus \{k\} : k \in M \setminus J\} \subset J$ . Therefore,  $J = \cap \{M \setminus \{k\} : k \in M \setminus J\}$  which proves (iii).

(iii)  $\Rightarrow$  (iv) Taking  $J = \{k\}$ , by (iii)  $J = \{k\} = \cap \{G : G \text{ is Nano JD open and } k \in G\}$ . This proves (iv).

(iv)  $\Rightarrow$  (i) Let  $k, l \in M$  with  $l \neq k$ . Then  $l \notin \{k\} = \cap \{G : G \text{ is Nano JD open and } k \in G\}$ . Hence there exists a Nano JD open set  $G$  containing  $k$  but not  $l$ . Similarly there exists a Nano JD open set  $H$  containing  $l$  but not  $k$ . Thus,  $M$  is Nano JD  $T_1$  space.

**Theorem 3.2 .4:** Let  $h : (M, \tau(P)) \Rightarrow (N, \tau(Q))$  be a bijection

- (i) If  $h$  is a Nano JD continuous and  $N$  is Nano  $T_1$  space, then  $M$  is a Nano JD  $T_1$  space.
- (ii) If  $h$  is a Nano JD irresolute and  $N$  is Nano JD  $T_1$  space.
- (iii) If  $h$  is a Nano JD open and  $M$  is Nano  $T_1$  space, then  $N$  is a Nano JD  $T_1$  space.

**Proof:**

(i) Suppose  $h$  is Nano JD continuous bijection and  $N$  is Nano  $T_1$  space. Let  $k_1, k_2 \in M$ , with  $k_1 \neq k_2$ . Let  $l_1 = h(k_1)$  and  $l_2 = h(k_2)$ . Since,  $h$  is one to one,  $y_1 \in y_2$ . Since  $N$  is Nano  $T_1$  space, there exists an Nano open set  $G$  and  $H$  in  $N$  such that  $l_1 \in G$  but  $l_2 \notin G$  and  $l_2 \in H$  but  $l_1 \notin H$ . Since  $h$  is bijection,  $k_1 \in h^{-1}(G)$  but  $k_2 \notin h^{-1}(G)$  and  $k_1 \notin h^{-1}(H)$  but  $k_2 \in h^{-1}(H)$ . Since  $h$  is Nano JD continuous,  $h^{-1}(G)$  and  $h^{-1}(H)$  are Nano JD open sets in  $M$ . Thus  $M$  is Nano JD  $T_1$  space.

(ii) Suppose  $h$  is a Nano JD irresolute bijection and  $N$  is Nano JD  $T_1$  space. Let  $k_1, k_2 \in M$ , with  $k_1 \neq k_2$ . Let  $l_1 = h(k_1)$  and  $l_2 = h(k_2)$ . Since  $h$  is one to one,  $l_1 \neq l_2$ . Since  $N$  is Nano  $T_1$  space, there exists an Nano open set  $G$  and  $H$  in  $N$  such that  $l_1 \in G$  but  $l_2 \notin G$  and  $l_2 \in H$  but  $l_1 \notin H$ . Since  $h$  is bijection,  $k_1 \in h^{-1}(G)$  but  $k_2 \notin h^{-1}(G)$  and  $k_1 \notin h^{-1}(H)$  but  $k_2 \in h^{-1}(H)$ . Since  $h$  is Nano JD irresolute,  $h^{-1}(G)$  and  $h^{-1}(H)$  are Nano JD open sets in  $M$ . Thus  $M$  is Nano JD  $T_1$  space.

