

# An Application of Fractal-Based Lightning for SFR Calculation of High Voltage Substations

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## Abstract

**Objectives:** To simulate the lightning more close to reality and investigate the performance of the lightning protection system of high voltage substation. **Methods/Statistical Analysis:** The method which is used for the system analysis is a new fractal-based model of simulated lightning by which the zigzag movement of lightning downward leader as well as branching nature is simulated. A new charge distribution is proposed for branched lightning channel. In addition, the branch fading during the lightning downward movement is also simulated in order to simulate the lightning more close to the reality. **Findings:** The results of the simulated lightning using the proposed method show that the proposed fractal method simulates the lightning close to the reality. The striking distances of a transmission line structure for different lightning return stroke currents are obtained by the proposed method and compared to those of Electrogeometric Model (EGM) and Leader Progression Model (LPM). The results show that how the proposed method obtains a wide range of possible results statistically. The lightning shielding system performance of a practical high voltage substation is investigated based on the proposed model in two different scenarios by which the effect of substation instruments on the lightning protection system is investigated. The results reveal that the presence of the substation instruments completely affects the shielding system performance. Using the propose method, the random behavior of the lightning is well modeled and the shielding failure of the protection system is investigated in a statistical manner. The obtained results are compared to those of EGM, and LPM. The results are discussed and superiority of the proposed simulated lightning model is concluded. **Application/Improvements:** The proposed method simulates the different features of the lightning including 1- branching nature, 2- zigzag movement, 3- random behavior, 4- branch fading, 5- branched charge distribution.

**Keywords:** Fractal Concept, High Voltage Substation, Lightning, Shielding Failure

## 1. Introduction

Direct lightning strokes to the transmission lines and high voltage substations, as two vital components of power systems, are a main cause of interruptions in power systems<sup>1-3</sup>. Therefore, consideration of shielding system against direct lightning strokes is a key factor in designing of these two important parts of the system. Because of several transmission lines connection, and the presence of high cost instruments in a substation, the shielding of high voltage substations is far more important than the transmission lines<sup>4</sup>. Thus, a high accurate method for lightning shielding system designing and analysis is needed.

The early methods used for lightning shielding system designing were Geometrical methods in which a locus is considered for each object. If the lightning enters to a locus, the corresponding object would be struck by the lightning<sup>5</sup>. Since these methods did not consider lots of lightning phenomena, a new group of methods known as Electrogeometrical Model (EGM), which are based on the concept of striking distance as a function of lightning current, were introduced and developed based on lots of observations and experiments<sup>6-10</sup>. However, because of unpredictable behavior of lightning, the SFR predicted by EGM was associated with large errors, especially for the high voltage structures<sup>1</sup>. A more physical method known as Leader Progression Model (LPM) was introduced in<sup>8</sup>.

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In this method, the downward leader movement, the lightning leader channel, the upward leader inception and propagation and lots of other parameters are considered. However, the most important issue related to this method is that the next jump of the downward leader in the space is selected based on the minimum absolute value of potential of points which can be considered as the next jump<sup>11</sup>. It means no branching is considered in the simulations. Moreover, the tortuosity of lightning is not modeled very well using LPM<sup>12</sup>. In addition, unique results are obtained for a specified situation in different simulations. This is not coincident with natural behavior of lightning which has a completely different behavior in even a same situation.

Based on observations and experiments, lightning has a random behavior even in a specific situation<sup>13,14</sup>. This is because of probabilistic movement of lightning downward leader which is a result of some various uncertain factors such as background noise, and spatial electric field<sup>12</sup>. This characteristic of lightning is not modeled by the foregoing methods.

A probabilistic simulation of breakdown based on fractal geometry was introduced by Niemeyer and Wiesmann for gas dielectric<sup>15,16</sup>. The model was used for other dielectrics in<sup>17,18</sup>. In this method, the next trajectory of the leader is selected randomly based on the concept of fractal geometry. The model was first used for lightning simulation in<sup>19</sup>. By this method, the tortuosity and the branching nature of lightning is well modeled. Moreover, different trials of lightning simulation using the fractal method lead to different results, which is coincident to the reality of lightning. A controversial issue related to this model is the charge distribution along the lightning leader channel. In most of previous works, a simple model such as uniform model is used in<sup>12</sup> and<sup>20</sup>. A high accurate charge distribution was proposed by<sup>21</sup>. However, this model was introduced for single branch lightning<sup>22</sup>. Proposed a charge distribution for branching channel based on a linearization of Cooray model. The distribution is a function of branches leader tip height and length.

A new model for charge distribution of branching downward leader is proposed. The model is based on Cooray model without any linearization. Moreover, the probability of branch fading is also taken into account. The proposed model is applied on striking distance calculation of a transmission line and comparing the results with those of LPM, and different models of EGM. Moreover, the model is employed for investigation of

lightning shielding system performance of a practical high voltage substation in the presence of almost all the instruments. The results are discussed and compared to those of LPM and different models of EGM. The superiority of the proposed model can be concluded from the results comparisons and discussions.

## 2. Proposed Model

### 2.1 Fractal-Based Stochastic Lightning Model

The question of calculating the length of coastline could be considered as the first spark of fractal concept. In 1961, Lewis Fry Richardson presented an equation based on power law for calculation of coastline according to the scale of map. After that, the concept of fractal geometry was introduced by<sup>23</sup>. This concept was employed for Dielectric Breakdown Model (DBM) for gas dielectric<sup>15,16</sup> and some other dielectrics<sup>17,18</sup>.

