INDIAN JOURNAL OF SCIENCE AND TECHNOLOGY

RESEARCH ARTICLE

OPEN ACCESS

Received: 29-08-2023

Accepted: 05-03-2024

Published: 31-05-2024

Editor: ICDMMMDE-2023 Special Issue Editors: Dr. G. Mahadevan & Prof. Dr. P. Balasubramaniam

Citation: Kaviya S, Mahadevan G, Sivagnanam C (2024) Generalizing TCCD-Number For Power Graph Of Some Graphs. Indian Journal of Science and Technology 17(SP1): 115-123. https://doi.org/10.17485/IJST/v17sp1.243

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Funding: None

Competing Interests: None

Published By Indian Society for Education and Environment (iSee)

ISSN
Print: 0974-6846
Electronic: 0974-5645

Generalizing TCCD-Number For Power Graph Of Some Graphs

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Abstract

Objective: Finding the triple connected certified domination number for the power graph of some peculiar graphs. **Methods:** A dominating set S with the condition that every vertex in S has either zero or at least two neighbors in $V-S$ and $<S>$ is triple connected is a called triple connected certified domination number of a graph. The minimum cardinality among all the triple connected certified dominating sets is called the triple connected certified domination number and is denoted by $\gamma_{TCC}(G)$. The upper bound and lower bound of γ_{TCC} for the given graphs is found and then proved the upper bound and lower bound of γ_{TCC} were equal. **Findings:** We found the (TCCD)-number for the power graph of some peculiar graphs. Also, we have generalized the result for path, cycle, ladder graph, comb graph, coconut tree graph, triangular snake, alternate triangular snake, quadrilateral snake and tadpole graph. **Novelty:** The triple connected certified domination is a new parameter in which the certified domination holds the triple connected in induced S.

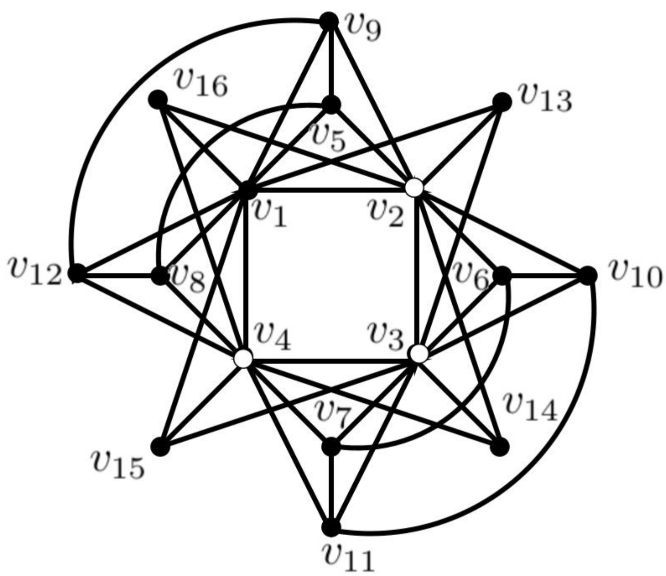
Keywords: Domination Number; Power Graphs; Triple Connected; Certified Domination; Triple Connected Certified Domination

1 Introduction

The graphs considered here is finite, non-trivial, simple and undirected. The graph $G=(V,E)$where V and E are vertex and edge set and the elements of V and E are called as vertices and edges of G. A dominating set D is a triple connected dominating set (TCD-set), if the $\langle D\rangle$ is triple connected. The smallest cardinality of a TCD set is called the triple connected domination number and is denoted by $\gamma_{tc}(G)$ [1]. A dominating set D of a graph G is said to be a certified dominating set (CD-set), if for every vertex $v\in S$ has zero or k neighbors in $V-S$ where $k\geq 2$. The smallest cardinality among all the CD sets is called the certified domination number and is denoted by $\gamma_{cer}(G)$ [2]. Motivated by the certified domination and triple connected domination here, we introduce a new concept called triple connected certified domination. A dominating set S is said to be a TCCD-set, if for every vertex $v\in S$ has zero or k neighbors in $V-S$ where $k\geq 2$ and any three vertices of S lie on a path in $<S>$. The smallest cardinality of a TCCD-set is called the TCCD-number and is denoted by $\gamma_{TCC}(G)$ [3]. Numerous recent works, like extracting the certified domination number for wide classes of graphs and product graphs. This work has a combined parameter called triple connected certified domination. Enormous articles deal with certified domination, and the power graphs have yet to be achieved. This article deals with determining the TCCD number of the power graph of some peculiar graphs. We have obtained TCCD number for some special graphs [3]. Also extracted the TCCD number on product graphs like Cartesian, Strong, Lexicographic and Corona products and no result exist for tensor product as it fails to satisfy the triple connected certified domination property [4]. Herein, we have extracted the exact γ_{TCC}values of power graph of some special graphs. Helm graph is formed by attaching P_2 on the outer cycle of wheel graph [5]. Closed helm is formed by connecting the pendent vertex of helm graph to form a cycle. A coconut tree is obtained by attaching m new pendent edges to the end vertex of P_n [6]. Web graph is formed by attaching P_2 to the vertices of outer cycle of closed helm graph [5]. Every pair of adjacent vertices of a path is replaced by C_4 is called Quadrilateral snake and a triangular snake is which every adjacent vertices are replaced by C_3 [7]. The edge of a path $\{v_n v_{n+1} : n \equiv 1(mod\ 2)\}$ is occupied by a triangle is alternate triangular snake. The end vertex of P_n is attached by an edge to any vertex of a cycle is tadpole graph or dragon graph [8]. Every vertex of $P_n$is connected by P_2 is comb graph. The end vertices of P_2 is attached by K_n is barbell graph. [5] $K_1 + nP_2$ is friendship graph [9]. $(h,q)-$Banana tree is a graph obtained by connecting one leaf of each of n copies of an $m-$ star graph (star graph with m vertices) with a single root vertex that is distinct from all the stars. $h-$Bistar graph is formed by attaching the apex (vertex with maximum degree) of two copies of $K_{1,n}$ by an edge. Book graph is formed by multiple C_3 sharing an edge. Fan graph is the corona product of K_1 and P_n [10]. Section 2, determines the exact γ_{TCC} values of the certain graphs by increasing the distance between the vertices of the considered graph and generalized the results for power graph of some special graphs. The power graph which has same vertices as in G and any two vertices in G^k is adjacent if they are connected by path of length less than or equal to k, $k \geq 2$ in G. [11,12].