(iii) Suppose  $h$  is a Nano JD open bijection and  $M$  is Nano  $T_1$  space. Let  $l_1 \neq l_2 \in N$ . Since  $h$  is a bijection there exists  $k_1, k_2 \in M$  such that  $h(k_1) = l_1$  and  $h(k_2) = l_2$  with  $k_1 \neq k_2$ . Since  $M$  is Nano  $T_1$  space, there exists open sets  $G$  and  $H$  in  $M$  such that  $k_1 \in G$  and  $k_2 \notin G$  and  $k_2 \in H$  but  $k_1 \notin H$ . Since  $h$  is Nano JD open,  $h(G)$  and  $h(H)$  are Nano JD open sets in  $N$  such that  $l_1 = h(k_1) \in h(G)$  and  $l_2 = h(k_2) \in h(H)$ . Since  $h$  is a bijection  $l_2 = h(k_2) \notin h(M)$  and  $l_1 = h(k_1) \in h(H)$  and  $l_2 = h(k_2) \in h(N)$ . Since  $h$  is a bijection  $l_2 = h(k_2) \notin h(G)$  and  $l_1 = h(k_1) \notin h(H)$ . Thus  $N$  is Nano JD  $T_1$  space.

**Theorem 3.2 .5:** A Nano topological space  $(M, \tau(P))$  is Nano JD  $T_1$  if and only if the singletons are Nano JD closed.

**Proof:**

Let  $(M, \tau(P))$  be Nano JD  $T_1$  space and  $e$  be any point of  $M$ . Suppose  $c \in (e)^c$  then  $e \neq c$  and so there exists a Nano JD open set  $G$  such that  $c \in G$  but  $e \notin G$ . Consequently,  $c \in G \subset (e)^c$  that is  $(e)^c = \bigcap \{G : c \in (e)^c\}$  which is Nano JD open. Hence,  $(e)$  is Nano JD closed.

Conversely, Let  $e, c$  be two distinct points of  $M$ . Then  $c \in (e)^c$  and  $(e)^c$  is Nano JD open set containing  $e$  but not  $c$ . Hence,  $M$  is Nano JD  $T_1$  space.

### 3.3 Nano JD- $T_2$ spaces

**Definition 3.3 .1:** A Nano topological space  $M$  is said to be Nano JD  $T_2$  space, if for each pair of distinct points of  $k, l$  of  $M$ , there exists disjoint Nano JD open sets  $G$  and  $H$  such that  $k \in G$  and  $l \in H$ .

**Theorem 3.3 .2:** Let  $h : (M, \tau(P)) \rightarrow (N, \tau(Q))$  is a bijection

(i) If  $h$  is a Nano JD continuous and  $N$  is Nano  $T_1$  space, then  $M$  is a Nano JD  $T_1$  space.

(ii) If  $h$  is a Nano JD irresolute and  $N$  is Nano JD  $T_1$  space.

(iii) If  $h$  is a Nano JD open and  $M$  is Nano  $T_1$  space, then  $N$  is a Nano JD  $T_1$  space.

**Proof:**

(i) Suppose  $h$  is Nano JD continuous bijection and  $N$  is Nano  $T_2$  space. Let  $k_1, k_2 \in M$ , with  $k_1 \neq k_2$ . Let  $l_1 = h(k_1)$  and  $l_2 = h(k_2)$ . Since,  $h$  is one to one,  $l_1 \neq l_2$ . Since  $N$  is Nano  $T_2$  space, there exists a Nano open set  $G$  and  $H$  containing  $l_1$  and  $l_2$  respectively. Since  $h$  is Nano JD continuous bijection,  $h^{-1}(G)$  and  $h^{-1}(H)$  are Nano JD open sets in  $M$  containing  $k_1$  and  $k_2$  respectively. Thus  $M$  is Nano JD  $T_2$  space.

(ii) Proof is similar to (i)

(iii) Suppose  $h$  is a Nano JD open bijection and  $M$  is Nano  $T_2$  space. Let  $l_1 \neq l_2 \in N$ . Since  $h$  is a bijection there exists  $k_1, k_2 \in M$  such that  $l_1 = h(k_1)$  and  $l_2 = h(k_2)$  with  $k_1 \neq k_2$ . Since  $M$  is Nano  $T_2$  space, there exists Nano disjoint open sets  $G$  and  $H$  in  $M$  such that  $k_1 \in G$  and  $k_2 \in H$ . Since  $h$  is Nano JD open in  $N$  such that  $l_1 = h(k_1) \in h(M)$  and  $l_2 = h(k_2) \in h(N)$ . Since  $h$  is a bijection  $l_2 = h(k_2) \notin h(G)$  and  $l_1 = h(k_1) \notin h(H)$ . Thus  $N$  is Nano JD  $T_2$  space.