For the next jump determination of the lightning using fractal model, a probability for each potential point is considered as follow:

$$p(i,j) = \begin{cases} \frac{(|E(i,j)| - E_{crit})^i}{\sum_{k=1}^{N_p} (|E(i,k)| - E_{crit})^i} & |E(i,j)| \geq E_{crit} \\ 0 & |E(i,j)| < E_{crit} \end{cases} \quad (1)$$

where  $i$  denotes for the leader tip point,  $j$ , and  $k$ , denote for the potential points beneath the downward leader,  $p(i,j)$  is the probability of potential point  $j$  to be selected as the next jump,  $E(i,j)$ , and  $E(i,k)$  (in kV/m), are the average electric field strength along the line connecting the leader tip point  $i$  to the potential points  $j$ , and  $k$ , respectively,  $E_{crit}$  (in kV/m), is the air critical breakdown electric field,  $N_p$  is the total number of potential points, and  $\eta$  is the developing probability index. The breakdown electric field of atmosphere is different in different height from the ground level as follow<sup>24</sup>:

$$E_{crit}^h = E_{crit}^0 \exp(-h/5) \quad (2)$$

where  $E_{crit}^0$  is the electrical breakdown in the ground level and  $h$  (in km), is the corresponding height.

### 2.2 Fractal Dimension

The parameter  $\mu$  determines the number of branches of simulated lightning. For bigger values of this parameter,

the electric field effect would be great and the number of branches is fewer compared to the cases with lower values of  $\mu$  where the next jump position would be selected more randomly. To choose an appropriate value of  $\mu$ , the dimension of fractal should be calculated. Fractal dimension is an important parameter in fractal simulation. The most well-known method for calculation of the dimension is Box-Counting which is as follow:

$$N(r) = \left(\frac{1}{r}\right)^D \quad (3)$$

where  $D$  is the fractal dimension,  $r$  is the measurement scale and  $N(r)$  is the number of lines, boxes, cubes, or any other geometrical shapes which cover the pattern. By plotting the  $\log(N(s))$  against  $\log(1/s)$ , the dimension can be calculated obtaining the slope of the plot as it represented in Equation 4. The fractal dimension means the value  $D$  is not an integer.

$$D = -\lim_{r \rightarrow 0} \frac{\log(N(r))}{\log(r)} \quad (4)$$

### 2.3 Potential Points for Next Jump

Determination of trajectory of downward leader in each step in the space is an important factor in lightning simulation. This part plays an important role in modeling of tortuosity and branching nature of simulation lightning. A 3-D model of next jump determination of lightning is proposed in<sup>11</sup>, where a hemisphere is drawn beneath the leader tip with the center of leader tip and the radius of next step length. Based on<sup>11</sup> and considering the stochastic nature of lightning, in the proposed model, the length of next step is calculated by the following equation.

$$L_l^i = \left(\frac{1}{k}\right) \times h_{lt} (1 + 0.1 \times \text{rand}) \quad (5)$$

where  $L_l^i$  is the leader length in step  $i$ ,  $h_{lt}$  is the leader tip height and  $\text{rand}$  is a random number in the range of  $[0, 1]$ . The length of leader must be in the range of 10m to 100<sup>25</sup>. Whenever it violates, the boundaries are applied.

The next jump would be selected among some potential points on the surface of the hemisphere based on the absolute value of electric field between the leader tip and the potential points. For a better simulation of lightning tortuosity, the potential test points are selected randomly based on Figure 1, as follow:

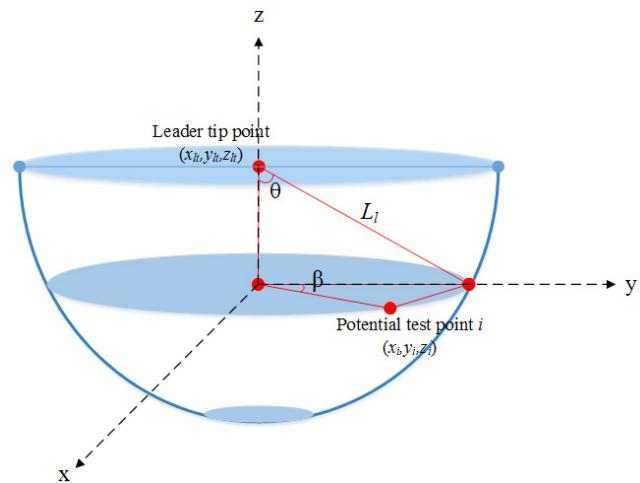
$$z_i = z_{lt} - L_l \times \cos \theta \quad (6)$$

$$y_i = y_{lt} \pm L_l \times \cos \beta \quad (7)$$

$$x_i = x_{lt} \pm L_l \times \sin \beta \quad (8)$$

$$\theta = (0^\circ, 30^\circ, 60^\circ) \pm 5^\circ \times \text{rand} \quad \beta = 360^\circ \times \text{rand} \quad (9)$$

where  $\text{rand}$  is a uniform random variable in the range of  $[0, 1]$ . To consider the repulsive electrostatic forces, a potential point close to another branch is discarded<sup>21</sup>. In this paper, whenever the distance of a potential point to a leader tips is less than the leader tip next step length, the potential point discarded.



