

## RESEARCH ARTICLE



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\* **Corresponding author.**

[aisupriyadharshini@gmail.com](mailto:aisupriyadharshini@gmail.com)

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## Inverse Detour Eccentric Domination in Graphs

A Mohamed Ismayil<sup>1</sup>, R Priyadharshini<sup>2\*</sup>

**1** Associate Professor, PG & Research Department of Mathematics, Jamal Mohamed College (Autonomous) (Affiliated to Bharathidasan University), Tiruchirappalli, 620 020, Tamil Nadu, India

**2** Research Scholar, PG & Research Department of Mathematics, Jamal Mohamed College (Autonomous) (Affiliated to Bharathidasan University), Tiruchirappalli, 620 020, Tamil Nadu, India

## Abstract

**Objective:** To determine the inverse detour eccentric domination number, inverse independent detour eccentric domination number and inverse total detour eccentric domination number for well-known graphs. **Methods:** Method of proving by existential statement and proving by different cases are used to prove the theorem and by determining the proposed numbers using the least cardinality. **Findings:** Inverse detour domination number and other numbers are determined and the relation between the proposed number and other existing numbers was found. **Novelty:** The detour distance is used to find the Inverse detour eccentric domination number and other numbers.

**Keywords:** Eccentric Dominating Set; Detour Eccentric Dominating Set; Detour Eccentric Domination Number; Inverse Detour Eccentric Dominating Set; Inverse Detour Eccentric Domination Number.

## 1 Introduction

In this study, the inverse detour eccentric dominating set and its numbers are obtained for various well-known graphs. Ore initiated the study of dominating sets in graphs. V.R. Kulli discussed the domination number and the inverse domination number. M. Bhanumathi and R. Meenal Abirami<sup>(1)</sup> developed the idea of upper eccentric domination in graphs. The detour eccentric domination number of several graphs was initiated by A. Nellai Murugan et.al.

**Research gap:** The detour eccentric domination in graphs was first introduced by A. Mohamed Ismayil and R. Priyadharshini in 2019<sup>(2)</sup>. Accurate eccentric domination in graphs and equal eccentric domination in graphs were investigated by A. Mohamed Ismayil and A. Riyaz Ur Rehman in 2020<sup>(3,4)</sup>. Inverse eccentric domination in graphs was studied by R. Jahir Hussain and A. Fathima Begam in 2021<sup>(5)</sup>. G. Upender Reddy et.al<sup>(6)</sup> described the bipolar single-valued neutrosophic detour distance. Detour D-eccentric domination in graphs was established by A. Prasanna and N. Mohamedazarudeen in 2022<sup>(7)</sup>. Serigio R. Canoy, Jr and Anabel E. Gamorez<sup>(8)</sup> defined monophonic eccentric domination number of graphs. In 2022, P. Titus et.al<sup>(9)</sup> established the connected monophonic eccentric domination number of a graph. In 2023, G. Priscilla Pacifica and J. Ajitha Fancy<sup>(10)</sup> elaborated on the connected restrained

detour number of a graph. Domination plays a vital role in network theory, but detour eccentric dominating set is used to determine the military networking and the proposed concepts are used to manage the military in an ordinary network. Here, let  $G = (V, E)$  be a connected simple graph and  $|V| > 1$ .

## 2 Methodology

**Definition 2.1.** Let  $G = (V, E)$  be represents any graph with  $V$  denoting the vertices set and  $E$  denoting the edges set. If  $u, v \in V$  in a connected graph  $G$ , the distance  $D(u, v)$  is defined by the length of the longest  $u - v$  path in  $G$  and is said to be the detour distance. The detour eccentricity of each vertex  $u$  of  $G$  is defined by  $e_D(u) = \max\{D(u, v) : v \in V\}$ . When  $D(u, v) = e_D(u)$ , a vertex  $v$  of  $G$  is said to be the detour eccentric vertex of  $u$ . The set  $D \subseteq V$  in a graph  $G$  is a  $D$ -set of  $G$  and for every,  $v \in V - D$  there exists at least one eccentric point  $u$  of  $v$  in  $D$ , then the set is called an Eccentric Dominating set (ED-set). For every  $v$  in the dominating set  $D \subseteq V$  of a graph  $G$ , if there exists at least one detour eccentric vertex  $u \in V - D$  the set is known as a Detour ED-set (DED-set).

