INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 29-08-2023

Accepted: 05-03-2024

Published: 25-05-2024

Editor: ICDMMMDE-2023 Special Issue Editors: Dr. G .Mahadevan and Prof. Dr. P. Balasubramaniam

Citation: Jackson S, Sivasankar J (2024) A New Class of Generalized Closed Sets in Pentapartitioned Neutrosophic Topological Spaces. Indian Journal of Science and Technology 17(SP1): 14-20. https://doi.org/10.17485/IJST/v17sp1.113

* **Corresponding author**.

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Funding: None

Competing Interests: None

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Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

A New Class of Generalized Closed Sets in Pentapartitioned Neutrosophic Topological Spaces

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Abstract

Objective: The motive of this research paper is to give a new notion called Pentapartitioned Neutrosophic Generalized Pre Open and Closed sets. **Methods:** To get the Pentapartitioned Neutrosophic Generalized Pre Open and Closed sets, PN topology is needed. Further, there is a need to find the Pre Open and Pre Closed sets and then we use our definition to get the required objective. **Findings:** The Pentapartitioned Neutrosophic Generalized Pre sets give a more finer collection of weak sets. Moreover, we can find the interior, exterior, and frontier of the resultant set. **Novelty:** Finding the generalized part of Pentapartitioned Neutrosophic sets is a subtle work to do. So, the investigation procedure of its characterization is an interesting aspect.

Keywords: Pentapartitioned Neutrosophic Generalized Pre Closed Set; Pentapartitioned Neutrosophic Frontier; Pentapartitioned Neutrosophic Pre Frontier of a Pentapartitioned Neutrosophic Set; Pentapartitioned Neutrosophic Set and Pentapartitioned Neutrosophic Pre

1 Introduction

Pentapartitioned Neutrosophic Generalized Pre Open sets and Pentapartitioned Neutrosophic Generalized Pre Closed sets are the main ideas presented in this research paper. Neutrosophic sets, a new type of generalized intuitionistic fuzzy sets, were introduced in 2005 by Florentin Smarandache. Further Salama and Alblowi opened a new topological space involving Neutrosophic sets. Our paper takes the Five Single Valued Neutrosophic Logic(FSVNL) or simply Pentapartitioned Neutrosophic fuzzy logic as the core. In 2020, Rama Mallick and Surapathi Pramanik[(1)] introduced the Pentapartitioned Neutrosophic sets and their properties. In the path of them, Suman Das and Binod Chandra Tripathy[(2)] deeply investigated the topology formed by the Pentapartitioned Neutrosophic Sets in 2020. In this research paper, we study some of the characteristics of Pentapartitioned Neutrosophic Pre *Open* and *Closed* sets like, interior, closure, frontier, and exterior.

Research gap: In the most recent body of research, many authors made their studies in Pentapartitioned Neutrosophic Topological Spaces but no study on Pentapartitioned Neutrosophic Generalized Pre open and closed sets of pentapartitioned neutrosophic topological space has been reported.

2 Methodology

This section presents Pentapartitioned Neutrosophic Sets and topology, null set, absolute set and basic properties to formulate the Generalized sets. Throughout this study, τ_{PN} stands for Topology induced by a Pentapartitioned Neutrosophic set.

Definition 2.1 [3] Consider Z as a non-empty set. A Neutrosophic set Rover $Z$is defined as $R = \{(r, T_R(r), I_R(r), F_R(r))/r \in Z\}$ where $T, I, F : Z \to [0,1]$, represents the degree of Truth Membership, Indeterminancy Membership and False Membership functions respectively and $0 \leq T_R(r) + I_R(r) + F_R(r) \leq 3$ for each $r \in Z$.

Definition 2.2 [4] ConsiderZas a universal set. A Pentapartitioned Neutrosophic Set(PN set) R over Z is a 6-tuple of the form, $R = \{(r, T_R(r), C_R(r), G_R(r), U_R(r), F_R(r))/r \in Z\}$ where $T, C, G, U, F : Z \to [0,1]$, represents the degree of the Truth Membership, Contradiction Membership, Ignorance Membership, Unknown Membership and Falsity Membership functions respectively and $0 \leq T_R(r)+C_R(r) + G_R(r) + U_R(r)+F_R(r) \leq 5$ for each $r \in Z$.

Definition 2.3 [5] ConsiderR, S be two Pentapartitioned Neutrosophic sets over Z.

i) $R \subseteq S$ if and only if $T_R(r) \leq T_S(r), C_R(r) \leq C_S(r), G_R(r) \geq G_S(r), U_R(r) \geq U_S(r), F_R(r) \geq F_S(r)$ for each $r \in Z$.

ii)$R \cup S = \{(r, max(T_R(r), T_S(r)), max(C_R(r), C_S(r)), min(G_R(r), G_S(r)),$
$min(U_R(r), U_S(r)), min\ (F_R(r), F_S(r)))/r \in Z\}$.

iii) $R \cap S = \{(r, min(T_R(r), T_S(r)), min(C_R(r), C_S(r)), max(G_R(r), G_S(r)),$

$$max\ (U_R(r), U_S(r)), max\ (F_R(r), F_S(r)))/r \in Z\}.$$

iv) If$R = \{(r, T_R(r), C_R(r), G_R(r), U_R(r), F_R(r))/r \in Z\}$,
then$R^C = \{(r, F_R(r), U_R(r), 1-(G_R(r)), C_R(r), T_R(r))/r \in Z\}$.

v) $R \not\subseteq S$ if atleast one of the following occurs $T_R(r) \geq T_S(r), C_R(r) \geq C_S(r), G_R(r) \leq G_S(r), U_R(r) \leq U_S(r), F_R(r) \leq F_S(r)$ for any $r \in Z$.

vi)$R \neq S$ if $R \not\subseteq S$ and $S \not\subseteq R$.

