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Modelling of Multiple Steady-state Behaviour and Control of a Continuous Bioreactor

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Abstract

Objectives: To simulate the steady-state multiplicity behaviour exhibited by a continuous bioreactor following substrate inhibition kinetics and design an efficient control scheme corresponding to its unstable (optimum) operating condition. **Methods/Statistical Analysis:** A rigorous first principles based nonlinear model of continuous bioreactor following substrate inhibition kinetics is presented. IMC PID controller has been designed at the unstable steady-state (optimum) operating point to control the biomass concentration in the bioreactor, by manipulating the (inlet) Dilution rate. **Findings**: The bioreactor has been shown to exhibit steady-state multiplicity (three steady-states) corresponding to different operating conditions. One of the steady-states has been shown to be unstable as computed from the Eigen values of state space model matrix. **Application/Improvements:** IMC based PID controller parameters have been tuned based on various performance criteria such as IAE, ISE, ITAE, settling time, % overshoot and rise time so as to provide optimum performance of the bioreactor, based on an unstable transfer function.

Keywords: Continuous Bioreactor, Controller Tuning, IMC PID Controller, Substrate Inhibition Kinetics, Unstable Transfer Function

1. Introduction

In recent years, there had been development of various biochemical reactor models in order to optimize the process for production of various products from agricultural, pharmaceuticals, sector etc. In industrial waste water treatment, Membrane Bioreactor Technology is widely used. With this wide usage of bioreactor, there is a need to optimize and improve the bioreactor efficiency by proper control. Biochemical control presents a challenge due to the nonlinear performance presented by system. Various variables such as pH, temperature, dissolved oxygen, substrate concentration required; etc. had been used for the designing of biochemical control. Biochemical reactor should be designed to minimize the cost while keeping the product of high quality and ensuring that biological as well as industrial requirement are fulfilled. Several relations have been developed between the substrate concentration and the growth rate coefficient in order to develop a dynamic model for the control of biochemical reactor in which Monod and Substrate Inhibition Kinetics are common. In case of Substrate Inhibition, numbers of steady state solutions are three in which two are stable but the conditions required for it are not optimum.

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The third steady state is highly unstable but provides optimum operating conditions. Hence there is a need for designing a controller with proper parameter for stabilizing it.

Various works establishing On-line Control Tuning Methods are available for systems that are unstable. In[1] proposed identification of model parameters for a first order process which is unstable and transfer function for time delay using step response data with proportional controller in a closed loop system. They observed that the method gave less overshoot as well as less settling time. Using the model reference, internal model control and synthesis method, a precise design formula[2] had been developed for feedback controllers. In[3] proposed scrutinized bifurcation and stability characteristics of bioreactor using an unstructured model with cell recycle following Substrate Inhibition Kinetics. Step-Change in the PI or PID controller is used for calculating parameters for first order system. Further it is also used in computing time delay transfer function model[4]. In case of non-linear process[5] suggested AdIMC and its application on fermenter process. Further improvements were made on tuning of PID controllers by[6] optimizing it using step as well as pulse response. In[7] proposed demonstrated the practical implications of PID controller involving parameter estimation by reaction curve method as well as by least-square algorithm for a closed loop arrangement. In[8] proposed designing of parallel cascade controllers for unstable processes in closed loop configuration.

Parameter estimation using discrete indirect estimation for the discrete-time adaptive control and its applications for continuous stirred tank bioreactor are discussed in[9]. In[10] developed Adaptive Model Predictive Control for calculation of variation Dissolved Oxygen which represents non-linear time varying system. In[11] proposed presented a complete analysis of five different control schemes on the basis of disturbance rejection and observed that using feed substrate concentration as manipulating variable in Substrate-Limited growth scheme in case of turbidostatic. Similar kind of analysis was performed by[12] by checking local controllability, stability criteria and steady state gains.

A direct synthesis method was used in the work of[13] for the designing of proportional integral controller with fractional order. In[14] proposed feedback strategy to control fed batch bioreactor by Observer based robust controller by maintaining the by-product concentration at minimum level. In[15] proposed studied the performance of digital control used for stabilizing the biochemical process in comparison to other PID controllers as shown in Figure 1. In order to achieve Maximum Production Rate,[16]gave design of a proportional integral controller on the basis of Monod Kinetics and evaluated its performance in a closed loop configuration. In[17] developed a mathematical model for enzymatic reaction for substrate as well as product inhibition in two interconnected CSTRs.