**Definition 2.2.** Let  $D \subseteq V$  represent the minimum DED-set in graph  $G$ . A DED-set  $D'$  of  $G$  is called an Inverse DED-set (IDED-set) about  $D$  if  $V - D$  contains  $D'$ . If a subset  $D'' \subset D'$  is not an IDEDED-set, then the IDEDED-set  $D'$  is called a minimal IDEDED-set. The IDEDED number, indicated by  $\gamma_{Ded}^{-1}(G)$  is the least cardinality of a minimal IDEDED-set  $D'$  ( $G$ ) and the corresponding set is called minimum IDEDED-set. The upper IDEDED number, indicated by  $\Gamma_{Ded}^{-1}(G)$  is the greatest cardinality of a minimal IDEDED-set of  $D'$  ( $G$ ).

**Example 2.3.** In Figure 1, the minimum DED-set is  $D_1 = \{a_1, a_7\}$  and some of the IDEDED-sets are  $D_2 = \{a_3, a_5\}$ ,  $D_3 = \{a_2, a_4, a_6\}$ .  $D_1$  is a minimum DED-set but  $D_2$  is a minimum IDEDED-set concerning  $D_1$ .  $D_3$  is a minimal IDEDED-set. Therefore, the IDEDED number  $\gamma_{Ded}^{-1}(G)$  is 2 and  $\Gamma_{Ded}^{-1}(G)$  is 3.

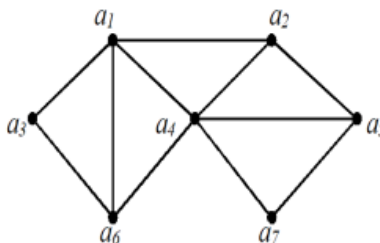


Fig 1. Graph 1

**Definition 2.4.** Let  $D$  be a minimum IDEDED-set of  $G$ . An IDEDED-set  $D'$  of  $G$  is called an Inverse Independent Detour Eccentric Dominating set (IIDED-set) concerning  $D$  if  $V - D$  contains such a set. The least cardinality of a minimal IIDED-set  $D'$  is called the IIDED number and it is denoted by  $i_{Ded}^{-1}(G)$ . The greatest cardinality of a minimal IIDED-set of  $D'$  is called the upper IIDED number and it is denoted by  $I_{Ded}^{-1}(G)$ .

**Example 2.5.** Consider the graph  $G$  as shown in the Figure 2. The minimum independent detour eccentric dominating set of  $G$  is  $\{x_3, x_4\}$  and some of the IIDED-sets are  $\{x_1, x_8\}$ ,  $\{x_2, x_7, x_6\}$ .

Therefore,  $i_{Ded}^{-1}(G) = 2$  and  $I_{Ded}^{-1}(G) = 3$ , are the minimum IIDED number and upper IIDED number respectively.

**Definition 2.6.** Let  $D$  be a minimum total DED-set of  $G$ . Total detour eccentric dominating set  $D'$  of  $G$  is known as an inverse total detour eccentric dominating set (ITDED-set) concerning  $D$  if  $V - D$  contains it. The minimum number of vertices in an Inverse Total Detour Eccentric Dominating set of  $G$  is known as the ITDED number or  $t_{Ded}^{-1}(G)$  of  $G$ . The maximum cardinality of an ITDED of  $G$  is represented by the upper ITDED number  $T_{Ded}^{-1}(G)$ .

**Example 2.7.** Consider the graph  $G$  given in Figure 3. The minimum total detour eccentric dominating set of  $G$  is  $\{a_1, a_7\}$  and some of the ITDED-sets are  $\{a_5, a_3\}$ ,  $\{a_2, a_4, a_6\}$ .

Hence,  $t_{Ded}^{-1}(G)$  is 2 and  $T_{Ded}^{-1}(G)$  is 3, indicating an ITDED number.