Definition 2.4 [6] A Pentapartitioned Neutrosophic set over Z is called as Null Pentapartitioned Neutrosophic set if, $T_D(a) = 0, C_D(a) = 0, G_D(a) = 1, U_D(a) = 1, F_D(a) = 1$. We denote the Null Pentapartitioned Neutrosophic set by 0_{PN}. That is $0_{PN} = \{(a, 0, 0, 1, 1, 1)/a \in Z\}$.

Definition 2.5 [6] A Pentapartitioned Neutrosophic set over Z is called as Absolute Pentapartitioned Neutrosophic set if, $T_D(a) = 1, C_D(a) = 1, G_D(a) = 0, U_D(a) = 0, F_D(a) = 0$. We denote the Absolute Pentapartitioned Neutrosophic set by 1_{PN}. That is $1_{PN} = \{(a, 1, 1, 0, 0, 0)/a \in Z\}$.

Definition 2.6 [2] Consider Z be the universal set. Then the collection $\tau_{PN}$of PentapartitionedNeutrosophic sets over $Z$is known as a Pentapartitioned Neutrosophic Topology on Z, if the following conditions are satisfied.

1. $0_{PN}, 1_{PN} \in \tau_{PN}$
2. $P, Q \in \tau_{PN}$then$P \cap Q \in \tau_{PN}$
3. $\{P_i/i \in I\} \in \tau_{PN}$then $\cup_{i \in I} P_i \in \tau_{PN}$

The pair (Z, τ_{PN}) is named as the Pentapartitioned Neutrosophic Topological Space. Each element of $\tau_{PN}$is called a Pentapartitioned Neutrosophic *Open*Sets.If $L \in \tau_{PN}$, then $L^C$is called as Pentapartitioned Neutrosophic (for simply PN)*Closed*Set.

Definition 2.7 [7] Consider(Z, τ_{PN}) a Pentapartitioned Neutrosophic Topological space. Let W be a Pentapartitioned Neutrosophic set over Z. Then PN-interior of W is the Union of PN-*Open* sets of (Z, τ_{PN}) contained in W and is denoted by PN-int(W). That is PN-int(W) = $\cup \{U/U \subseteq W \text{ and } U \text{ is a } PN-Open \text{ set in } (Z, \tau_{PN})\}$.

Definition 2.8 [2] Consider(Z, τ_{PN}) a Pentapartitioned Neutrosophic Topological space. Let W be Pentapartitioned Neutrosophic set over Z. Then PN-closure of W is the Intersection of PN-*closed* sets of (Z, τ_{PN}) containing W and is denoted by PN-cl(W). That is PN-cl(W) = $\cap \{U/W \subseteq U \text{ and } U \text{ is a } PN-Closed \text{ set in } (Z, \tau_{PN})\}$.

Remark 2.9 If W is Pentapartitioned Neutrosophic *Open* set in a PN Topological space(Z, τ_{PN}), then PN-int(W)=W. If W is Pentapartitioned Neutrosophic *Closed* set in a PN Topological space (Z, τ_{PN}), then PN-cl(W)=W.

Definition 2.10 [8] Consider(Z, τ_{PN}) a Pentapartitioned Neutrosophic Topological Space. A Pentapartitioned Neutrosophic set R over Z is known as,

(i) PN regular *open* if R=PN-int [PN-cl(R)]

(ii) PN regular *closed* if R=PN-cl [PN-int (R)].

Definition 2.11 [2] Consider$(Z,\ \tau_{PN})$ a Pentapartitioned Neutrosophic Topological space. Then a Pentapartitioned Neutrosophic set Rover $Z$is known as,

(i) PN-Semi *Open* set if and only if R ⊆PN-cl [PN-int (R)].

(ii) PN-α *Open* set if and only if R ⊆ PN-int (PN-cl (PN-int (R))).

Definition 2.12 Consider$(Z,\ \tau_{PN})$ a Pentapartitioned Neutrosophic Topological space. Then a Pentapartitioned Neutrosophic Set Rover $Z$is known as,

(i) PN-*Semi*Closed set if and only if PN-int [PN-cl (R)]⊆ R.

(ii) PN-α *Closed* set if and only if PN-cl (PN-int (PN-cl (R))) ⊆ R.

3 Results and Discussion

3.1 PN Pre Interior and PN Pre Closure of a PN set

Definition 3.1. 1 A PN set M is taken from PN Topological space $(Z,\ \tau_{PN})$. Then M is called PN Pre *Closed*Set in $(Z,\ \tau_{PN})$ if and only if PN-cl [PN-int (M)]⊆ M.

Definition 3.1.2 [2]A PN set M is taken from PN Topological space $(Z,\ \tau_{PN})$. Then M is called PN Pre *Open* set in $(Z,\ \tau_{PN})$ if and only if M ⊆PN-int [PN-cl (M)].