In[18] proposed implemented model predictive parameter on nonlinear continuous bioreactor. They investigated its effect based on predictor horizon which is considered an important parameter for MPC controller. Recently[19] proposed temperature control in bioreactor using novel control algorithm that combines IMC-PID and fractional mathematics. They further modified it by introducing an additional control loop having proportional gain for reducing offset.

In[20] proposed presented the usage of PID algorithm for control of temperature, pH and Dissolved Oxygen. Their PID controller was based on NARX model whose parameters were tuned using the famous Levenberg-Marquardt algorithm. They observed improvement in response and delay time as well as reduced the residual error. Among the ways for detection of fault in model, developed model based diagnosis in a nonlinear bioreactor.

Performance analysis on exergy parameter was done for a continuous stirred bioreactor intended for fermentation of acetate and ethanol from syngas through famous[21].

In[22] developed a generalized mathematical model to analyze the steady state behaviour of gas-liquid reactors which were agitated mechanically in non-isothermal

conditions. The gas absorption is taken into account with inter-phase mass transfer reaction effects considering various parameters. In[23] proposed a controller with the aim of regulating the behaviour of continuous bioreactor in dynamic condition for biogenic production of hydrogen.

In[24] suggested modelling with cross flow filtration for a continuous biochemical reactor. The modelling of non-isothermal CSTR dynamics by using components methods based on linear and nonlinear principal were examined by[25] and further analysed on their ability to calculate multiple steady states and usage for adaptive on-line process control. In[26] suggested modelling of an Upflow Anaerobic Sludge Blanket reactor lab plant used for the treatment of synthetic substrate and wastewater containing starch using ADM1.

In[27] proposed a strategy based on a combinatorics approach for the bifurcation analysis of cybernetic models used for computing all probable consequences of competition amongst diverse enzyme systems. The analysis of the model was done for certain ranges of bifurcation parameters and the results show that multiple stable steady states existed. In[28] proposed studied the arrays of diffusively coupled reactors and the consequence of the network structure on the instability induced due to diffusion.

In[29] proposed followed the concept of steady state multiplicity to estimate the physiological multiplicity and population heterogeneity linked to two experimentally described systems – a biological system using bacteriophage 2 and the other using E. coli lactose operon as its nutrient. In earlier work of by[30] proposed nonlinear analysis of hybrid system in order to describe the fed-batch conversion to 1, 3-propanediol from glycerol with substrate open loop inputs and pH logic control. Their recent work in 2014 focuses on its optimal control. In[31] proposed studied about quadratic autocatalysis in an extended CSTR.

In a study by[32], they introduced a parameter in the back-stepping design technique for preventing the control system from functioning at a particular point in a biochemical reactor. In[33] projected the saturated Proportional Integral control of continuous biochemical reactors using Haldane kinetics. In 2012 they proposed feedback control with saturated linearized output for three-state continuous bioreactors class through inhibited kinetics. The experiments done by[34] showed that self-tuning Dissolved Oxygen concentration control by using the set profile which has five step changes performed 90% better than the performance of the same control strategy which utilized the constant set value of DO concentration.

In[35] proposed presented a paper in which the stabilization by PID controller of second-order unstable processes is performed via the necessary and sufficient criteria of stability analysis. In[36] proposed recommended stabilization of unstable steady states of a biochemical CSTR with kinetics of predator-prey system by employing continuous P or PI controllers. In[37] gave a synopsis of existing results of state and parameter methods for chemical as well as biochemical processes.

In[38] proposed investigated the multiplicity behaviour of six reaction systems of increasing complexity to chain growth polymerization from one reactant, first-order reaction.

In[39] proposed calculated the competition dynamics amongst three microbial populations within a spatially heterogeneous network of four interconnected biochemical reactors.