## 3 Results and Discussion

### 3.1 Inverse detour eccentric domination number of a graph

#### Observations 3.1.1

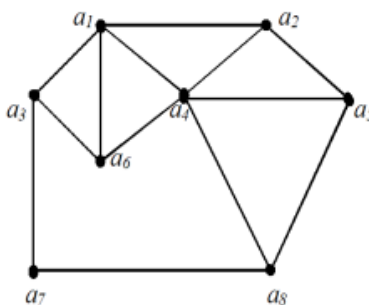


Fig 2. Graph 2

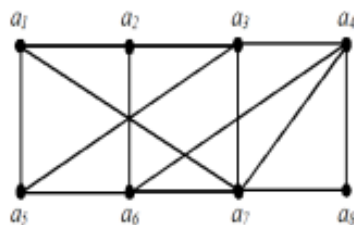


Fig 3. Graph 3

Let  $K_n$ ,  $K_{m,n}$  and  $C_n$  are complete graph, complete bipartite graph, and cycle graph respectively. Then,

- i.  $\gamma_{Ded}^{-1}(K_n) = 1 = \gamma_{Ded}(K_n)$
- ii.  $\gamma_{Ded}^{-1}(K_{m,n}) = 2 = \gamma_{Ded}(K_{m,n})$
- iii.  $\gamma_{Ded}^{-1}(C_n) = \lfloor \frac{n}{3} \rfloor = \gamma_{Ded}(C_n)$

**Note 3.1.2**

1. IDED-set does not exist for the graph having pendent (except  $P_2$ ) and isolated vertices.
2. Every graph, without pendants and isolated vertices, contains an IDED-set since the complement of any minimal IDED-set is also an IDED-set.
3. The graph is disconnected,  $\gamma(G) = \gamma_{ed}(G) = \gamma_{Ded}(G) = \gamma_{Ded}^{-1}(G)$ .

Hereafter, the notation  $G$  denotes only a connected graph without pendent (except  $P_2$ ) and without isolate vertices.

**Theorem 3.1.3.** If a graph  $G$  has a minimum IDED-set  $D$ , then every vertex in  $D$  must have at least one detour eccentric vertex in  $V - D$  for there to be an IDED number of  $G$  with respect to  $D$ .

**Proof:** If a graph  $G$  has an IDED number with respect to  $D$ , then let  $D$  be the minimum DED-set of that graph. Allow  $D' \subseteq V - D$ . Then  $D'$  has the minimum DED-set. As a result, each vertex in  $D$  has at least one detour eccentric vertex in  $D'$ .

On the other hand, let  $D$  be a minimum DED-set of  $G$  where each vertex has at least one detour eccentric vertex in  $V - D$ . We know that the set  $D$  of  $G$  is being dominated by  $V - D$ . Therefore,  $V - D$  has a DED-set of  $G$  with respect to  $D$ . This implies that set  $D'$  in  $V - D$  contains the minimum DED-set. As a result,  $D'$  is the IDED-set with the minimum cardinality with respect to  $D$ .

**Theorem 3.1.4.** Let  $W_n$  be a wheel graph with  $n$  vertices, then

$$\gamma_{Ded}^{-1}(W_n) = \begin{cases} 1, & \text{if } n = 4 \\ \lfloor \frac{n+1}{3} \rfloor & \text{if } n \geq 5 \end{cases}$$

**Proof: Case 1.**  $n = 3k + 2$ ,  $k = 1, 2, 3, \dots$

Let  $\{v_1, v_2, v_3, \dots, v_n\}$  represent the vertices of  $W_n(G)$ .  $D' = \{v_1, v_4, v_7, \dots, v_{3k-1}, v_{3k+2}\}$  is the minimum IDED-set of  $W_n(G)$ . A DED-set of  $W_n(G)$  must contain the central vertex ( $v_2$ ) and another one vertex  $w_n$  in  $V(w_n)$ . Then  $D = \{v_2, w_n\}$  is the  $\gamma_{Ded}$ -set of  $W_n$ .