Definition 3.1. 3 For any PN set M in a PN Topological Space $(Z,\ \tau_{PN})$, PN Pre-interior of M is defined as the Union of all PN Pre *Open* sets contained in M. That is,

PN Pre – int $M = \cup \{D \subseteq (Z,\ \tau_{PN}) / \ D \subseteq M \ and\ D\ is\ a\ PN\ Pre\ Open\ set\}$

Definition 3.1. 4 For any PN set A in a PN Topological Space $(Z,\ \tau_{PN})$, PN Pre-closure of M is defined as the Intersection of all PN Pre *Closed* sets containing M. That is,

PN Pre – cl $M = \cap \{D \subseteq (Z,\ \tau_{PN}) / \ M \subseteq D\ and\ D\ is\ a\ PN\ Pre\ Closed\ set\}$

Proposition 3.1.5 Consider$M$ a PN set over a PN Topological space $(Z,\ \tau_{PN})$. Then

1. [PN Pre – int $M]^{C}$ = PN Pre – cl M^{C}

2. [PN Pre – cl $M]^{C}$ = PN Pre – int M^{C}

Proof:

1. By considering$M$as a PN set over a PN Topological space $(Z,\ \tau_{PN})$,we have PN Pre – int M $=\cup \{G/\ G\ is\ a\ PN\ Pre\ Open\ set\ and\ G \subseteq M\}$.

Then [PN Pre – int $M]^{C} = [\cup \{G/\ G\ is\ a\ PN\ Pre\ Open\ set\ and\ G \subseteq M\}]^{C} = \cap \{G^{C}/G^{C} is\ a\ PN\ Pre\ Closed\ set\ and\ G^{C} \supseteq M^{C}\}$. Taking G^{C}as S, we get, [PN Pre – int $M]^{C} = \cap \{S/\ S\ is\ a\ PN\ Pre\ Closed\ set\ and\ S \supseteq M^{C}\}$. Thus, [PN Pre – int $M]^{C}$=PN Pre – cl M^{C}.

2. By considering$M$as a PN set over a PN Topological space $(Z,\ \tau_{PN})$. Now, PN Pre – cl $M = \cap \{H/\ H\ is\ a\ PN\ Pre\ Closed\ set\ and\ H \supseteq M\}$. Then [PN Pre – cl $M]^{C} = [\cap \{H/\ H\ is\ a\ PN\ Pre\ Closed\ set\ and\ H \supseteq M\}]^{C} = \cup \{H^{C}/H^{C} is\ a\ PN\ Pre\ Open\ set\ and\ H^{C} \subseteq M^{C}\}$. Taking H^{C}as S, we get [PN Pre – cl $M]^{C} = \cup \{S/\ S\ is\ a\ PN\ Pre\ Open\ set\ and\ S \subseteq M^{C}\}$. Thus, [PN Pre – cl $M]^{C}$=PN Pre – int M^{C}.

Theorem 3.1 .6 Consider $(Z,\ \tau_{PN})$ a PN Topological space. Then for any two PN sets L and Q over a PN Topological space $(Z,\ \tau_{PN})$ we have, PN Pre – int ($L \cap Q$) = (PN Pre – int L) ∩ (PN Pre – int Q).

Proof:

Let $(Z,\ \tau_{PN})$ be a PN Topological space. Let L and Q be any two PN sets over a PN Topological space $(Z,\ \tau_{PN})$. By the property of PN sets, $L \cap Q \subseteq L$ and $L \cap Q \subseteq Q$. Then we have PN Pre – int ($L \cap Q$) ⊆(PN Pre – int L) and PN Pre – int ($L \cap Q$) ⊆ (PN Pre – int Q). Therefore PN Pre – int ($L \cap Q$) ⊆ (PN Pre – int L) ∩ (PN Pre – int Q) —–(1)

Now, (PN Pre – int L) ⊆ L and (PN Pre – int Q) ⊆ Q. Then (PN Pre – int L) ∩ (PN Pre – int Q)⊆ $L \cap Q$. By we have PN Pre – int [(PN Pre – int L) ∩ (PN Pre – int Q)] ⊆ PN Pre – int ($L \cap Q$). Thus (PN Pre – int L) ∩ (PN Pre – int Q) ⊆ PN Pre – int ($L \cap Q$) —–(2)

From (1) and (2) we get,

PN Pre – int ($L \cap Q$) = (PN Pre – int L) ∩ (PN Pre – int Q)

Theorem 3.1 .7 Consider$(Z,\ \tau_{PN})$ a PN Topological space. Then for any two PN sets L and Q over a PN Topological space $(Z,\ \tau_{PN})$ we have, PN Pre – int ($L \cup Q$) ⊇ (PN Pre– int L) ∪ (PN Pre – int Q).

Proof:

Consider$(Z,\ \tau_{PN})$as a PN Topological space. Let L and Q be any two PN sets over a PN Topological space $(Z,\ \tau_{PN})$. By the property of PN sets we have $L \subseteq L \cup Q$ and $Q \subseteq L \cup Q$. Then we have (PN Pre – int L) ⊆ PN Pre – int ($L \cup Q$) and (PN Pre – int Q) ⊆ PN Pre – int ($L \cup Q$). Therefore (PN Pre – int L) ∪ (PN Pre – int Q) ⊆PN Pre – int ($L \cup Q$).

Theorem 3.1 .8 Consider$(Z,\ \tau_{PN})$ a PN Topological space. Then for any two PN sets L and Q over a PN Topological space $(Z,\ \tau_{PN})$ we have, PN Pre – cl ($L \cup Q$) = (PN Pre – cl L) ∪ (PN Pre – cl Q).