Definition 1.1: [3] A dominating set S is said to be a triple connected certified dominating set (TCCD-set), if for every vertex$v \in S\,(N(v) \cap (V-S)| = 0$ *or* k where $k \geq 2$ and any three vertices of S lie on a path in $< S >$. The minimum cardinality of a TCCD-set is called the triple connected certified domination number (TCCD-number) and is denoted by $\gamma_{TCC}(G)$.

Example 1.1:



A graph with $\gamma_{TCC}(G) = 3$

Fig 1.

The lightened vertices forms a TCCD set of minimum cardinality and hence $\gamma_{TCC}(G) = 3$.

2 Methodology

Although the studies on certified domination have been increasing, the results for special types of graphs and product graphs have been determined. This article attains a domination parameter called triple connected certified domination by imposing the constraint triple connected on $< S >$, where the concept is used to obtain the TCCD number for some elementary graphs and product related graphs. The case of distance in graphs has yet to be attempted. The distance between the vertices is increased for the considered graph, and the increased distance is estimated by length less than or equal to k, where $k \geq 2$. This article uses the parameter certified domination with condition triple connected to extract the exact γ_{TCC} value for distance in some peculiar graphs. Calculating both the upper and lower bound of the following graphs and proving the obtained bound values are the same.

3 Results and Discussion

3.1 TCCD-NUMBER OF SOME SPCEIAL TYPES OF GRAPHS

OBSERVATION 3.1:

1. If G is a $h-$a bistar graph or barbell graph or fan graph or book graph or friendship graph or complete graph or helm graph or closed helm graph then $\gamma_{TCC}(G^k) = 3, k \geq 2$.
2. If G is $(h,q)-$Banana tree, then $\gamma_{TCC}(G) = 3$ if $k \leq h$.
3. If G is web graph, then $\gamma_{TCC}(G) = 3,\ k \geq 3$.

Theorem 3.1:[3] For a path P_a, $a \geq 7$ then $\gamma_{TCC}\left(P_a^2\right) = \begin{cases} 3 & \text{if } a = 7 \\ \left\lfloor \frac{a}{2} \right\rfloor - 1 & \text{if } a \geq 8 \end{cases}$

Theorem 3.2:[3] For a path $P_a,\ a \geq 5$ then $\gamma_{TCC}\left(P_a^3\right) = \begin{cases} 3 & \text{if } 5 \leq a \leq 10, \\ \left\lfloor \frac{a}{3} \right\rfloor & \text{if } a \equiv 2(\bmod 3), \\ \left\lfloor \frac{a}{3} \right\rfloor - 1 & \text{if } a \equiv 0 \,\text{or}\, 1(\bmod 3). \end{cases}$

Theorem 3 .3: If $k \geq 3$, then $\gamma_{TCC}\left(P_h^k\right) = \begin{cases} 3 & \text{if } 5 \leq h \leq 3k+1, \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{if } h \geq 3k+2 \text{ and } h \equiv 0 \text{ or } 1(\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor & \text{otherwise.} \end{cases}$

Proof: Let $V(P_h^k) = (v_1, v_2, v_3, \ldots, v_h\}$ and $E\left(P_h^k\right) = \{v_i v_{i+p} :\ 1 \leq i \leq h-p,\quad 1 \leq p \leq k\}$.
Consider the set $S_1 = \{v_i : i \equiv 1\ (modk)\} -\ \{v_1\}$.
Assume $S = \left(\begin{array}{ll} S_1 - \{v_h\} & if h \equiv 1\ (modk), \\ S_1 & otherwise. \end{array}\right.$

Clearly S is a TCCD-set of P_h^k and hence $\gamma_{TCC}\left(P_h^k\right) = |S| \leq \begin{cases} 3 & \text{if } 5 \leq h \leq 3k+1 \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{if } h \geq 3k+2 \text{ and } h \equiv 0 \text{ or } 1(\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor & \text{otherwise.} \end{cases}$