**Case 2.**  $n = 3k + 3$ ,  $k = 1, 2, 3, \dots$

$D' = \{v_1, v_4, v_7, \dots, v_{3k}, v_{3k+3}\}$  is the minimum IDEDED-set of  $W_n(G)$ . A DED-set of  $W_n(G)$  must contain the central vertex ( $v_2$ ) and another one vertex  $w_n$  in  $V(w_n)$ . Then  $D = \{v_2, w_n\}$  is the  $\gamma_{Ded}$  - set of  $W_n$ .

**Case 3.**  $n = 3k + 1$ ,  $k = 1, 2, 3, \dots$

$D' = \{v_1, v_4, v_7, \dots, v_{3k-2}, v_{3k+3}\}$  is the minimum IDEDED-set of  $W_n(G)$ . A DED-set of  $W_n(G)$  must contain the central vertex ( $v_2$ ) and another vertex  $w_n$  in  $V(w_n)$ . Then  $D = \{v_2, w_n\}$  is the  $\gamma_{Ded}$  - set of  $W_n$ .

Therefore, from the cases  $\gamma_{Ded}^{-1}(W_n) = \lfloor \frac{n+1}{3} \rfloor$ ,  $n \geq 5$ .

**Observations 3.1.5.** For any graph  $G$

i.  $\gamma_{Ded}(G) \leq \gamma_{Ded}^{-1}(G)$ , if IDEDED-set exists.

ii.  $\gamma_{Ded}(G) + \gamma_{Ded}^{-1}(G) \leq n$ .

**Theorem 3.1.6.** Let  $D$  be a minimum DED-set of  $G$ . If the induced subgraph  $\langle N[v] \rangle$  for each vertex  $v \in D$  is a connected graph with a detour eccentric point  $u$  in  $V - D$ , then  $\gamma_{Ded}(G) = \gamma_{Ded}^{-1}(G)$ .

**Proof:** Let  $D = \{v_1, v_2, \dots, v_n\}$  be a minimum DED-set of  $G$ . Given  $\langle N[v_i] \rangle$  is a complete graph of order 2 and  $u \in E_D(v_i)$ , Then

$$(N[V_1]) \cap (N[V_2]) = \emptyset$$

**Case 1.** Let for every  $u_1 \in N(v_1)$  and  $u_2 \in N(v_2)$

If  $u_1 = u_2$  then  $\langle N[v_1] \rangle$  and  $\langle N[v_2] \rangle$  is not complete.

Hence  $u_1 \neq u_2$

Hence  $(N[V_1]) \cap (N[V_2]) = \emptyset$

**Case 2.** Let  $u_1 \in E_D(v_1)$  and  $u_2 \in E_D(v_2)$

$\Rightarrow v_1 \in E_D(u_1)$  and  $v_2 \in E_D(u_2)$

$\Rightarrow \forall v \in V - D$  such that exactly one eccentric vertex  $u \subseteq D$ .

$G$  is a disconnected graph

Therefore,  $\gamma(G) = \gamma_{Ded}(G)$  and  $\gamma(G) = \gamma_{Ded}^{-1}(G)$

Hence  $\gamma_{Ded}(G) = \gamma_{Ded}^{-1}(G)$

**Theorem 3.1.7.** Let the family of minimum DED-sets of  $G$  be denoted by  $\tau$ .  $\gamma_{Ded}(G) + \gamma_{Ded}^{-1}(G) = n$  if each minimum IDEDED-set  $D \in \tau$ ,  $V - D$  is independent.

**Proof:**  $V - D$  is a minimum IDEDED-set of  $G$  if  $V - D$  is independent for any minimum DED-set  $D$ . Hence,  $\gamma_{Ded}(G) + \gamma_{Ded}^{-1}(G) = n$ .

**Theorem 3.1.8.** If an  $(n, m)$  graph  $G$ , then  $\frac{2n-m}{3} \leq \gamma_{Ded}^{-1}(G)$ .

**Proof:** Assuming that  $D$  is a minimum DED-set and that  $D' \subseteq V - D$  is the IDEDED-set of  $G$  such that  $|D| = \gamma_{Ded}(G)$  and  $|D'| = \gamma_{Ded}^{-1}(G)$ . If  $D$  is a DED-set of  $G$ , then  $D$  and  $D'$  are connected by at least  $\gamma_{Ded}^{-1}(G)$  edges.