The study by[40] emphases on the interpretation of multiple steady-states and related stability as well as instability characteristic over a varying range of operating conditions. The bifurcation analysis of cybernetic models done by[41] tells the steady-state behaviour under certain operating conditions and can be used in planning of optimum biochemical reactor operation. The study by[42] examines the steady-state behaviour of biological conversion systems by common kinetics, which performs two successive reactions by two groups of microbes. In[43] presented the work which discusses the connection between software tools primarily intended for mathematical modelling, simulation as well as investigation of chemical reactors and the general method for hazard identification, the HAZOP study. A sample kinetic model of glycolysis by

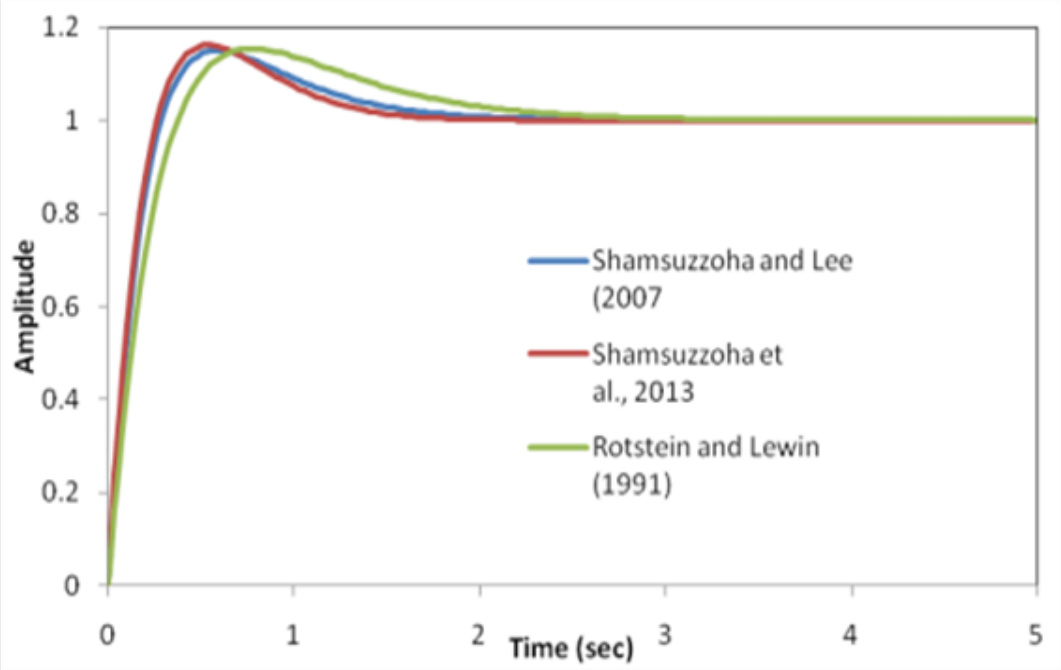

Figure 1. Comparison of response based on tuning methods.

bacteria has been framed and a minimal set of assumptions has been hypothesized that allows the devising of a rough mathematical model by[44].

The work by[45] use subspace-based identification method for transfer function model identification for an unstable bioreactor. In[46] proposed used the values obtained, in sulphide response analysis for sulphide control in sulfate reducing biochemical reactors by means of a pS electrode, for calculating the parameters of a PID controller. In[47] suggested that continuous bioreactors in series with recirculation can be modelled by using a system of linear equations of the form Ax = 0 and using this representation, singular value decomposition can be used as analysis tool. A mathematical model of a three-phase fluidized bed biofilm reactor for aerobic processes was presented by[48]. The relationship between biofilm growth and boundaries of fluidized bed existence was also shown. The article by[49] presented control scheme using inferential predictions which was founded on ADALINE soft sensor for fermentation process control. PID and 2-DOF-PID were used by means of ADALINE and retrained ADALINE which in turn resulted in RAN2PID inferential controller.

2. Modelling and Simulation

A continuous stirred bioreactor is shown in the Figure 2. The contents of the reactor are mixed thoroughly, ensuring uniform concentrations. The substrate is taken as input for the process which act as a food for growth of biomass and it's propagation. The concentration of biomass is denoted by x_1(kg/m^3) and concentration of substrate is denoted by x_2 (kg/m^3).

The dynamic model was developed and material balances on the biomass (cells) and the substrate (feed source for the cells) were written.

Accumulation rate = flow in - flow out + rate of generation

$$\frac{dVx_1}{dt} = Fx_{1f} - Fx_1 + Vr_1 \quad (1)$$

(Biomass Material balance)

x_{1f}: feed stream biomass concentration,
F: volumetric flow rate.