Assume a dominating set $D \subseteq\ P_h^k$ exists of cardinality at most
$d = \begin{cases} 2 & \text{if } 5 \leq h \leq 3k+1 \\ \left\lfloor \frac{h}{k} \right\rfloor - 2 & \text{if } h \equiv 0 \text{ or } 1(\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{otherwise.} \end{cases}$
Whose induced subgraph $< D >$ is not triple connected, then we have
$\gamma_{TCC}\left(P_h^k\right) \geq d+1 = \begin{cases} 3 & \text{if } 5 \leq h \leq 3k+1, \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{if } h \geq 3k+2 \text{ and } h \equiv 0 \text{ or } 1(\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor & \text{otherwise.} \end{cases}$
Hence the result follows.

Theorem 3 .4:[3] For a cycle C_a, $a \geq 5$then
$\gamma_{TCC}\left(C_a^2\right) = \begin{cases} 3 & \text{if } a = 5 \text{ or } 6 \text{ or } 7 \\ \left\lfloor \frac{a}{2} \right\rfloor - 1 & \text{if } a \geq 8. \end{cases}$

Theorem 3.5:[3] For a cycle $C_a, a \geq 5$ then $\gamma_{TCC}\left(C_a^3\right) = \begin{cases} 3 & \text{if } 5 \leq a \leq 10 \\ \left\lfloor \frac{a}{3} \right\rfloor & \text{if } a \equiv 2(\bmod 3) \\ \left\lfloor \frac{a}{3} \right\rfloor - 1 & \text{if } a \equiv 0 \,\text{or}\, 1(\bmod 3). \end{cases}$

Theorem 3 .6: If $k \geq 3$,then $\gamma_{TCC}(C_h^k) = \begin{cases} 3 & \text{if } 5 \leq h \leq 3k+1 \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{if } h \geq 3k+2 \text{ and } h \equiv 0 \text{ or } 1(\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor & \text{otherwise.} \end{cases}$

Proof: Let $V(C_h^k) = \{v_1, v_2 \ldots,\ v_h\}$ and let
$E(C_h^k) = \{v_i v_{i+p} :\ 1 \leq i \leq h-p,\quad 1 \leq p \leq k\} \cup$
$\{v_j v_l : j = h-(p-l),\ \ 1 \leq l \leq p,\ 1 \leq p \leq k\}$.
Consider the set $S = \{v_i : i \equiv 1\,(modk)\}$.

Assume

$$S=\left(\begin{array}{ll} S_1-\{v_{h-(k-1)}\} & if\ h\equiv 0\,(modk)\,, \\ S_1-\{v_h,\ v_{h-k}\} & if\ h\equiv 1\,(modk)\,, \\ S_1-\{v_{h-i}\} & if\ h\equiv i+1\,(modk)\ where\ i=1,2,\ldots k-2. \end{array}\right.$$

Clearly S is a TCCD-set of C_h^k and hence $\gamma_{TCC}\left(C_h^k\right)=|S|\leq\begin{cases} 3 & \text{if } 5\leq h\leq 3k+1 \\ \left\lfloor\frac{h}{3}\right\rfloor-1 & \text{if } h\geq 3k+2 \text{ and } h\equiv 0 \text{ or } 1(\bmod k) \\ \left\lfloor\frac{h}{3}\right\rfloor & \text{otherwise.}\end{cases}$

Assume a dominating set $D\subseteq C_h^k$ exists of cardinality at most

$$d=\begin{cases} 3 & \text{if } 5\leq h\leq 3k+1 \\ \left\lfloor\frac{h}{3}\right\rfloor-1 & \text{if } h\equiv 0 \text{ or } 1(\bmod k) \\ \left\lfloor\frac{h}{3}\right\rfloor & \text{otherwise.}\end{cases}$$

Whose induced subgraph $<D>$ is not triple connected, then we have

$$\gamma_{TCC}\left(C_h^k\right)\geq d+1=\begin{cases} 3 & \text{if } 5\leq h\leq 3k+1, \\ \left\lfloor\frac{h}{k}\right\rfloor-1 & \text{if } h\geq 3k+2 \text{ and } h\equiv 0 \text{ or } 1(\bmod k), \\ \left\lfloor\frac{h}{k}\right\rfloor & \text{otherwise.}\end{cases}$$

Hence the result follows.