Proof:

Consider$(Z,\ \tau_{PN})$ a PN Topological space. Let L and R be any two PN sets over a PN Topological space $(Z,\ \tau_{PN})$. We have PN Pre – cl ($L \cup R$) = PN Precl ($[(L \cup R)^{C}]^{C}$). Then PN Pre – cl ($L \cup R$) = [PN Pre – int $(L \cup R)^{C}]^{C}$. Also we have PN Pre – cl ($L \cup R$) = [PN Pre – int ($L^{C} \cap R^{C}$)]^{C}. This implies PN Pre – cl ($L \cup R$) = [PN Pre – int (L^{C}) ∩ PN Preint (R^{C})]^{C}. Again we have PN Pre – cl ($L \cup R$) = [PN Pre – int(L^{C})]^{C} ∪ [PN Pre – int (R^{C})]^{C}. Thus we have PN Pre – cl ($L \cup R$) = [PN Pre – cl (L)] ∪ [PN Pre – int (R)].

Theorem 3.1.9 Consider$(Z,\ \tau_{PN})$ a PN Topological space. Then for any two PN sets P and Q over a PN Topological space $(Z,\ \tau_{PN})$ we have, PN Pre – cl ($P \cap Q$) ⊆ (PN Pre – cl P) ∩ (PN Pre – cl Q).

Proof:

Consider$(Z,\ \tau_{PN})$ be a PN Topological space. Let P and Q be any two PN sets over a PN Topological space $(Z,\ \tau_{PN})$. By the property of PN sets,$P \cap Q \subseteq P$ and $P \cap Q \subseteq Q$. Then we have PN Pre – cl ($P \cap Q$)⊆ (PN Pre – cl P) and PN Pre – cl ($P \cap Q$) ⊆ (PN Pre – cl Q). This implies PN Pre – cl ($P \cap Q$) ⊆ (PN Pre – cl P) ∩ (PN Pre – cl Q).

Definition 3.1.10 A PN set M is taken from PN Topological Space $(Z,\ \tau_{PN})$. Then PN exterior of M in (Z, τ_{PN}) is defined as PN exterior (M) = PN -int(M^{C})

Definition 3.1.11 A PN set M is taken from PN Topological Space $(Z,\ \tau_{PN})$. Then PN

Pre exterior of M in $(Z,\ \tau_{PN})$ is defined as PN Pre exterior (M) = PN Pre -int(M^{C})

Definition 3.1.12 A PN set M is taken from PN Topological Space $(Z,\ \tau_{PN})$. Then PN frontier of M in $(Z,\ \tau_{PN})$ is defined as PN frontier (M) = PN -cl(M)∩ PN -cl(M^{C})

Definition 3.1.13 A PN set M is taken from PN Topological Space $(Z,\ \tau_{PN})$. Then PN Pre frontier of M in $(Z,\ \tau_{PN})$ is defined as PN Pre frontier (M) = PN Pre -cl(M)∩ PNPre -cl(M^{C})

3.2 PN Generalized Pre Closed Set

Definition 3.2 .1 A PN set M is taken from PN Topological space $(Z,\ \tau_{PN})$. Then M is said to be a PN Generalized Pre *Closed* set in $(Z,\ \tau_{PN})$ if PN Pre-cl(M)$\subseteq K$whenever $M \subseteq K$and $K$is a PN *open* set.

Theorem 3.2 .2 Every PN Pre *Closed* set in a PN Topological space $(Z,\ \tau_{PN})$is PNGeneralized Pre *Closed* set.

Proof:

Let J be a PN Pre *Closed* set. Then PN Pre-cl(J)=J —–(i). And if $J \subset K$ by (i), we get PN Pre-cl(J)$\subset K$. From definition 3.2.1, we conclude J is a PN Generalized Pre *Closed* set. Converse part of Theorem 3.2.2 is need not be True.

Theorem 3.2 .3 Every PN *Closed* set in a PN Topological space $(Z,\ \tau_{PN})$is PN GeneralizedPre *Closed* set.

Proof:

Consider J be a PN *Closed* set. Then PN -cl(J)=J —–(i).We know, always PN -int(J)$\subseteq J$. This implies PN -cl(PN -int(J))⊆PN -cl(J) by (i) we get, PN -cl(PN -int(J))⊆J. Therefore, J is a PN Pre *Closed* set. Also from Theorem 3.2.2 we get, J is a PN Generalized Pre *Closed* set.

Converse part of Theorem 3.2.3 is need not be True.

Example 3.2 .4 Consider$X = \{p\}$ and $E,\ F$ be two Pentapartitioned Neutrosophic sets over Xsuch that, $E = \{(p,\ 0.5,\ 0.4,\ 0.9,\ 0.8,\ 0.7)\}$and $F = \{(p,\ 0.6,\ 0.7,\ 0.7,\ 0.3,\ 0.4)\}$. Then $\tau_{PN} = \{0,\ 1,\ E,\ F\}$ forms a Pentapartitioned Neutrosophic Topology. Also,$E^{C} = \{(p,\ 0.7,\ 0.8,\ 0.1,\ 0.4,\ 0.5)\}$$F^{C} = \{(p,\ 0.4,\ 0.3,\ 0.3,\ 0.7,\ 0.6)\}$. Since E and F are PN *open* sets, PN-int(E)=E, PN-int(E^{C})=E, PN-int(F)=F, PN-int(F^{C})=0, PN-cl(E) =E^{C}, PN-cl(E^{C}) = E^{C}, PN-cl(F) =1 and PN-cl(F^{C}) = F^{C}.