Assume  $(V - D) - D' = \emptyset$ . As a result,  $\gamma_{Ded}(G) + \gamma_{Ded}^{-1}(G) = n$ . Since  $n \geq 2$  and  $G$  are connected, hence  $\gamma_{Ded}^{-1}(G) \geq \gamma_{Ded}(G)$ , we obtain  $\gamma_{Ded}^{-1}(G) \geq \frac{n}{2} \geq \frac{2n-m}{3}$ .

Consider  $(V - D) - D' \neq \emptyset$ . Every vertex in  $(V - D) - D'$  has at least one edge to  $D$  and at least one edge to  $D'$  if  $D'$  is a DED-set of  $G$ . Hence, there are at least  $2|(V - D) - D'|$  edges from  $(V - D) - D'$ .

Hence,  $m \geq 2|(V - D) - D'| + \gamma_{Ded}^{-1}(G)$

$= 2(n - \gamma_{Ded}(G) - \gamma_{Ded}^{-1}(G)) + \gamma_{Ded}^{-1}(G)$ .

$\gamma_{Ded}^{-1}(G) + 2\gamma_{Ded}(G) \geq 2n - m$ .

Since  $\gamma_{Ded}^{-1}(G) \geq \gamma_{Ded}(G)$ , it follows that  $\frac{2n-m}{3} \leq \gamma_{Ded}^{-1}(G)$ .

**Theorem 3.1.9.** If the maximum independent detour eccentric domination number for every graph  $G$  is  $\beta_{Ded}(G)$ , then  $\gamma_{Ded}^{-1}(G) \leq \beta_{Ded}(G)$ .

**Proof:** Let  $D$  be a minimum DED-set of  $G$ . Let  $S$  be an independent detour eccentric domination set in  $\langle V - D \rangle$ . If consider the following two cases.

**Case 1:** Suppose  $(V - D) - S = \emptyset$ . Then  $V - D = S$  is an independent IDEDED-set of  $G$ .

Therefore,  $\gamma_{Ded}^{-1}(G) = |V - D|$

$= |S|$

$= \beta_{Ded}(G)$ .

**Case 2:** Assume  $(V - D) - S \neq \emptyset$ . Then, each vertex in  $(V - D) - S$  has at least one vertex that it is adjacent to in  $S$ .  $S$  is an IDEDED-set of  $G$  if every vertex in  $D$  is adjacent to at least one vertex and at least one detour eccentric point in  $S$ . If not, then let  $D' \subset D$  be a set of vertices of  $S$ . Every vertex in  $D'$  must be adjacent to at least one vertex in  $(V - D) - S$  and at least one detour

eccentric point in  $(V - D) - S$  since  $D$  is a minimum DED-set. Assume that every vertex in  $D'$  is adjacent to at least one vertex in  $S' \subset (V - D) - S$ . It is obvious that  $|S'| \leq |D'|$  and  $S \cup S'$  is an IDEDED-set.

Therefore, *Errorconverting from MathML to LaTeX*

$$\leq |S \cup D'| \\ \leq \beta_{Ded}(G).$$

**Theorem 3.1.10.** For any graph  $G$ , if  $\beta_{Ded}(G) = \Gamma(G)$ , then  $\gamma_{Ded}^{-1}(G) \leq \beta_{Ded}(G)$ .

**Proof:** Let  $D'$  be a minimum DED-set in  $V - D$  and let  $D$  be any  $\gamma_{Ded}$ -set of  $G$ . According to the definition,  $\gamma_{Ded}^{-1}(G) \leq |D'| \leq \Gamma(G)$ . But  $\beta_{Ded}(G) = \Gamma(G)$  and hence  $\gamma_{Ded}^{-1}(G) \leq \beta_{Ded}(G)$ .

### 3.2 Inverse independent and inverse total detour eccentric domination number

**Theorem 3.2.1.**  $i_{Ded}^{-1}(K_n) = 1$ , if  $K_n$  is a complete graph of order  $n \geq 2$ .

**Proof:** Let  $\{u_i\}$ ,  $\forall i$  be any vertex of  $K_n$ . Then  $E(v_i) = \{u_1, u_2, \dots, u_{i-1}, u_{i+1}, \dots, u_{n-1}\}$ ,  $\forall i$ .