Theorem 3.7: If $k\geq 2$, then

$$\gamma_{TCC}(L_h^k)=\left(\begin{array}{ll} 3 & if\ 3\leq h\leq 3k-1, \\ \left\lfloor\frac{h}{k}\right\rfloor & if\ h\geq 3k. \end{array}\right.$$

Proof: Let $V(L_h^k)=(v_1,\ v_2\ldots,v_h,u_1,u_2\ldots,u_h\}$ and let

$E\left(L_h^k\right)=\{\{v_i\,u_{\,i\,+(p-1)}:1\leq i\ \leq h-(p-1)\}\cup\{u_iv_{i+p-1}:1\leq i\ \leq h-(p-1),2\leq$

$p\leq k\}\cup\{v_iv_{i+p}:1\leq i\ \leq h-p\}\cup\{u_iu_{i+p}:1\leq i\ \leq h-p\}\,1\leq p\leq k\}$

$\cup(v_iv_{i+p}:1\leq i\leq h-p\}\cup\{u_iu_{i+p}:1\leq i\leq h-p\}1\leq p\leq k\}.$

Assume $S=\{v_i\ :i\equiv 0\ \ (mod\ k)\}$. Clearly S is a TCCD-set of L_h^k and hence

$$\gamma_{TCC}\left(L_h^k\right)=|S|\leq\left(\begin{array}{ll} 3 & if\ 3\leq h\leq 3k-1, \\ \left\lfloor\frac{h}{k}\right\rfloor & if\ h\geq 3k. \end{array}\right.$$

Assume a dominating set $D\subseteq\ L_h^k$ exists of cardinality at most

$$d=\begin{cases} 2 & \text{if } 3\leq h\leq 3k-1 \\ \left\lfloor\frac{h}{k}\right\rfloor-1 & \text{if } h\geq 3k.\end{cases}$$

Whose induced subgraph $<D>$ is not triple connected, then we have $\gamma_{TCC}\left(L_h^k\right)\geq d+1=\left(\begin{array}{ll} 3 & if\ 3\leq h\leq 3k-1 \\ \left\lfloor\frac{h}{k}\right\rfloor & if\ h\geq 3k. \end{array}\right.$

Hence the result follows.

Theorem 3.8: If $k\geq 2$, then $\gamma_{TCC}\left((P_h\odot K_1)^k\right)=\left(\begin{array}{ll} 3 & if\ 3\leq h\leq 3k-1, \\ \left\lfloor\frac{h}{k}\right\rfloor & if\ h\geq 3k. \end{array}\right.$

Proof: Let $V\left((P_h\odot K_1)^k\right)=\{v_1,v_2\ldots,v_n,u_1,u_2\ldots,u_q\}$and let

$E\left((P_h\odot K_1)^k\right)=\{\{v_iu_{i+(p-1)}:1\leq i\leq h-(p-1)\}\cup\{u_iu_{i+(p-2)}:1\leq i\leq h-(p-2),\ p\geq 3\}\cup\{v_{i+(p-1)}u_i:1\leq i\leq h-(p-1)\,,\,2\leq p\leq k\}\cup\{v_iv_{i+p}:1\leq i\leq h-p\}\ 1\leq p\leq k\}.$

Assume $S=\{v_i:i\ \equiv 0(\bmod k)\}$ Clearly S is a TCCD-set of $(P_h\odot K_1)^k$and hence

$$\gamma_{TCC}\left((P_h\odot K_1)^k\right)=|S|\leq\begin{cases} 3 & \text{if } 3\leq h\leq 3k-1 \\ \left\lfloor\frac{h}{k}\right\rfloor & \text{if } h\geq 3k\end{cases}$$

Assume a dominating set $D\subseteq(P_h\odot K_1)^k$ exists of cardinality at most

$$d=\begin{cases} 2 & \text{if } 3\leq h\leq 3k-1 \\ \left\lfloor\frac{h}{k}\right\rfloor-1 & \text{if } h\geq 3k.\end{cases}$$

Whose induced subgraph$< D >$ is not triple connected, then we have $\gamma_{TCC}\left((P_h\odot K_1)^k\right)\geq d+1=\left(\begin{array}{ll} 3 & if\ 3\leq h\leq 3k-1, \\ \left\lfloor\frac{h}{k}\right\rfloor & if\ h\geq 3k. \end{array}\right.$

Hence the result follows.

Theorem 3.9: If $k \geq 2$, then

$\gamma_{TCC}\left((CT_{l,e})^k\right) = \begin{cases} 3 & \text{if } 3 \leq e \leq 3k \\ \lceil \frac{e}{k} \rceil - 1 & \text{if } e \geq 3k+1 \end{cases}$

Proof: Let $V\left((CT_{l,e})^k\right) = (r_1,\ r_2 \ldots, r_e, s_1, s_2 \ldots, s_l\}$and

let$E\left((CT_{l,e})^k\right) = \left\{\left\{r_i r_{i+p} : 1 \leq i\ \leq e-p\right\} \cup \left\{r_i s_j : 1 \leq j \leq l, 1 \leq i\ \leq p\right\} 1 \leq p \leq k\right\}$.

Take $S = \{r_i : i \equiv 0\ (mod\ k)\}$.

Assume $S = \left(\begin{array}{cc} S_1 - \{r_e\} & if\ e \equiv 0\ \ (mod\ k) \\ S_1 & otherwise. \end{array} \right.$.