Now $T = \{(p,\ 0.5,\ 0.6,\ 0.3,\ 0.7,\ 0.6)\}$ is a Pentapartitioned Neutrosophic set over X. Since PN-int(T) =E, PN-cl(T) =$E^{C}$and PN-cl(PN-int(T))=PN-cl(E)=E^{C}. Then $E^{C} \not\subseteq T$. Thus T is not a PN Pre *Closed* set. Also $E^{C}$is the smallest PN Pre *Closed* set containing T, therefore PN Pre-cl T=E^{C}. Also, $T \subseteq 1$ then we get PN Pre-cl $T = E^{C} \subseteq 1$, where 1 is PN *Open* set. Therefore, T is a PN Generalized Pre *Closed* set.

Now $M = \{(p,\ 0.6,\ 0.5,\ 0.4,\ 0.4,\ 0.6)\}$ is a Pentapartitioned Neutrosophic set over X. Since PN-int(M) =E, PN-cl(M) =E^{C}then M is not a PN *Closed* set. And PN-cl(PNint(M))=PN-cl(E)=E^{C}. Then $E^{C} \not\subseteq M$. Thus M is not a PN Pre *Closed* set. Also $E^{C}$is the smallest PN Pre *Closed* set containing M, therefore PN Pre-cl M=E^{C}. Also, $M \subseteq 1$ then we get PN Pre-cl $M = E^{C} \subseteq 1$, where 1 is PN *Open* set. Therefore, M is a PN Generalized Pre *Closed* set.

Definition 3.2.5 A PN set $M$is taken from PN Topological space $(Z,\ \tau_{PN})$. Then M is called PN Generalized Pre *Open* set in $(Z,\ \tau_{PN})$ if K ⊆PN Pre-int(M) whenever $K \subseteq M$and $K$is a PN *closed* set.

Example 3.2.6 Consider $X = \{p\}$ and D,E be two PentapartitionedNeutrosophic sets over Xsuch that, $D = \{(p,\ 0.4,\ 0.3,\ 0.6,\ 0.7,\ 0.5)\}$and $E = \{(p,\ 0.5,\ 0.5,\ 0.5,\ 0.4,\ 0.2)\}$. Then $\tau_{PN} = \{0,\ 1,\ D,\ E\}$ forms a Pentapartitioned

NeutrosophicTopology. Also,$D^C = \{(p,\ 0.5,\ 0.7,\ 0.4,\ 0.3,\ 0.4)\}E^C = \{(p,\ 0.2,\ 0.4,\ 0.5,\ 0.5,\ 0.5)\}$. Since$D$ and E are PN *open* sets, PN-int(D)=D, PN-int(D^C)=D, PN-int(E)=E, PN-int(E^C)=0, PN-cl(D) =D^C, PN-cl(D^C) = D^C, PN-cl(E) =1 and PN-cl(E^C) = E^C.

Now $M = \{(p,\ 0.7,\ 0.5,\ 0.6,\ 0.1,\ 0.3)\}$ is a Pentapartitioned Neutrosophic set over X. Since PN-int(M) =D, PN-cl(M) =1 and PN-int(PN-cl(M))=PN-int(1)=1. Then $M \subseteq 1$.Thus M is a PN Pre $Open$ set. Also M is the largest PN Pre $Open$ set contained in M, therefore PN Pre-int M=M. Hence, if $M \subseteq U$ then we get PN Pre-int $M \subseteq U$, where U is PN $Closed$ set. Therefore, M is a PN Generalized Pre $Open$ set.

Remark 3.2.7 Complement of a PN Generalized Pre $Closed$ set is PN Generealized Pre $Open$ set and viceversa.

Theorem 3.2.8 Every PN Pre $Open$ set in a PN Topological space $(Z,\ \tau_{PN})$ is PNGeneralized Pre $Open$ set.

Proof:

Let J be a PN Pre $Open$ set. Then PN Pre-int(J)=J —–(i). And if $M \subset J$ by (i), we get M ⊂PN Pre-int(J). Thus we conclude J is a PN Generalized Pre $Open$ set. Converse part of Theorem 3.2.7 is need not be True.

Theorem 3.2.9 Every PN $Open$ set in a PN Topological space $(Z,\ \tau_{PN})$is PN GeneralizedPre $Open$ set.

Proof:

Let J be a PN $Open$ set. Then PN -int(J)=J —–(i).We know, always J ⊆PN -cl(J). This implies PN -int(J)⊆PN -int(PN -cl(J)) by (i) we get, J ⊆PN -int(PN -cl(J)). Therefore, J is a PN Pre $Open$ set. Also from Theorem 3.2.8 of this section we get, J is a PN Generalized Pre Open set. Converse part of Theorem 3.2.8 is need not be True.