**Figure 1.** Potential test point determination.

### 2.4 Charge Distribution along the Channel

As mentioned before, charge distribution of lightning channel is a controversial issue. The charge distribution of the downward leader increases through the channel towards the leader tip<sup>21</sup>. Uniform, linear decay and exponential decay are some of well-known charge distribution introduced so far. However, these models are not very accurate. Cooray introduced an appropriate charge distribution model in which the characteristics of channel charge distribution are satisfied. This model which is suitable for single branch lightning is a function of the lightning current, leader tip height and the height of cloud as follow:

$$\rho(\xi) = a_0 \left(1 - \frac{\xi}{H - z_0}\right) G(z_0) I_p + \frac{I_p (a + b\xi)}{1 + c\xi + d\xi^2} H(z_0) \quad (10)$$

$$Z_0 >= 10$$

$$G(z_0) = 1 - \left(\frac{z_0}{H}\right)^{\alpha} \quad (11)$$

$$H(z_0) = 0.3\alpha + 0.7\beta \quad (12)$$

$$\alpha = e^{-(z_0 - 10)/75} \quad (13)$$

$$\beta = \left( 1 - \frac{z_0}{H} \right) \quad (14)$$

where  $a_0 = 1.476 \times 10^{-5}$ ,  $a = 4.857 \times 10^{-5}$ ,  $b = 3.909 \times 10^{-6}$ ,  $c = 0.522$ , and  $d = 3.73 \times 10^{-3}$ .  $\rho(\xi)$  is the charge density [C/m],  $\xi$  is the length along the leader channel ( $\xi=0$  for the leader tip),  $z_0$  is the height of downward leader tip [m],  $I_p$  is the lightning return stroke current [kA], and  $H$  is the height of cloud. Thus, the charge density can be written as follow:

$$\rho(\xi) = f(\xi, z_0, I_p) \quad (15)$$

In the case of branching lightning, the values for leader tip height and branch lightning current are not unique values. A charge distribution model for branched channel was introduced by Shi et. al. based on the linearized approximation of Cooray model in<sup>21</sup>. The model calculates the charge distribution of the channel based on the branch leader tip height and the length of the branch.

In this paper, a new model for charge distribution of leader channel is proposed based on Cooray model without any linearization as follows:

$I_p$  is a parameter which  $\rho$  depends on. Thus, first of all, the amount of lightning current flows through a branch must be calculated. As the downward leader goes down, the lightning current divides in the different branches as depicted in Figure 2. However, it is difficult to found the exact value of this current. In order to calculate the amount of lightning current of each branch in this paper, it is assumed that the current flows into the branches based on fractal model as follows:

$$I_{p123...ij} = I_{p123...i} \times X_j \quad (16)$$

where  $X_j$  is the current portion parameter which determines the percentage of upper branch lightning current flows to a lower branch. This parameter can be calculated using Equation 17 based on fractal approach:

$$\chi_j = \frac{j_j^\mu}{\sum_{k=1}^{N_{ps}} j_k^\mu} \quad (17)$$

where  $\Phi_j$  is the electric potential of point  $j$  and  $N_{ps}$  is the number of candidate potential test points which are selected as the next jump position.

Besides the branch's lightning current determination, the other important step of proposed model is to determine the leader tip height which is an important parameter

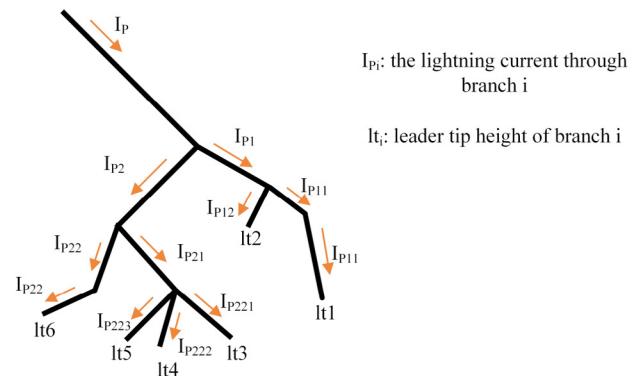


Figure 2. Branching downward leader.

in Cooray model. As it can be seen from the Figure 2, there is more than one value for leader tip height. Thus, it is an important problem to consider an appropriate value for leader tip height for obtaining the charge distribution charge of each branch. In this paper, first of all, the corresponding leader tips for each branch are determined. For more explanation, referring to Figure 2, in the charge distribution calculation of branch 1, all the leader tips are integrated. But for the branch with the lightning current of  $I_{p21}$ , only the leader tips  $lt3$ ,  $lt4$ , and  $lt5$  are integrated. In order to identify the corresponding leader tips for each branch, the Dijkstra's path-finding algorithm is employed. Using the algorithm, the path between the receiving end of a branch and a leader tip is found. The leader tip would be corresponding to the branch, if the finding path is a downward path. In other words, the path which goes up from the receiving end to the upper nodes and after that reaches the leader tip is not accepted. As the corresponding leader tips of the branches are obtained, based on the superposition principle, the charge density of a branch can be calculated as follow:

$$\rho_b^k(\xi) = \sum_{lt=1}^{N_{lt}^{cb}} \zeta_{lt}^b f(\xi, z_{lt}^b, I_p^b) \quad (18)$$

where  $z_{lt}^b$  is the height of leader tip  $lt$ ,  $I_p^b$  is the lightning current of the branch  $b$ , and  $N_{lt}^{cb}$  is the total number of corresponding leader tips to the branch  $b$ .  $\zeta_{lt}^b$  is the parameter describes the percentage of effectiveness of leader tip  $lt$  to the charge density of branch  $b$ . This parameter can be calculated as follow:

$$\zeta_{lt}^b = \frac{j_{lt}}{\sum_{k=1}^{N_{lt}^{cb}} j_k} \quad (19)$$

It must be highly noted that whenever a branch does not have a corresponding leader tip in the current step, it means there was no further movement in the previous step; thus, this branch must be eliminated.