Clearly S is a TCCD-set of $(CT_{l,e})^k$ and hence$\gamma_{TCC}\left((CT_{l,e})^k\right) = |S| \leq \begin{cases} 3 & \text{if } 3 \leq e \leq 3k \\ \lceil \frac{e}{k} \rceil - 1 & \text{if } e \geq 3k+1 \end{cases}$

Assume a dominating set $D \subseteq (CT_{l,e})^k$ exists of cardinality at most $d = \begin{cases} 2 & \text{if } 3 \leq e \leq 3k \\ \lceil \frac{e}{k} \rceil - 2 & \text{if } e \geq 3k+1 \end{cases}$

Whose induced subgraph $< D >$ is not triple connected, we

have $\gamma_{TCC}\left((CT_{l,e})^k\right) \geq d+1 = \left(\begin{array}{cc} 3 & if\ 3 \leq e \leq 3k \\ \lceil \frac{e}{k} \rceil - 1 & if\ e \geq 3k+1. \end{array} \right.$

Hence the result follows.

Theorem 3.10: If $k \geq 2$, then $\gamma_{TCC}(TS_h^k) = \begin{cases} 3 & \text{if } h \leq 3k \\ \lceil \frac{h}{k} \rceil - 1 & \text{if } h \geq 3k+1 \end{cases}$

Proof: Let $V(TS_h^k) = \{v_1,\ v_2 \ldots, v_{h+1},\ u_1,\ u_2,\ \ldots,\ u_h\}$ and let

$$E\left(TS_h^k\right) = \left\{\left\{v_i v_{i+p} : 1 \leq i\ \leq (h+1)-p\right\} \cup \left\{v_i u_{i+(p-1)} : 1 \leq i\ \leq (h+1)-p\right\} \cup \left\{v_{i+p} u_i : 1 \leq i\ \leq (h+1)-p\right\} 1 \leq p \leq k\right\} \cup \left\{u_i u_{i+(p-1)} : 1 \leq i \leq (h+1)-p, 2 \leq p \leq k\right\}.$$

Take $S_1 = \{v_i : i \equiv 1\ (mod\ k)\} - \{v_1\}$.

Assume $S = \left(\begin{array}{cc} S_1 - \{v_{h+1}\} & if\ h \equiv 0\ \ (mod\ k) \\ S_1 & otherwise. \end{array} \right.$

Clearly S is a TCCD-set of TS_h^k and hence $\gamma_{TCC}\left(TS_h^k\right) = |S| \leq \begin{cases} 3 & \text{if } h \leq 3k \\ \lceil \frac{h}{k} \rceil - 1 & \text{if } h \geq 3k+1 \end{cases}$

Assume a dominating set $D \subseteq (TS_h^k)$ exists of cardinality at most$d = \begin{cases} 2 & \text{if } h \leq 3k \\ \lceil \frac{h}{k} \rceil - 2 & \text{if } h \geq 3k+1 \end{cases}$

Whose induced subgraph $< D >$ is not triple connected, then we have

$\gamma_{TCC}\left(TS_h^k\right) \geq d+1 = \begin{cases} 3 & \text{if } h \leq 3k \\ \lceil \frac{h}{k} \rceil - 1 & \text{if } h \geq 3k+1 \end{cases}$

Hence the result follows.

Theorem 3.11: If $k \geq 2$, then $\gamma_{TCC}(ATS_h^k) = \begin{cases} 3 & \text{if } 3 \leq h \leq 3k, \\ \lfloor \frac{h}{k} \rfloor - 1 & \text{if } h \geq 3k+1 \text{ and } h \equiv 0(\bmod k) \\ \lfloor \frac{h}{k} \rfloor & \text{otherwise.} \end{cases}$

Proof: Let $V\left(ATS_h^k\right) = \left\{v_1, v_2 \ldots, v_{h+1}, u_1, u_2, \ldots, u_{\lfloor \frac{h+1}{2} \rfloor}\right\}$ and let

$$E\left(ATS_h^k\right) = \left\{\left\{v_i v_{i+p} : 1 \leq i\ \leq (h+1)-p\right\} \cup \left\{v_i u_j : 1 \leq j \leq \left\lceil \frac{h+1}{2} \right\rceil, i = 2j+(p-1),\quad 1 \leq i\ \leq h+1\right\} \cup \left\{v_i u_j : 1 \leq j \leq \left\lceil \frac{h+1}{2} \right\rceil\right.\right.$$

$$\left. i = 2j-p, 1 \leq i\ \leq h+1\right\} 1 \leq p \leq k\} \cup \left\{u_j u^{-1}_{j+\lceil \frac{p}{2}} : 1 \leq j \leq \left\lfloor \frac{h+1}{2} \right\rfloor - \left(\left\lceil \frac{p}{2} \right\rceil - 1\right), 3 \leq p \leq k\right\}$$

Take

$S_1 = \{v_i : i \equiv 1 \ (mod\ k)\} - \{v_1\}$.

Assume $S = \begin{pmatrix} S_1 - \{v_{h+1}\} & if\ h \equiv 0 \ \ (mod\ k), \\ S_1 & otherwise. \end{pmatrix}$

Clearly S is a TCCD-set of (ATS_h^k) and hence

$$\gamma_{TCC}\left(ATS_h^k\right) = |S| \le \begin{cases} 3 & \text{if } 3 \le h \le 3k, \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{if } h \ge 3k+1 \text{ and } h \equiv 0 (\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor & \text{otherwise.} \end{cases}$$

Assume a dominating set $D \subseteq (ATS_h^k)$ exists of cardinality at most

$$d = \begin{cases} 2 & \text{if } 3 \le h \le 3k \\ \left\lfloor \frac{h}{k} \right\rfloor - 2 & \text{if } h \equiv 0 (\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{otherwise.} \end{cases}$$

Whose induced subgraph $< D >$ is not triple connected, then we have $\gamma_{TCC}\left(ATS_h^k\right) \ge d+1 = \begin{cases} 3 & \text{if } 3 \le h \le 3k, \\ \left\lfloor \frac{h}{k} \right\rfloor - 1 & \text{if } h \ge 3k+1 \text{ and } h \equiv 0 (\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor & \text{otherwise.} \end{cases}$

Hence the result follows.