Example 3.2.10 Consider$X = \{p\}$ and $E,\ F$ be two Pentapartitioned Neutrosophic sets over Xsuch that, $E = \{(p,\ 0.5,\ 0.4,\ 0.9,\ 0.8,\ 0.7)\}$and $F = \{(p,\ 0.6,\ 0.7,\ 0.7,\ 0.3,\ 0.4)\}$. Then $\tau_{PN} = \{0,\ 1,\ E,\ F\}$ forms a Pentapartitioned Neutrosophic Topology. Also,$E^C = \{(p,\ 0.7,\ 0.8,\ 0.1,\ 0.4,\ 0.5)\}F^C = \{(p,\ 0.4,\ 0.3,\ 0.3,\ 0.7,\ 0.6)\}$. Since E and F are PN *open* sets, PN-int(E)=E, PN-int(E^C)=E, PN-int(F)=F, PN-int(F^C)=0, PN-cl(E) =E^C, PN-cl(E^C) = E^C, PN-cl(F) =1 and PN-cl(F^C) = F^C.

Now $R = \{(p,\ 0.5,\ 0.5,\ 0.5,\ 0.5,\ 0.5)\}$ is a Pentapartitioned Neutrosophic set over X. Since PN-int(R) =E, PN-cl(R) =$E^C$and PN-int(PN-cl(R))=PN-int(E^C)=E. Then $R \nsubseteq E$. Thus R is not a PN Pre $Open$ set. Also E is the largest PN Pre $Open$ set containied in R, therefore PN Pre-int R=E. Also, $0 \subseteq R$ then we get $0 \subseteq E$=PN Pre-int R, where 0 is PN $Closed$ set. Therefore, R is a PN Generalized Pre $Open$ set.

Now $S = \{(p,\ 0.7,\ 0.7,\ 0.3,\ 0.5,\ 0.6)\}$ is a Pentapartitioned Neutrosophic set over X. Since PN-int(S) =E, PN-cl(S) =E^Cthen S is not a PN $Open$ set. And PN-int(PNcl(S))=PN-int(E^C)=E. Then $S \nsubseteq E$. Thus S is not a PN Pre $Open$ set. Also E is the largest PN Pre $Open$ set containied in S, therefore PN Pre-int S=E. Also, $0 \subseteq S$ then we get $0 \subseteq E$=PN Pre-int S, where 0 is PN $Closed$ set. Therefore, S is a PN Generalized Pre $Open$ set.

Definition 3.2.11 The PN Generalized Pre-interior of any PN set M in a PN Topological Space $(Z,\ \tau_{PN})$ is defined as the Union of all PN Generalized Pre $Open$ sets Contained in M and denoted by PN Generalized Pre – int M.

That is, PN Generalized Pre – int $M = \cup\{K \subseteq (Z,\ \tau_{PN})/\ K \subseteq M$ and K is a PN Generalized Pre $Open$ set}

Definition 3.2.12 The PN Generalized Pre-closure of any PN set M in a PN Topological Space $(Z,\ \tau_{PN})$ is defined as the Intersection of all PN Generalized Pre $Closed$ sets Containing$M$ and denoted by PN Generalized Pre – cl$M$.

That is, PN Generalized Pre – cl$M = \cup\{K \subseteq (Z,\ \tau_{PN})/\ M \subseteq K$ and K is a PN GeneralizedPre $Closed$ set}

Proposition 3.2.1 3 Let J and K be two PN sets in a PN Topological Space $(Z,\ \tau_{PN})$.Then we have the following results.

(i) $J \subseteq$ PN Generalized Pre -cl (J).

(ii) J is a PN Generalized Pre $Closed$ set in $(Z,\ \tau_{PN})$ if and only if PN Generalized Pre -cl (J) = J.

(iii) PN Generalized Pre $-$ cl(PN Generalized Pre $-$ cl(J)) = PN Generalized Pre $-$ cl(J)

(iv) If $J \subseteq K$ then PN Generalized Pre -cl (J) $\subseteq$ PN Generalized Pre -cl (K).

Proof:

(i) By Definition 3.2.11, PN Generalized Pre -cl (J) = ∩ {S/ S is a PN Generalized Pre$Closed$ set and $J \subseteq S$}. This implies $J \subseteq$ PN Generalized Pre -cl (J). Thus (i) holds.

(ii) Suppose PN Generalized Pre -cl (J) = J. Then ∪ {R/ R is a PN Generalized Pre $Closed$ set and $J \subseteq R$} = J. This implies that J is a PN Generalized Pre $Closed$ set in $(Z,\ \tau_{PN})$.

Conversely, suppose J is a PN Generalized Pre $Closed$ set in $(Z,\ \tau_{PN})$. Then ∪{R/ R is a PN Generalized Pre $Closed$ set and $J \subseteq R$} = J. Therefore, by Definition 3.2.11, PN Generalized Pre -cl (J) = J. Thus (ii) holds.

(iii) For any PN set J, PN Generalized Pre -cl (J) is a PN Generalized Pre $Closed$ set in $(Z,\ \tau_{PN})$. Using (ii) we get, PN Generalized Pre -cl (PN Generalized Pre -cl (J)) = PNGeneralized Pre -cl (J). Thus (iii) holds.

(iv) Let $J \subseteq K$. Then $J \subseteq K \subseteq$ PN Generalized Pre -cl (K). From this we get $J \subseteq$ PN Generalized Pre -cl (K). This implies PN Generalized Pre -cl (J) ⊆PN Generalized Pre -cl (PN Generalized Pre -cl (K)). Therefore by (iii) of this proposition PN Generalized Pre -cl (PN Generalized Pre -cl (K)) = PN Generalized Pre -cl (K). Hence we get PN Generalized Pre -cl (J)⊆ PN

Generalized Pre -cl (K). Thus (iv) holds.