Accumulation rate = flow in - flow out + Rate of consumption

$$\frac{dVx_2}{dt} = Fx_{2f} - Fx_2 + Vr_2 \quad (2)$$

(Substrate Material balance)

x_{2f}: feed stream substrate concentration,

Also , $r_1 = \mu x_1$ (3)

Where,

μ : Specific growth rate coefficient

However, μ is variable and depends on the substrate concentration. The unit of μ is time$^{-1.}$

Yield is defined as:

$$Y = \frac{mass of cells\ produced}{mass of substrate\ consumed} = \frac{r_1}{r_2} \quad (4)$$

$$r_2 = \frac{r_1}{Y} \Rightarrow r_2 = \frac{\mu x_1}{Y} \quad (5)$$

Assuming a constant volume reactor, we get:

$$\frac{dx_1}{dt} = \frac{F}{V}x_{1f} - \frac{F}{V}x_1 + r_1 \quad (6)$$

$$\frac{dx_2}{dt} = \frac{F}{V}x_{2f} - \frac{F}{V}x_2 + r_2 \quad (7)$$

Defining $\frac{F}{V} = D$, the dilution rate, we find:

$$\frac{dx_1}{dt} = Dx_{1f} - Dx_1 + r_1 \quad (8)$$

$$\frac{dx_2}{dt} = Dx_{2f} - Dx_2 + r_2 \quad (9)$$

Generally, we assume that there is no biomass in the feed stream, so $x_{1f} = 0$. The bioreactor modelling equations are then written in the following form:

$$\frac{dx_1}{dt} = (\mu - D)x_1 \quad (10)$$

$$\frac{dx_2}{dt} = D(x_{2f} - x_2) - \frac{\mu x_1}{Y} \quad (11)$$

Several relations have been developed between the substrate concentration and the growth rate coefficient in order to develop a dynamic model for the control of biochemical reactor in which Monod and Substrate Inhibition Kinetics are common.

In case of Substrate Inhibition, numbers of steady state solutions are three in which two are stable but the conditions required for it are not optimum. The third steady state is highly unstable but provides optimum operating conditions. Hence there is need for designing a controller with proper parameter for stabilizing it.

Monod Model is represented by a hyperbolic function in growth rate coefficient, which is represented by equation:

$$\mu = \frac{\mu_{max} x_2}{k_m + x_2} \quad (12)$$

In substrate Inhibition, toxicity of substrate is also considered and hence is represented by the Equation 13:

$$\mu = \frac{\mu_{max} x_2}{k_m + x_2 + k_1 {x_2}^2} \quad (13)$$

In both condition we get two types of solutions which are washout (trivial) and non-trivial solutions.

Washout solution comes out to be at:

$$x_{1s} = 0; x_{2fs} = x_{2s}$$

Non-trivial steady solutions for Substrate Inhibition for substrate are obtained as:

$$x_{2s}=\frac{-(1-\frac{\mu_{max}}{D_s})\pm\sqrt{(1-\frac{\mu_{max}}{D_s})^2-4k_1k_m}}{2k_1} \quad (14)$$

And for biomass concentration, it is:

$$x_{1s}=Y(x_{2fs}-x_{2s}) \quad (15)$$

The requirement for solution to be true if:

$$D_s<\frac{\mu_{max}}{1+2\sqrt{k_1k_m}}$$

The stability of these steady states can be explained by Jacobian Matrix of the modelling equations and obtaining its Eigen Values.

The matrix so obtained is:

$$A=\begin{bmatrix}\mu_s-D_s & x_{1s}\mu_s' \\ \frac{-\mu_s}{Y} & -D_s-\frac{\mu_s' x_{1s}}{Y}\end{bmatrix} \quad (16)$$

Stability analysis for the non-trivial steady state solution tells that:

$$x_{2s}=\frac{(\frac{\mu_{max}}{D_s}-1)-\sqrt{(1-\frac{\mu_{max}}{D_s})^2-4k_1k_m}}{2k_1} \quad (17)$$

is always stable.

And,

$$x_{2s}=\frac{(\frac{\mu_{max}}{D_s}-1)+\sqrt{(1-\frac{\mu_{max}}{D_s})^2-4k_1k_m}}{2k_1} \quad (18)$$

is always unstable.