Theorem 3.12: If $k \ge 2$, then $\gamma_{TCC}(Q_e^k) = \begin{pmatrix} 3 & if\ 2 \le e \le 3k-2, \\ \frac{e}{k} & if\ e \ge 3k-1\ and\ e \equiv 0\ (mod\ k), \\ \lceil \frac{e}{k} \rceil & if\ e \ge 3k-1\ and\ e\ \equiv k-1\ (mod\ k), \\ \lfloor \frac{e}{k} \rfloor & otherwise. \end{pmatrix}$

Proof: Let $V(Q_e^k) = \{v_1,\ v_2 \ldots, v_{e+1}, u_1, u_2,\ \ldots,\ u_{2e}\}$ and let

$E\left(Q_e^k\right) = \{\{v_i v_{i+t} : 1 \le i\ \le (e+1) - t\}$

$\cup \{u_i u_j : 1 \le i\ \le (2(e+1) - (2t-3)), j = i + (t + (t-4))$

$j = i + (t + (t-4)) + 1, 1 \le j \le 2(e+1) - 2, i$ is even $\} \cup \{u_i u_j : 1 \le i$

$\le (2(e+1) - (2t-3)), j = (i-1) + (t + (t-4))$

$j = (i-1) + (t + (t-4)) + 1, 1 \le j \le 2(e+1) - 2, i$ is odd $\} 1 \le t \le k\}$

$\cup \{v_i u_j : 1 \le i\ \le h+1, j = 2i-2, j = 2i-1, 1 \le j \le 2(e+1) - 2\}$

$\cup \{u_i u_j : i = 2r + (t-2), j = i+1, 1 \le r \le e + (1-t), 1 \le t \le 2\} \cup \{v_i u_j : j$

$= 2i + (2t-4), 2i + (2t-3), \quad (2i + (2t-4) - (4t-4)),$

$(2i + (2t-3) - (4t-4)), 1 \le j \le 2(m+1) - 2, 1 \le i\ \le e+1, 2 \le t \le k\}$

Take $S = \{v_i : i \equiv 0\ (mod\ k)\}$. Clearly S is a TCCD-set of (Q_e^k) and hence

$\gamma_{TCC}\left(Q_e^k\right) = |S| \le \begin{pmatrix} 3 & if\ 2 \le e \le 3k-2, \\ \frac{e}{k} & if\ e \ge 3k-1\ and\ e \equiv 0\ (mod\ k), \\ \lceil \frac{e}{k} \rceil & if\ e \ge 3k-1\ and\ e\ \equiv k-1\ (mod\ k), \\ \lfloor \frac{e}{k} \rfloor & otherwise. \end{pmatrix}$ Assume a dominating set $D \subseteq (Q_e^k)$exists of cardinality at most

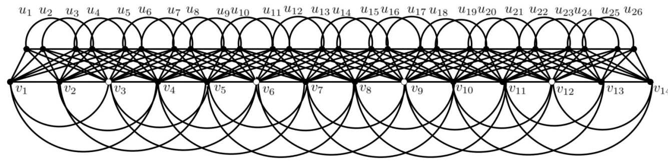
$d = \begin{pmatrix} 2 & if\ 2 \le e \le 3k-2, \\ \frac{e}{k} - 1 & if\ e \ge 3k-1\ and\ e \equiv 0\ (mod\ k), \\ \lceil \frac{e}{k} \rceil - 1 & if\ e \ge 3k-1\ and\ e\ \equiv k-1\ (mod\ k), \\ \lceil \frac{e}{k} \rceil - 1 & otherwise. \end{pmatrix}$ Whose induced subgraph $< D >$ is not triple connected, then we have

$$\gamma_{TCC}\left(Q_e^k\right) \ge d+1 = \begin{pmatrix} 3 & if\ 2 \le e \le 3k-2, \\ \frac{e}{k} & if\ e \ge 3k-1\ and\ e \equiv 0\ (mod\ k), \\ \lceil \frac{e}{k} \rceil & if\ e \ge 3k-1\ and\ e\ \equiv k-1\ (mod\ k), \\ \lfloor \frac{e}{k} \rfloor & otherwise. \end{pmatrix}$$

Hence the result follows.

Example 3.1:

Here the set of lightened vertices forms a TCCD set of minimum cardinality and hence $\gamma_{TCC}\left(Q_{13}^3\right) = 4$.



$\gamma_{TCC}(Q^3_{13}) = 4$

Fig 2.