Proposition 3.2.1 4 Consider J and K the two PN sets in a PN Topological Space (Z, τ_{PN}). Then we have the following results.

(i) PN Generalized Pre -int (J) $\subseteq J$.

(ii) J is a PN Generalized Pre *Open* set in (Z, τ_{PN}) if and only if PN Generalized Pre -int (J) = J.

(iii)PN Generalized Pre -int (PN Generalized Pre -int (J)) = PN Generalized Pre -int (J)

(iv) If $J \subseteq K$ then PN Generalized Pre -int (J) $\subseteq$ PN Generalized Pre -int (K)

Proof:

(i) By Definition 3.2.12, PN Generalized Pre -int (J) = $\cap\{K/\ K$ is a PN Generalized Pre*Open* set and $K \subseteq J\}$. This implies $J \subseteq$ PN Generalized Pre -int (J). Thus (i) holds.

(ii) Suppose PN Generalized Pre -int (J) = J. Then $\cup\ \{K/\ K$ is a PN Generalized Pre *Open* set and $K \subseteq J\}= J$. This implies that J is a PN Generalized Pre *Open* set in (Z, τ_{PN}).

Conversely, suppose J is a PN Generalized Pre *Open* set in (Z, τ_{PN}). Then $\cup\ \{K/\ K$ is a PN Generalized Pre *Open* set and $K \subseteq J\} = J$. Therefore, by Definition 3.2.12, PN Generalized Pre -int (J) = J. Thus (ii) holds.

(iii) For any PN set J, PN Generalized Pre -int (J) is a PN Generalized Pre *Open* setin (Z, τ_{PN}). Using (ii) we get, PN Generalized Pre -int (PN Generalized Pre -int (J)) = PN Generalized Pre -int (J). Thus (iii) holds.

(iv) Let $J \subseteq K$. Then PN Generalized Pre -int (J) $\subseteq J \subseteq K$. From this we get PN Generalized Pre -int (J)$\subseteq K$. This implies PN Generalized Pre -int (PN Generalized Pre -int (J)) $\subseteq$ PN Generalized Pre -int (K). Therefore by (iii) of this proposition PN Generalized Pre -int (PN Generalized Pre -int (J)) = PN Generalized Pre -int (J). Hence we get PN Generalized Pre -int (J)$\subseteq$ PN Generalized Pre -int (K). Thus (iv) holds.

Theorem 3.2.1 5 Let G be a PN Generalized Pre *Closed* set in (Z, τ_{PN}) and $G \subseteq H \subseteq$ PNPre -cl (G), then H is a PN Generalized Pre *Closed* set in (Z, τ_{PN}).

Proof:

Let G be a PN Generalized Pre *Closed* set in (Z, τ_{PN}). Then PN Pre -cl (G) $\subseteq R$ whenever $G \subseteq R$ where R is a PN *Open* set in (Z, τ_{PN}). Now by hypothesis, $G \subseteq H \subseteq$ PN Pre -cl(G). This implies $H \subseteq$ PN Pre -cl (G) $\subseteq R$. Also PN Pre -cl (H) $\subseteq$ PN Pre -cl (PN Pre -cl (G)) = PN Pre -cl (G). Thus we get, PN Pre -cl (H)$\subseteq$PN Pre -cl (G)$\subseteq R$. Hence PN Pre -cl (H)$\subseteq R$ whenever $H \subseteq R$. Therefore, H is a PN Generalized Pre *Closed* set in (Z, τ_{PN}).

Theorem 3.2.16 Consider J a PN Generalized Pre *Open* set in (Z, τ_{PN}) and PN Pre -int(J)$\subseteq L \subseteq J$, then L is a PN Generalized Pre *Open* set in (Z, τ_{PN}).

Proof:

Consider J a PN Generalized Pre *Open* set in (Z, τ_{PN}). Then $H \subseteq$ PN Pre -int (J) whenever $H \subseteq J$ where H is a PN *Closed* set in (Z, τ_{PN}). Now by hypothesis, PN Pre -int (J)$\subseteq L \subseteq J$. This implies $H \subseteq$PN Pre -int (J) $\subseteq L$ Also PN Pre -int (L) $\supseteq$ PN Pre -int (PN Pre-int (J))=PN Pre -int (J). Thus we get PN Pre -int (L) $\supseteq$PN Pre -int (J)$\supseteq H$. HencePN Pre -int (L)$\supseteq H$ whenever $L \supseteq H$. Therefore, L is a PN Generalized Pre *Open* set in(Z, τ_{PN}).

Proposition 3.2.1 7 Consider J a PN set over a PN Topological space (Z, τ_{PN}). Then

(i) [PN Generalized Pre – int $J]^{\mathrm{C}}$ = PN Generalized Pre – clJ^C

(ii) [PN Generalized Pre – cl $J]^{\mathrm{C}}$ = PN Generalized Pre – int J^C

Proof:

(i) Consider J a PN set over a PN Topological space (Z, τ_{PN}). Now, PN Generalized Pre – int $J = \cup\{H/\ H$ is a PN Generalized Pre *Open* set and $H \subseteq J\}$. Then [PN GeneralizedPre – int $J]^{\mathrm{C}} = [\cup\ \{H/\ H$ is a PN Generalized Pre *Open* set and $H \subseteq J\}]^{\mathrm{C}}$= $\cap\ \{H^C/H^C$is a PN Generalized Pre *Closed* set and $H^C \supseteq J^C\}$. Taking H^Cas R, we get, [PN Generalized Pre – int $J]^{\mathrm{C}} = \cap\{R/\ R$ is a PN Generalized Pre *Closed* set and $R \supseteq J^C\ \}$. Thus, [PN Generalized Pre – int $J]^{\mathrm{C}}$=PN Generalized Pre – cl J^C.