The dynamic (State-Space) Model was developed. Its linearization gives:

$$\dot{z}=Az+Bu, y=Cx \quad (19)$$

$$z_1=x_1-x_{1s}, z_2=x_2-x_{2s} \quad (20)$$

Matrix A is defined as

$$A=\begin{bmatrix}\frac{\partial f_1}{\partial x_{1|s}} & \frac{\partial f_1}{\partial x_{2|s}} \\ \frac{\partial f_2}{\partial x_{1|s}} & \frac{\partial f_2}{\partial x_{2|s}}\end{bmatrix} \quad (21)$$

Where,

$$f_1=\frac{dx_1}{dt}=(\mu-D)x_1 \quad (22)$$

$$f_2=\frac{dx_2}{dt}=D(x_{2f}-x_2)-\frac{\mu x_1}{Y} \quad (23)$$

$$\frac{\partial f_1}{\partial x_{1|s}}=\mu_s-D_s, \frac{\partial f_1}{\partial x_{2|s}}=x_{1s}\mu_s' \quad (24)$$

$$\frac{\partial f_2}{\partial x_{1|s}}=\frac{-\mu_s}{Y}, \quad \frac{\partial f_2}{\partial x_{2|s}}=-D_s-\frac{\mu_s' x_{1s}}{Y} \quad (25)$$

$$where, \mu_s = \frac{\partial \mu(x_2)}{\partial x_2}_{|s}$$

For Substrate Inhibition Model:-

$$\mu = \frac{\mu_{max} x_2}{k_m + x_2 + k_1 x_2^2} \quad (26)$$

$$\frac{\partial \mu}{\partial x_2} = \frac{\mu_{max} x_2}{k_m + x_2 + k_1 x_2^2} - \frac{\mu_{max} x_2 (1 + 2k_1 x_2)}{(k_m + x_2 + k_1 x_2^2)^2} \quad (27)$$

$$\frac{\partial \mu}{\partial x_{2|s}} = \frac{\mu_{max}(k_m - k_1 x_{2s}^2)}{(k_m + x_2 + k_1 x_{2s}^2)^2} \quad (28)$$

Matrix B is defined as

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_{1|s}} & \frac{\partial f_1}{\partial u_{2|s}} \\ \frac{\partial f_2}{\partial u_{1|s}} & \frac{\partial f_2}{\partial u_{2|s}} \end{bmatrix} \quad (29)$$

Where,

$$u_1 = D - D_s, u_2 = x_2 - x_{2fs} \quad (30)$$

$$\frac{\partial f_1}{\partial u_{1|s}} = -x_1, \quad \frac{\partial f_1}{\partial u_{2|s}} = 0 \quad (31)$$

$$\frac{\partial f_2}{\partial u_{1|s}} = x_{2f} - x_{2fs}, \frac{\partial f_2}{\partial x_{2|s}} = D_s \quad (32)$$