**Theorem 3.3 .3:** The following statements are equivalent for a Nano topological space  $(M, \tau(P))$

(i)  $M$  is Nano JD  $T_2$  space

(ii) Let  $k \in M$  for each  $k \neq l$ , there exists a Nano JD open set  $G$  such that  $k \in G$  and  $l \notin NJDcl(G)$ .

(iii) For each  $k \in M$ ,  $\bigcap \{NJDcl(G) : G \in NJDO(M) \text{ and } k \in G\} = \{k\}$

**Proof:**

(i)  $\Rightarrow$  (ii) Suppose  $M$  is a Nano JD  $T_2$  space. Let  $k \in M$  and  $l \in M$  with  $k \neq l$ . Then there exists disjoint Nano JD open sets  $G$  and  $H$  such that  $k \in G$  and  $l \in H$ . Since  $H$  is Nano JD open,  $M \setminus H$  is Nano JD closed and  $G \subset M \setminus H$ . This implies that  $NJDcl(G) \subset M \setminus H$ . Since,  $l \notin M \setminus H$ ,  $l \notin NJDcl(G)$ .

(ii)  $\Rightarrow$  (iii) If  $k \neq l$  then there exists a Nano JD open set  $G$  such that  $k \in G$  and  $l \notin NJDcl(G)$ . Hence  $l \notin \bigcap \{NJDcl(G) : G \in NJDO(M) \text{ and } k \in G\}$ . This proves (iii).

(iii)  $\Rightarrow$  (i) Let  $k \neq l$  in  $M$ . Then  $k \notin \bigcap \{NJDcl(G) : G \in NJDO(M) \text{ and } k \in G\}$ . This implies that there exists a Nano JD open set  $G$  such that  $k \in G$  and  $l \notin NJDcl(G)$ . Then  $H = M \setminus NJDcl(G)$  is Nano JD open and  $l \in H$ . Now,  $G \cap H = G \cap (M \setminus NJDcl(G)) \subset G \cap (M \setminus NJDcl(G)) = \emptyset$ . This proves (i).

## 4 Conclusion

This study introduces Nano JD-*T*spaces ( $i=0,1,2$ ) using Nano JD open set and investigated their properties. Various characterization of these spaces are discussed. Also, the relationship among themselves and with known separation axioms are studied.

## Declaration

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## References

- 1) Renuka J, Jesthi J. Some Separation Axioms in Nano Ideal topological Spaces. *Journal of Namibian Studies*. 2023;35(1). Available from: <https://doi.org/10.59670/jns.v35i.4029>.
- 2) Mohammed NH, Shihab AA. M Nano –Separation Axioms  $M_N-T_i$  Spaces. *Tikrit Journal of Pure Science*. 2023;28(3):109–113. Available from: <https://dx.doi.org/10.25130/tjps.v28i3.1436>.
- 3) Srividhya P, Indira T. On Nano Regular B-Connectedness in Nano Topological Spaces. *International Journal of Aquatic Science*. 2021;12(2):79–90. Available from: [https://www.journal-aquaticscience.com/article\\_131789\\_68b809f97f0e9c6186ade700d460a353.pdf](https://www.journal-aquaticscience.com/article_131789_68b809f97f0e9c6186ade700d460a353.pdf).
- 4) Selvaraj XA, Balakrishna U. Nano Z Separation Axioms. In: Springer Proceedings in Mathematics & Statistics; vol. 384. Springer International Publishing. 2022;p. 83–92. Available from: [https://doi.org/10.1007/978-3-030-96401-6\\_7](https://doi.org/10.1007/978-3-030-96401-6_7).
- 5) Jenavee KS, Asokan R, Nethaji O. New Separation Axioms . *Nano Topological Spaces Turkish Journal of Computer and Mathematics Education*. 2022;13(03):133–139. Available from: <https://doi.org/10.33773/jum.974278>.