## 2.5 Branch Fading

One of the features of the lightning which is shown by high speed cameras is fading branches as the downward leader approaches the ground. During the lightning downward leader movement, some branches are started from the main channel and disappeared after a while. It is difficult to apperceive the physical reason of this phenomenon. As a theory, if the charge of leader tip decreases to a value lesser than the critical value (for example  $1\mu\text{C}$ ), no air breakdown and in a consequence, no further movement occur. Since, the branch fades and the charge along the branches disappeared in the space. In this paper, the branch fading is also simulated in the proposed stepped leader model. Whenever the leader tip charge of a branch becomes lesser than a critical value (here  $1 \mu\text{C}$ ), the branched would be eliminated in the next step of simulations. It should be also mentioned that since there is no further movement of the faded branch, all the branches which connect the faded branch to the main channel are also eliminated in the further steps (i.e., no corresponding leader tip would be found for these branches in the next steps).

## 2.6 Upward Leader Modeling

As the downward leader approaches to the ground, the congestion of positive charges increases on the sharp points dramatically. This causes increase in electric field and air breakdown as a consequence. Therefore, an upward leader starts. If the upward leader formed from a point is sustainable, the point would be the lightning striking point. The model of upward leader, the inception and formation required condition, and the criterion of stable upward leader identification are based on the model proposed by Becerra<sup>26</sup>.

To sum up, the following features can be considered for the proposed model, 1- The physical phenomena of the lightning as the downward leader moves down are modeled, 2- The effect of lightning channel charge is considered, 3- The effect of background electric field of the cloud charge is also taken into consideration, 4- Using the proposed potential points determination, the tortuosity of the lightning is well modeled, 5- Using

the fractal approach, the branching nature of lightning is simulated, 6- Considering the leader tip charge, the branch fading feature of actual lightning is modeled in the simulation, 7- Employing the fractal approach, the stochastic nature of lightning is modeled (different results for different trials in a specific situation), 8- The upward leader formation and propagation are considered for striking point determination.

The flowchart of the proposed model is depicted in Figure 3.

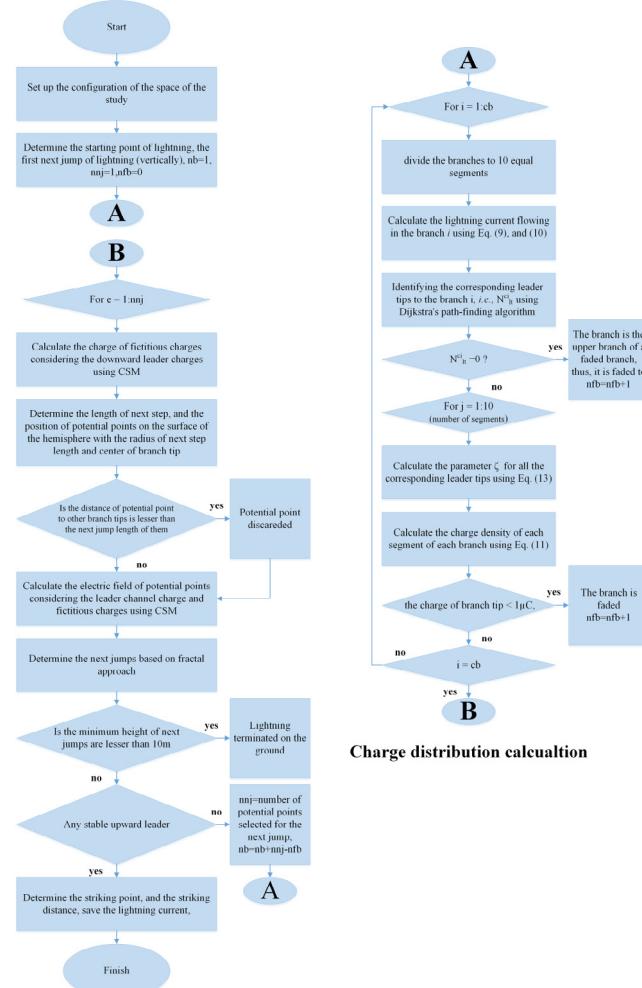
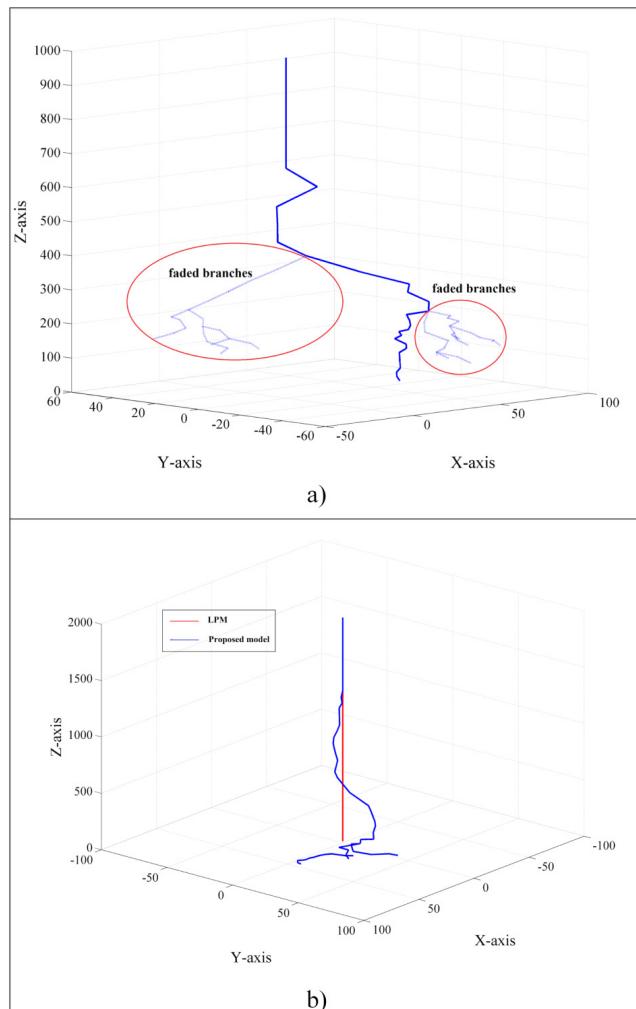