Theorem 3.13: For a tadpole graph $(T_{h,q})$, $k \geq 2$

i) If $h \equiv 0\ (mod\ k)$ then $\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) = \begin{cases} \frac{h}{k} + \frac{q}{k} - 1 & if\ q \equiv 0\ (mod\ k), \\ \frac{h}{k} + \lfloor \frac{q}{k} \rfloor - 1 & if\ q \equiv 1\ (mod\ k), \\ \frac{h}{k} + \lceil \frac{q}{k} \rceil - 1 & otherwise. \end{cases}$

ii) If $h \equiv 1\ (mod\ k)$ then $\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) = \begin{cases} \lfloor \frac{h}{k} \rfloor + \frac{q}{k} - 1 & if\ q \equiv 0\ (mod\ k), \\ \lfloor \frac{h}{k} \rfloor + \lfloor \frac{q}{k} \rfloor - 1 & if\ q \equiv 1\ (mod\ k), \\ \lfloor \frac{h}{k} \rfloor + \lceil \frac{q}{k} \rceil - 1 & otherwise. \end{cases}$

iii) If $h \not\equiv 0, 1\ (mod\ k)$ then $\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) = \begin{cases} \lfloor \frac{h}{k} \rfloor + \frac{q}{k} & if\ q \equiv 0\ (mod\ k) \\ \lfloor \frac{h}{k} \rfloor + \lfloor \frac{q}{k} \rfloor & if\ q \equiv 1\ (mod\ k) \\ \lfloor \frac{h}{k} \rfloor + \lceil \frac{q}{k} \rceil & otherwise. \end{cases}$

Proof: Case(i): For $n \equiv 0\ (mod\ k)$.

Let $V\left(\left(T_{h,q}\right)^k\right) = \{v_1, v_2 \ldots, v_h, u_1, u_2 \ldots, u_q\}$ and let

$E\left(\left(T_{h,q}\right)^k\right) = \{\{v_i v_{i+p} : 1 \leq i \leq h - p\} \cup \{u_i u_{i+p} : 1 \leq i \leq h - p\} \cup \{u_j u_l : j = h - (p - l), 1 \leq l \leq p\} 1 \leq p \leq k\} \cup \{u_1 v_p : 1 \leq p \leq k\} \cup \{v_i u_j : 1 \leq i \leq p - 1, j = p - (i - 1), 2 \leq p \leq k\} \cup \{v_i u_j : 1 \leq i \leq p - 1, 2 \leq p \leq k, j = q + 2 - (l - (i - 1)), l \geq i, 2 \leq l \leq k\}$.

Take $S_1 = \{v_i : i \equiv 0\ (mod\ k)\} - \{v_h\}$ and $S_2 = \{v_j : j \equiv 1\ (mod\ k)\}$.

Assume

$S = \begin{cases} S_1 \cup S_2 - \{u_q\} & if\ q \equiv 1\ (mod\ k), \\ S_1 \cup S_2 & otherwise. \end{cases}$ Clearly S is a TCCD-set of $\left(\left(T_{h,q}\right)^k\right)$ and hence

$\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) = |S| \leq \begin{cases} \frac{h}{k} + \frac{q}{k} - 1 & if\ q \equiv 0\ (mod\ k), \\ \frac{h}{k} + \lfloor \frac{q}{k} \rfloor - 1 & if\ q \equiv 1\ (mod\ k), \\ \frac{h}{k} + \lceil \frac{q}{k} \rceil - 1 & otherwise. \end{cases}$

Assume a dominating set $D \subseteq \left(T_{h,q}\right)^k$ exists of cardinality at most

$d = \begin{cases} \frac{h}{k} + \frac{q}{k} - 2 & if\ q \equiv 0\ (mod\ k), \\ \frac{h}{k} + \lfloor \frac{q}{k} \rfloor - 2 & if\ q \equiv 1\ (mod\ k), \\ \frac{h}{k} + \lceil \frac{q}{k} \rceil - 2 & otherwise. \end{cases}$

Whose induced subgraph $< D >$ is not triple connected, then we have

$\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) \geq d + 1 = \begin{cases} \frac{h}{k} + \frac{q}{k} - 1 & if\ q \equiv 0\ (mod\ k), \\ \frac{h}{k} + \lfloor \frac{q}{k} \rfloor - 1 & if\ q \equiv 1\ (mod\ k), \\ \frac{h}{k} + \lceil \frac{q}{k} \rceil - 1 & otherwise. \end{cases}$

Case(ii): For $n \equiv 1\ (mod\ k)$.

Take $S_1 = \{v_i : i \equiv 0 (\mathrm{mod}\, k)\} - \{v_{h-1}\}$and $S_2 = \{v_j : j \equiv 1\ (mod\ k)\}$.