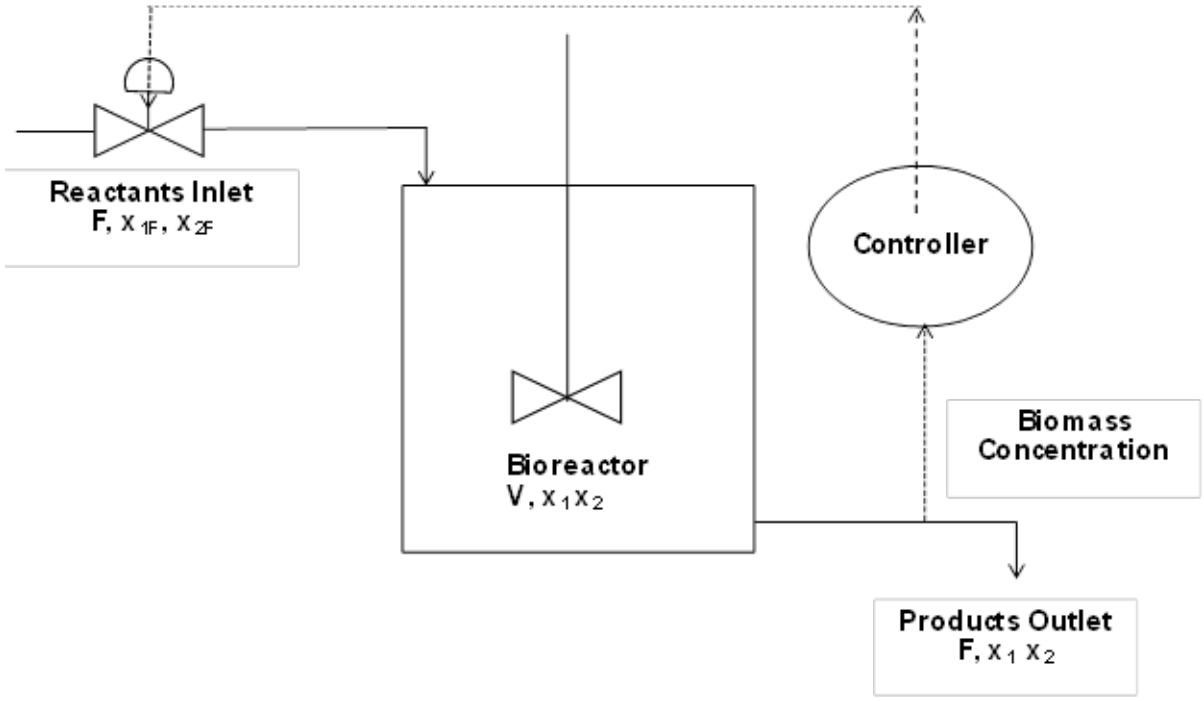

Figure 2. Block diagram of bioreactor.

Table 1. Parameters taken from experiments

Substrate Inhibition	
μ_{max}	0.53 hr^{-1}
k_m	0.12 g/L
k_1	0.4545 L/g
Y	0.4
x_{2fs}	4.0 g/L

Table 2. Steady state value at different conditions

Equilibrium 1 Washout	$X_{1s} = 0$	$X_{2s} = 4.0$	Stable
Equilibrium 2 Non Trivial	$X_{1s} = 0.9951$	$X_{2s} = 1.5123$	Unstable
Equilibrium 3 Non Trivial	$X_{1s} = 1.5302$	$X_{2s} = 0.1745$	Stable

$$B = \begin{bmatrix} -x_1 & 0 \\ x_{2f} - x_{2fs} & D_s \end{bmatrix}, C = \begin{bmatrix} c1 & 0 \\ 0 & 1 \end{bmatrix} \quad (33)$$

The parameters taken for the experiment are in Table 1.

In case of Medium Dilution Rate, i.e, Ds = 0.3, results shown in Table 2 are obtained by solving the steady state value in MATLAB.

3. Results and Discussions

Studies of tuning of PID controller have been done to control the biomass concentration in bioreactor by manipulating the dilution rate. The process transfer function of bioreactor is unstable second order system, which was reduced to first order unstable system with negative process gain. To control the unstable process, IMC based PID controller is used. Three methods of design of IMC-PID controller are selected from the literature Tables 3 and 4. The tuning parameter 'λ' was selected on the basis of responses of unit step-change in set point. Its performance was compared in the terms of integral time error (IAE, ISE, and ITAE) settling time (Ts), % overshoot (% OS), rise time (Tr), as shown in Figure 1. Transfer function of bioreactor linking dilution rate with biomass concentration at unsteady state point can be written as[50–52].

$$G_p = \frac{-0.9951s - 0.2985}{s^2 + 0.1302s - 0.0509} \quad (34)$$

Implementing the process model in gain and time constant form (and removing the common poles and zeros), the following unstable first order process transfer function was found:

Table 3. PID parameters using three different tuning techniques for process transfer function

Methods	K_c	T_I	T_D	λ
Rotstein and Lewin (1991) [51]	-5.2	0.8272		0.4
Shamsuzzoha et al. 2013 [52]	-6.8640	0.6158		0.3
Shamsuzzoha and Lee (2007) [53]	-7.2535	0.4919	0.00446	0.3

Table 4. Performance of different IMC based PID controller

Methods	IAE	ISE	ITAE	T_s (s)	T_r(s)	%OS
(Rotstein and Lewin, 1991) [51]	0.3074	0.1003	0.2115	2.5	0.4	15
(Shamsuzzoha et al. 2013) [52]	0.2067	0.06883	0.08909	1.5	0.25	16
(Shamsuzzoha and Lee, 2007) [53]	0.2282	0.07519	0.1164	1.8	0.28	15

$$G_p = \frac{5.8644}{-5.888s + 1} \quad (35)$$

(Bequette, 2012)

4. Conclusions

The mathematical model of a biochemical reactor exhibiting substrate inhibition kinetics has been developed. The steady state behaviour of the biochemical reactor was studied. It was seen that at dilution rate D = 0.3 unit, the unstable steady state obtained was stabilized with the help of control parameters given in. Afterwards, the dynamic behaviour of the reactor was studied. From the performance point of view, the IMC-PID controller parameter obtained using has better results than the other two tuning techniques. But has slightly greater percentage overshoot.

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