Figure 3. Flowchart of proposed lightning model.

## 3. Evaluation of the Proposed Model

In order to evaluate the proposed approach, some simulated lightning using the proposed model are depicted, an appropriate value for parameter  $\mu$  is selected based on the

fractal dimension, and the striking distance of a transmission line is calculated by different methods and compared.



**Figure 4.** Simulated lightning by the proposed model a) branch fading, b) tortuosity.

### 3.1 Simulated Lightning

As mentioned before, the tortuosity and branching nature of lightning are modeled by the proposed model. Moreover, the branch fading feature of the lightning is simulated using the proposed model. Figure 4, presents two lightning simulated by the proposed model. In the Figure 4, a) the branch fading simulation is presented. As it can be seen, some of the branches are diminished in the further steps. This is coincident with the natural lightning. In second figure, a downward leader is simulated by LPM and the proposed model. The lightning is started exactly from the center of the cloud and there is no structure on the ground. Since the situation is completely symmetrical,

the lightning which is simulated by LPM is completely vertical. However, because of random nature of lightning trajectory, a vertical movement is rarely occurred. On the contrary, the tortuosity and the branching nature of the lightning in a same situation are well modeled using the proposed model. Although the situation is completely symmetrical, the stochastic nature of the lightning movement makes it branched and zigzag.

### 3.2 Fractal Dimension

As mentioned before, the fractal dimension is highly depends on parameter  $\mu$ . 10 different trials of lightning are simulated by the proposed model for a specific situation and for different values of  $\mu$ . The results are listed in Table 1. Based on the studies of<sup>[19,27]</sup>, the fractal dimension of actual lightning lies on the range of [1.1 1.4]. It seems the value of 1.25 is an appropriate value for the fractal dimension. Thus, the parameter  $\mu$  is considered 2 in this paper.

**Table 1.** Statistical characteristics of fractal dimension for different values of  $\mu$  without any branch fading

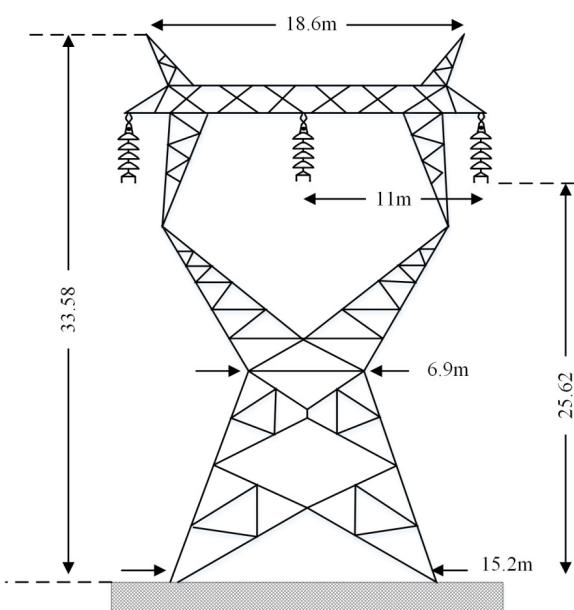
| $\mu$ | Min    | Max    | Average | STD (%) |
|-------|--------|--------|---------|---------|
| 0     | 1.2899 | 1.4416 | 1.3447  | 5.31    |
| 1     | 1.2629 | 1.3215 | 1.2812  | 2.08    |
| 2     | 1.2316 | 1.2656 | 1.2534  | 1.15    |
| 6     | 1.1869 | 1.2299 | 1.2114  | 1.33    |
| 9     | 1.1085 | 1.2205 | 1.1851  | 3.36    |

### 3.2 Striking Distances

In this part, the striking distances of a transmission line structure is calculated for different values of return stroke current by different methods including proposed model. The transmission line structure is the same with that illustrated in<sup>[1]</sup>. The length of the span and the sag of the conductors are considered the same with the structure of<sup>[1]</sup>. The tower characteristics are depicted in Figure 5. The values of striking distances obtained by LPM and different models of EGM are unique. In other words, different trials do not affect the value of striking distance obtained by these methods. However, the lightning has a random behavior even in a completely same situation. Therefore, the performances of the LPM and different models of EGM are not coincident with the reality of lightning.