Therefore, any singleton set other than  $\{u_i\}$  that is an independent detour eccentric dominating set.

Hence  $i_{Ded}^{-1}(K_n) = 1$ .

**Theorem 3.2.2.**  $i_{Ded}(G) \leq i_{Ded}^{-1}(G)$ , if a graph  $G$ . Moreover, the equality holds if  $G = K_p, C_p$  and  $P_2$ .

**Proof:** Every IDEDED-set is an independent detour eccentric dominating set, according to the definition.

Hence,  $i_{Ded}(G) \leq i_{Ded}^{-1}(G)$ .

**Corollary 3.2.3.**  $i_{Ded}^{-1}(C_n) = \lceil \frac{n}{3} \rceil$  if  $C_n$  is a cycle with order  $n \geq 3$ .

**Observation 3.2.4.**  $i_{Ded}(G) + i_{Ded}^{-1}(G) \leq n$ , if a graph  $G$ . Moreover, the equality holds if  $G = C_4, K_2$  or  $P_2$ .

**Observation 3.2.5.** Let  $D$  be a  $t_{Ded}$ -set of a connected graph  $G$  has at least 4 vertices if there is a  $t_{Ded}^{-1}(G)$ -set.

**Theorem 3.2.6.** Let  $D$  be a  $t_{Ded}$ -set of  $G$  has an ITDED-set. If and only if the following conditions are satisfied,

- (i)  $\langle V - D \rangle$  contains no isolated vertices,
- (ii)  $G$  does not have any terminal vertices and
- (iii)  $|D| \leq |V - D|$ .

**Proof:** (i) Let  $\langle V - D \rangle$  contain an isolated vertex and define  $D$  as a  $t_{Ded}$ -set of  $G$ . Thus, it is a contradiction that  $V - D$  does not contain a total detour eccentric dominating set of  $G$ .

Hence  $\langle V - D \rangle$  contains no isolated vertices.

(ii) Let  $v$  the terminal vertex of  $G$ . Thus, exactly one vertex  $u$  in  $G$  is adjacent to vertex  $v$ .

**Case 1.** If  $v$  is not in  $D$  and  $u$  is in  $D$ , an isolate vertex is contained in  $V - D$ . Hence, and in opposition with this statement,  $V - D$  does not contain a total detour eccentric dominating set.

**Case 2.**  $u$  must be in  $D$  if  $v$  is in  $D$ . Hence,  $v$  is not adjacent to any vertex of  $V - D$ . Hence, and in yet another contradiction,  $V - D$  does not contain a total detour eccentric dominating set of  $G$ . Therefore,  $G$  has no terminal vertices.

(iii) Assume that  $|D| > |V - D|$  and that every vertex of  $V$  is adjacent to some vertex of  $D$  and detour eccentric of some vertex of  $D$  since  $D$  is a  $t_{Ded}$ -set of  $G$ . There exists a vertex in  $D$  that is not adjacent to any vertex of  $V - D$  if every vertex of  $V - D$  is adjacent to exactly one vertex of  $D$  and detour eccentric to any vertex of  $D$ . Because of this contradiction,  $V - D$  does not contain a total detour eccentric dominating set. So,  $|D| \leq |V - D|$ .

On the other hand, suppose  $G$  satisfies the given conditions. By (i), every vertex of  $V - D$  is adjacent to at least one vertex of  $V - D$  and detour eccentric to at least one vertex in  $D$  and detour eccentric to at least one vertex in  $D$ . By (ii), every vertex of  $V - D$  is adjacent to at least one vertex of  $D$ . By  $t_{Ded} \leq |V - D|$  in (iii). Thus, concerning  $D$ ,  $V - D$  contains a total detour eccentric dominating set. Hence  $G$  has an ITDED-set.

**Remark 3.2.7**

Let  $K_n, C_{4n}$  and  $W_n$  are complete graph, cycle graph, and wheel graph respectively. Then,

(i)  $t_{Ded}^{-1}(K_n) = t_{Ded}^{-1}(K_{m,n}) = 2, m, n \geq 2$ .