Assume $S = \begin{cases} S_1 \cup S_2 - \{u_q\} & if\ q \equiv 1\ (mod\ k), \\ S_1 \cup S_2 & otherwise. \end{cases}$ Clearly S is a TCCD-set of $\left(\left(T_{h,q}\right)^k\right)$ and hence$\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) = (S| \leq$

$\begin{cases} \lfloor \frac{h}{k} \rfloor + \frac{q}{k} - 1 & if\ q \equiv 0\ (mod\ k), \\ \lfloor \frac{h}{k} \rfloor + \lfloor \frac{q}{k} \rfloor - 1 & if\ q \equiv 1\ (mod\ k), \\ \lfloor \frac{h}{k} \rfloor + \lceil \frac{q}{k} \rceil - 1 & otherwise. \end{cases}$

Assume a dominating set $D \subseteq \left(T_{h,q}\right)^k$of cardinality at most

$$d = \begin{cases} \left\lfloor \frac{h}{k} \right\rfloor + \frac{q}{k} - 2 & \text{if } q \equiv 0(\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor + \left\lfloor \frac{q}{k} \right\rfloor - 2 & \text{if } q \equiv 1(\bmod k), \\ \left\lfloor \frac{h}{k} \right\rfloor + \left\lceil \frac{q}{k} \right\rceil - 2 & \text{otherwise.} \end{cases}$$

Whose induced subgraph $< D >$ is not triple connected, then we have $\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) \geq d+1 = \left(\begin{array}{ll} \lfloor \frac{h}{k} \rfloor + \frac{q}{k} - 1 & if\ q \equiv 0\ (mod\ k), \\ \lfloor \frac{h}{k} \rfloor + \lfloor \frac{q}{k} \rfloor - 1 & if\ q \equiv 1\ (mod\ k), \\ \lfloor \frac{h}{k} \rfloor + \lceil \frac{q}{k} \rceil - 1 & otherwise. \end{array} \right.$

Case(iii): For $n \not\equiv 0, 1\ (mod\ k)$.

Take $S_1 = \{v_i : i \equiv 0\ (mod\ k)\}$ and $S_2 = \{v_j : j \equiv 1\ (mod\ k)\}$.

Assume $S = \left(\begin{array}{ll} S_1 \cup S_2 - \{u_q\} & if\ q \equiv 1\ (mod\ k), \\ S_1 \cup S_2 & otherwise. \end{array} \right.$

Clearly S is a TCCD-set of $\left(T_{h,q}\right)^k$ and

hence$\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) = |S| \leq \begin{cases} \left\lfloor \frac{h}{k} \right\rfloor + \frac{q}{k} & \text{if } q \equiv 0(\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor + \left\lfloor \frac{q}{k} \right\rfloor & \text{if } q \equiv 1(\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor + \left\lceil \frac{q}{k} \right\rceil & \text{otherwise.} \end{cases}$

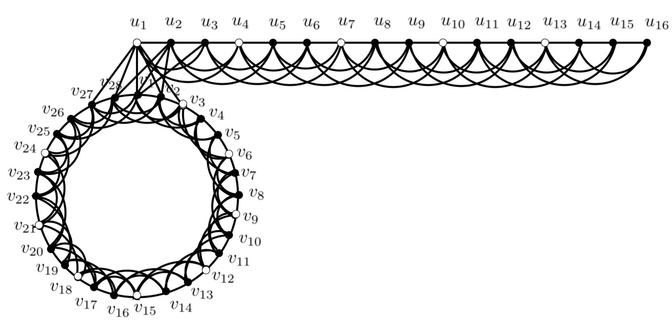
Assume a dominating set $D \subseteq \left(T_{h,q}\right)^k$of cardinality at most

$$d = \begin{cases} \left\lfloor \frac{h}{k} \right\rfloor + \frac{q}{k} - 1 & \text{if } q \equiv 0(\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor + \left\lfloor \frac{q}{k} \right\rfloor - 1 & \text{if } q \equiv 1(\bmod k) \\ \left\lfloor \frac{h}{k} \right\rfloor + \left\lceil \frac{q}{k} \right\rceil - 1 & \text{otherwise.} \end{cases}$$

Whose induced subgraph $< D >$ is not triple connected, then we have $\gamma_{TCC}\left(\left(T_{h,q}\right)^k\right) \geq d+1 = \left(\begin{array}{ll} \lfloor \frac{h}{k} \rfloor + \frac{q}{k} & if\ q \equiv 0\ (mod\ k) \\ \lfloor \frac{h}{k} \rfloor + \lfloor \frac{q}{k} \rfloor & if\ q \equiv 1\ (mod\ k) \\ \lfloor \frac{h}{k} \rfloor + \lceil \frac{q}{k} \rceil & otherwise. \end{array} \right.$

Hence the result follows.

Example 3.2:



$(T_{28,16})^3$

Fig 3.

The lightened vertices from the TCCD set and hence $\gamma_{TCC}(T_{28,16})^3 = 13$.

4 Conclusion

This study has generalized the results for the power graph of some special types of graphs with distance k where $k \geq 2$. In future this parameter triple connected certified domination will be discussed for grid graphs.

Declaration

Presented in 9th INTERNATIONAL CONFERENCE ON DISCRETE MATHEMATICS AND MATHEMATICAL MODELLING IN DIGITAL ERA (ICDMMMDE-2023) during March 23-25, 2023, Organized by the Department of Mathematics, The Gandhigram Rural Institute (Deemed to be University), Gandhigram - 624302, Dindigul, Tamil Nadu, India.

ICDMMMDE-23 was supported by GRI-DTBU, CSIR

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