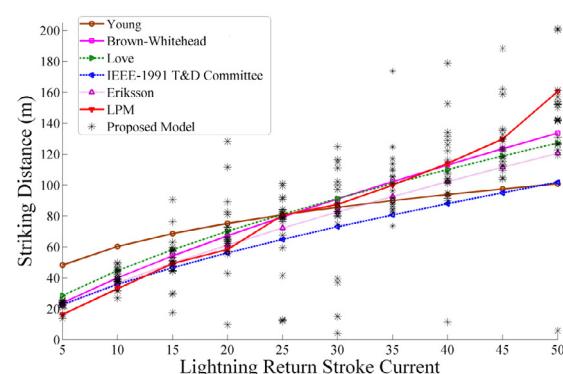
The striking distances obtained by 20 trials of the proposed model for different lightning return stroke

currents, are depicted and compared with those of LPM and different models of EGM in Figure 6. As it can be seen, different values are obtained for different trials of simulations, but in a reasonable range. The out of range values (very high or very low value) for striking distances may be because of two main reasons. 1- Because of stochastic parameter in the simulation model, the lightning may move away from the structure and terminated on the ground. In this situation, since there may be no test point at which the upward leader inception is checked, the lightning height reaches the value lower than 20 m. In the simulation model, if the lightning height reaches the value of lower than 20 m, it is assumed that the lightning terminated at the ground and the striking distance is considered the height of leader tip before the last step. Thus, the low value of striking distance is obtained. 2- Because of tortuosity of the lightning simulated by the proposed model, the side flashes are also may occur. In the case of side flashes, the striking distances may be much higher than the strikes when lightning strikes to the object from above. Since the lightning movement is considered completely vertical in different models of EGM the side flash could not be studied using these models. The tortuosity of lightning is not simulated very well by LPM, which results in rare occurrence of side flashes. However, since the zig-zag movement and branching nature of lightning are well modeled using the proposed model, the side flashes are also investigated by the proposed model.



**Figure 5.** Tower configuration.

To sum up, it can be stated that none of the models introduced for lightning performance investigations have the same results in a specific situation. This is also concluded in<sup>28</sup>, and can be considered because of stochastic nature of the lightning. As one can see, none of the models (EGM models, and LPM) can simulate the stochastic nature of lightning, while, the proposed model simulates this important feature of the lightning. In other words, a bigger range of conceivable striking distances can be obtained using the proposed model. This is a big positive point of proposed model by which the stochastic nature of lighting performance can be also investigated.



**Figure 6.** Striking distance of a transmission line obtained by different methods.

#### 4. SFR Calculation of Substation

In this section, the performance of a high voltage substation system protection against lightning is investigated using the proposed model. Since the substations are associated with lots of instruments, investigation of lightning system protection is a demanding work. In this paper, a practical 400/132 kV substation known as Sarbedaran substation located in Sabzevar, Iran, is studied. The protection system of this substation against direct lightning strokes is investigated using LPM in<sup>29</sup>. In this paper, the protection system of this substation is investigated using the proposed model which is more accurate than LPM. Since the electric field in the space of study is calculated by Charge Simulation Method (CSM), different parts of the system must be modeled by fictitious charges. The configuration of the substation and also the simulation model of different apparatuses as fictitious charges are illustrated in Figure 7<sup>29</sup>.

The investigation of the system protection is performed in two scenarios. In the first scenario, the

substation instruments are not considered in the study and the protection system investigated and the results are compared to those of LPM and different models of EGM. In the second scenario, almost all the substation apparatuses are modeled in the simulations and their effects on shielding system performance are investigated. The results of the proposed model for these two scenarios are compared with those of LPM. It should be mentioned that the influence of the substation instruments on SFR of substation could not be studied using the EGM.

Since the electric field and potential in the space of study are calculated using CSM, all the structures must be modeled by the fictitious charges such as ring charges, line charges, or segment ring charges. The Sarbedaran substation layout and the simulation model of different apparatuses are extracted from<sup>29</sup>. It should be mentioned that the under study area is considered pollutant so that charges can be appeared on the insulators through creepage.