(ii) Consider$J$ a PN set over a PN Topological space (Z, τ_{PN}). Now, PN Generalized Pre – cl $J = \cap\{G/\ G$ is a PN Generalized Pre *Closed* set and $G \supseteq J\}$. Then [PN Generalized Pre – cl $J]^{\mathrm{C}} = [\cap\ \{G/\ G$ is a PN Generalized Pre *Closed* set and $G \supseteq J\}]^{\mathrm{C}}$ = $\cup\ \{G^C/\ G^C$is a PN Generalized Pre *Open* set and $G^C \subseteq J^C\}$. Taking G^Cas R, we get, [PNGeneralized Pre – cl $J]^{\mathrm{C}} = \cup\{R/\ R$ is a PN Generalized Pre *Open* set and $R \subseteq J^C\}$. Thus, [PN Generalized Pre – cl $J]^{\mathrm{C}}$=PN Generalized Pre – int J^C.

Theorem 3.2.1 8 In a PN Topological Space (Z, τ_{PN}), Intersection of two PN GeneralizedPre *Closed* Set is again a PN Generalized Pre *Closed* set.

Proof:

Consider$S$ and R be two PN Generalized Pre *Closed* Sets. Then PN Pre -cl (S) $\subseteq M$ whenever $S \subseteq M$ where M is a PN *Open* set in (Z, τ_{PN}) andPN Pre -cl (R) $\subseteq M$ whenever $R \subseteq M$ where M is a PN Open set in (Z, τ_{PN}).PN Pre – cl ($S \cap R$)$\subseteq$ (PN Pre – cl S) $\cap$ (PN Pre – cl R). Since PN Pre -cl (S)$\subseteq M$ and PN Pre -cl (R) $\subseteq M$ we get, PN Pre – cl ($S \cap R$) $\subseteq$PNPre $-$ cl(S) $\cap$PNPre $-$ cl(R)$\subseteq M$ whenever $S \cap R \subseteq M$ where M is a PN Open setin (Z, τ_{PN}).Thus $S \cap R$is a PN Generalized Pre *Closed* set.

Theorem 3.2.19 In a PN Topological Space $(Z,\ \tau_{PN})$, Union of two PN Generalized Pre$Open$ Set is again a PN Generalized Pre $Open$ set.

Proof:

Consider$G$ and H be two PN Generalized Pre $Open$ Sets.Then $M\ \subseteq$ PN Pre -int (G) whenever $M\ \subseteq\ G$ where M is a PN Closed set in $(Z,\ \tau_{PN})$ and$M\ \subseteq$ PN Pre -int (H) whenever $M\ \subseteq\ H$ where M is a PN Closed set in $(Z,\ \tau_{PN})$.PN Pre – int $(G \cup H)$ $\supseteq$ (PN Pre – int G) $\cup$ (PN Pre – int H). Since $M\ \subseteq$ PN Pre -int (G) and $M\ \subseteq$ PN Pre -int (H) we get, $M\ \subseteq$(PN Pre – int G) $\cup$ (PN Pre – int H) $\subseteq$ PN Pre -int $(G\ \cup\ H)$ whenever $M\ \subseteq\ G\ \cup\ H$where M is a PN Closedset in $(Z,\ \tau_{PN})$. Thus $G\ \cup\ H$is a PN Generalized Pre $Open$ set.

Definition 3.2.20 A PN set M is taken from PN Topological Space $(Z,\ \tau_{PN})$. Then PN Generalized Pre frontier of M in $(Z,\ \tau_{PN})$ is defined as PN Generalized Pre frontier (M) = PN Generalized Pre -cl$(M)\cap$ PN Generalized Pre -cl(M^C)

Definition 3.2.21 A PN set M is taken from PN Topological Space $(Z,\ \tau_{PN})$. Then PN Generalized Pre exterior of M in $(Z,\ \tau_{PN})$ is defined as PN Generalized Pre exterior (M)= PN Generalized Pre -int (M^C).

4 Conclusion

The Pentapartitioned Neutrosophic Generalized Pre Open and Closed sets are defined in this study. In order to comprehend the concept of Pentapartitioned Neutrosophic Generalized Pre Open and Closed sets, numerous theorems and propositions were also investigated. Additionally, the exterior and frontier of a Pentapartitioned Neutrosophic set were the subject of this study. This will lead to further the investigation of generalized set continuity and connectedness in Pentapartitioned Neutrosophic Topological spaces.

Declaration

Presented in 9th INTERNATIONAL CONFERENCE ON DISCRETE MATHEMATICS AND MATHEMATICAL MODELLING IN DIGITAL ERA (ICDMMMDE-2023) during March 23-25, 2023, Organized by the Department of Mathematics, The Gandhigram Rural Institute (Deemed to be University), Gandhigram - 624302, Dindigul, Tamil Nadu, India. ICDMMMDE-23 was supported by: GRI-DTBU, CSIR.

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