(ii)  $t_{Ded}^{-1}(C_{4n}) = 2n, n \geq 1$ .

(iii)  $t_{Ded}^{-1}(W_n) = \frac{n+1}{2}$  if  $n \equiv 3 \pmod{4}$ ,

$= \lfloor \frac{n-1}{2} \rfloor$  otherwise.

**Theorem 3.2.8.** If a graph  $G$  with  $n \geq 4$  vertices,  $t_{Ded}(G) \leq t_{Ded}^{-1}(G)$  and this bound is sharp, if  $n = 4$ .

**Proof:** Clearly, every ITDED-set is a total detour eccentric dominating set of  $G$ . Thus,  $t_{Ded}(G) \leq t_{Ded}^{-1}(G)$ .

This bound is reached by the cycle  $C_4$  and the complete graph  $K_4$ , where  $t_{Ded}(C_4) = t_{Ded}(K_4) = 2$  and  $t_{Ded}^{-1}(C_4) = t_{Ded}^{-1}(K_4) =$

2.

**Theorem 3.2.9.** If a graph  $G$  with  $n \geq 4$  vertices,  $t_{Ded}(G) + t_{Ded}^{-1}(G) \leq n$  and this bound is sharp, if  $n = 4$ .

**Proof:** The proof follows from the definition of  $t_{Ded}^{-1}(G)$ .

**Observation 3.2.10.** If a graph  $G$  has a set called  $t_{Ded}^{-1}(G)$ , then the bounds  $\gamma(G) \leq t_{Ded}(G) \leq t_{Ded}^{-1}(G) \leq T_{Ded}^{-1}(G)$  are sharp if  $G = C_4$ .

**Theorem 3.2.11.** Let  $G$  be an  $n \geq 4$  graph. If  $n = 4$ , then  $2 \leq t_{Ded}^{-1}(G) \leq n - 2$ .

**Proof:** Theorem 3.2.13, states that,  $t_{Ded}^{-1}(G) \leq n - t_{Ded}(G)$ , and since  $2 \leq t_{Ded}(G)$ ,

$$t_{Ded}^{-1}(G) \leq n - 2$$

According to Theorem 3.2.12,

$$t_{Ded}(G) \leq t_{Ded}^{-1}(G)$$

and

Since

$$2 \leq t_{Ded}(G)$$

$$2 \leq t_{Ded}^{-1}(G)$$

Hence,  $2 \leq t_{Ded}^{-1}(G) \leq n - 2$ .

As can be seen, these bounds are sharp, with the cycle  $C_4$  and the complete graphs  $K_n$  and  $n \geq 4$  achieving the lower and upper bounds, respectively.

**Theorem 3.2.12.**  $G$  is a graph with the order  $|V| = 2m, m \geq 2$ . If  $|D| = 2$ , then let  $D$  be a  $t_{Ded}$ -set of  $G$ . If  $\langle V - D \rangle = (m - 1)K_2$ , then

- (i)  $t_{Ded}^{-1}(G) = n - 2$ , and
- (ii)  $t_{Ded}(G) + t_{Ded}^{-1}(G) = n$ .

**Proof:**

If  $|D| = 2$ , then let  $D$  be a  $t_{Ded}$ -set of  $G$ .  $t_{Ded}(G) = 2$  if  $D$  has two adjacent  $G$  vertices. When  $\langle V - D \rangle = (m - 1)K_2$ , it is an edge independent set. Every vertex of  $V$  is therefore adjacent to a vertex of  $V - D$ . Thus,  $V - D$  is both a minimum ITDED-set of  $G$  and a total detour eccentric dominating set in and of itself. Thus,

$$\begin{aligned} t_{Ded}^{-1}(G) &= |V - D| \\ &= |V| - |D| \\ &= n - 2. \end{aligned}$$

Since  $t_{Ded}(G) = 2$ ,

$$t_{Ded}(G) + t_{Ded}^{-1}(G) = n.$$

## 4 Conclusion

In this study, the inverse detour eccentric domination number, inverse independent detour eccentric domination number, and inverse total detour eccentric domination number are obtained for some well-known graphs using detour distance. Theorems related to the above concepts are confirmed and proved.

## Declaration

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