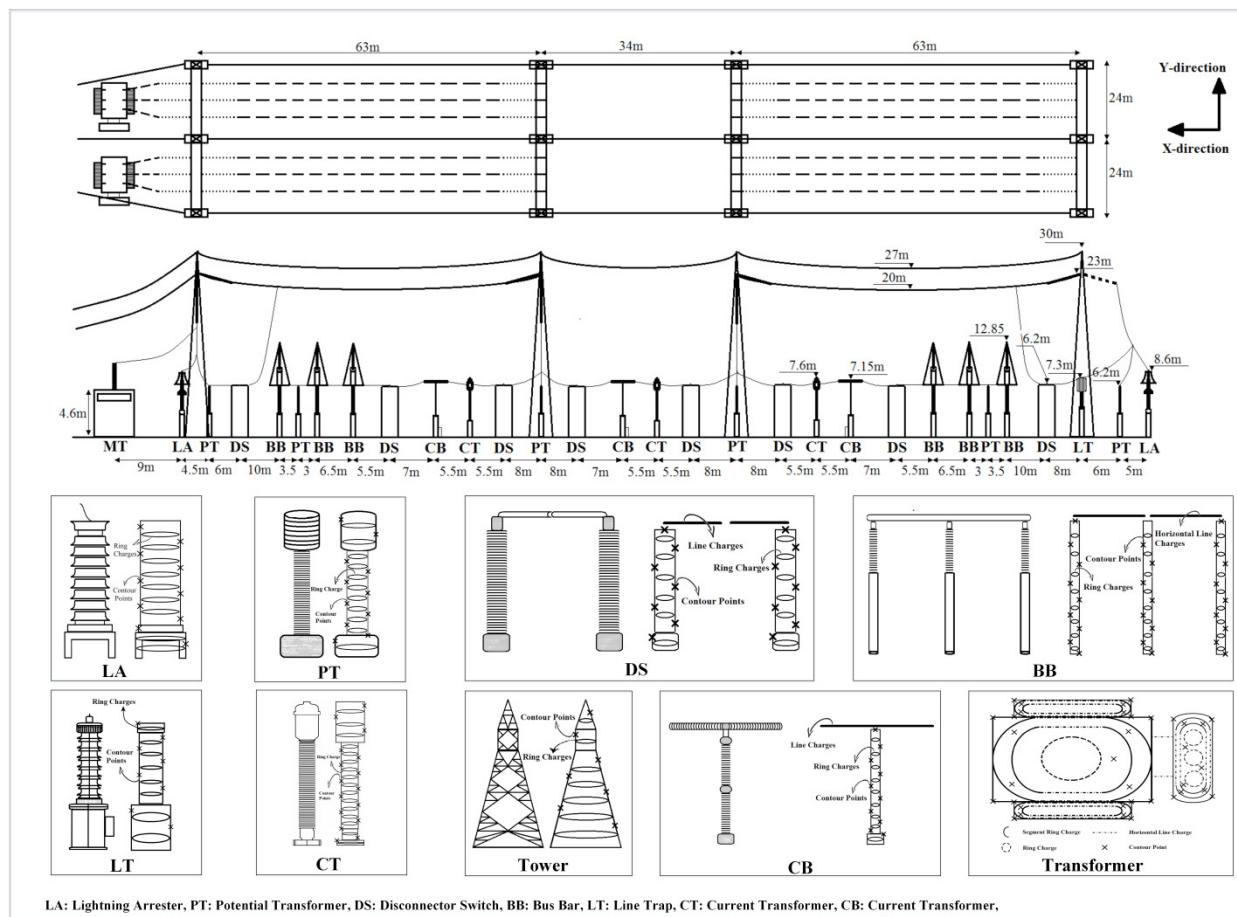
For the calculation of SFR of the substation using the proposed model and LPM, an extended meshed area

above the substation is defined like what performed in<sup>11</sup>. The dimension of the meshed area is considered so that any lightning started out of this area would defiantly terminate to the ground not to the substation area. Since the substation is symmetric in Y-axis, only  $\frac{1}{2}$  of meshed area is investigated.

For calculation of the SFR, for each mesh of the meshed area, the lightning with different return stroke current are simulated till the final jump and striking point determination. The lightning currents are considered from 3 kA to 35 kA with the step of 0.5 kA. For each mesh with the dimension of  $dx \times dy$ , the SFR is calculated using the following equation<sup>29</sup>:

$$SFR_i = 0.1 \times GFD \times \left( \frac{dx}{1000} \right) \times \left( \frac{dy}{1000} \right) \times [P(I_{max}) - P(I_c)] \quad (29)$$

where  $dx$  and  $dy$  are the mesh dimensions in m,  $I_{max}$  and  $I_c$  are maximum and minimum lightning current which cause shielding failure,  $P(I)$  is the probability distribution function lightning current exceeding from the value  $I$ , and  $GFD$  is the flash ground density of the area. The values of  $P(I)$  and  $GFD$  can be calculated as follows:



**Figure 7.** Configuration of Sarbedaran substation and the fictitious charge model of different apparatuses.

$$P(I) = \frac{1}{1 + \left(\frac{I}{31}\right)^{2.6}} \quad (21)$$

$$GFD = 0.04 \times T_d^{1.25} \quad (22)$$

$T_d$  is the number of thunderstorm day in the under-study area. For the investigation of lightning protection system of substations,  $T_d$  is considered two times the actual value of thunderstorms day of the area<sup>13</sup>. Thus, the total SFR can be calculated as follow:

$$SFR = 2 \times 2 \times \sum_{i=1}^{N_m} SFR_i \quad (23)$$

The dimensions of Sarbedaran substation is 48×180 m. Based on different EGM models,  $r_g$  (the distance out of which the lightning terminates at the ground) for the lightning current of 25 kA (a good approximation of maximum lightning current which causes shielding failure) is 70m. Thus, the meshed area is considered to be  $(70+48+70) \times (70+320+70)$  m. This area is divided into

120 meshes ( $8 \times 15$ ). The number of thunderstorm days i.e.,  $T_d$  is 5 based on<sup>30</sup>.

First of all, the number of shielding failure for the sub-station based on different models of EGM is investigated. Based on the height of shield wires and also the towers, the minimum protection height of the system protection for different values of lightning currents are listed in Table 2. As it can be seen, the minimum protection height never becomes lesser than 27.5m. Since the phase wires are placed at the height of 23m, perfect protection is achieved. This means all the apparatuses are protected.

As mentioned before, the effect of instruments could not be investigated using the EGM models. Now, the protection system is investigated using LPM and the proposed model. For this purpose, the simulations are performed in two scenarios. In the first scenario, only the shield and phase wires, the towers, the main transformer, and the bus bars are considered in the space of study. In the second one, in addition to the mentioned objects, other apparatuses are also simulated by fictitious charges.

**Table 2.** Minimum protection height of Sarbedaran substation for different lightning currents by means of different models of EGM

| Models [13]     | Minimum protection angle for different lightning current |      |      |      |      |      |      |
|-----------------|--|------|------|------|------|------|------|
|                 | 10kA   | 15kA | 20kA | 25kA | 30kA | 35kA | 40kA |
| Young           | 28.7   | 28.9 | 29.0 | 29.1 | 29.1 | 29.2 | 29.2 |
| Love            | 28.4   | 28.7 | 29.0 | 29.1 | 29.2 | 29.3 | 29.3 |
| Brown-whitehead | 28.2   | 28.7 | 28.9 | 29.1 | 29.2 | 29.3 | 29.4 |
| IEEE 1992       | 28.6   | 28.9 | 29.1 | 29.2 | 29.3 | 29.4 | 29.4 |
| IEEE 1995       | 27.9   | 28.4 | 28.7 | 28.9 | 29.0 | 29.1 | 29.2 |

**Table 3.** Results of shielding system investigation of Sarbedaran substation using LPM, and the proposed model

| NT             | LPM [34] |      |        |      |      |      |      |
|----------------|----------|------|--------|------|------|------|------|
|                | NSS      | NF   | NSB    | Ic   | Imax | SFR  |      |
| Scenario 1     | 3900     | 2322 | 50     | 0    | 6.5  | 25   | 2    |
| Scenario 2     | 3900     | 2420 | 52     | 0    | 6    | 26.5 | 2.1  |
| Proposed Model |          |      |        |      |      |      |      |
| Scenario 1     | Min      | 3900 | 2849   | 49   | 0    | 3.5  | 27   |
|                | Max      | 3900 | 2925   | 61   | 0    | 6    | 34.5 |
|                | Mean     | 3900 | 2887.4 | 56   | 0    | 5.2  | 31.2 |
| Scenario 2     | Min      | 3900 | 2882   | 49   | 0    | 4    | 24   |
|                | Max      | 3900 | 2925   | 54   | 0    | 6.5  | 30   |
|                | Mean     | 3900 | 2897   | 51.6 | 0    | 5.7  | 27.4 |

The lightning simulation of proposed model is performed for 10 different trials. The results of protection system performance analysis are listed in Table 3.

In<sup>29</sup> the authors concluded that the effect of substation instruments causes increase in the number of lightning strokes to the substation and also number of shielding failures. The authors said that this is because of positive charges beneath the downward negative lightning because of the presence of substation instruments. This seems to be reasonable. However, as it can be seen, there is no big difference between the results of two different scenarios. In other words, although the effect of substation apparatuses can be investigated using LPM, since the branching nature and tortuosity movement of the lightning are not well modeled, the lightning has approximately a vertical movement and the effect of substation instruments are not revealed very well.

On the other side, the simulated lightning by means of proposed model is a branched lightning with zigzag movement. Since the proposed model is based on fractal approach, the stochastic nature of lightning is also taken into consideration. In the first scenario where the substation instruments are not considered, the random part of simulation plays a big role in determination of next jump of the downward leader especially in the lower heights. This is due to close values of the electric field of potential points beneath the leader tip. On the contrary, in the presence of substation instruments, the large numbers of positive charges related to these apparatuses affect the electric field of potential points beneath the leader tip and the next jump of the leader is determined mostly based on the maximum electric field. In other words, when the substation apparatuses are not considered, the lightning performance is more stochastic in comparison to the scenario in which the substation instruments are taken into consideration. Thus, the number of lightning branches, the trajectory of downward leader, the number of upward leader inception from the objects, and the striking point are determined more randomly in the first scenario. This phrase can be concluded from the results of Table 3 where the minimum and maximum number of shielding failure of scenario 1 has a bigger difference compared to the scenario 2. Another point is that the results of proposed model in the second scenario are close to those of LPM. It means in the presence of substation instruments the lightning simulated by the proposed mode has a more deterministic behavior regards to the case where no instruments are considered in the substation.

Another important point which is concluded by the simulation of proposed model is the stochastic simulation of lightning. As it can be seen from the Table 3, Scenario 2, the number of shielding failure could be as high as 2.98 which is 28% higher than the mean value. This shows that a high value of SFR is possible. Thus, using the proposed method, the worst situation could be investigated and based on the occurrence of this situation an excellence protection system could be drawn. This is a great feature of the proposed model by which the stochastic nature of lightning could be studied. The previous models are not able to simulate the performance of the protection system against direct lightning stochastically.

Based on the simulation results, the important merits of proposed model can be listed as follow:

- The tortuosity of lightning is modeled in a good way,
- The branching nature of lightning is well modeled,
- Different trials of simulation lead to different results (stochastic nature),
- Branching fading is simulated,
- An appropriate value of  $\mu$  is selected in order to simulate the lightning close to reality,
- The accuracy of the proposed method is acceptable.
- Using the proposed model, a wide range of possible results are obtained. Thus, a better protection system could be designed using the proposed method.

## 5. Conclusion

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In this paper a new simulation model for lightning is proposed based on fractal approach. The tortuosity and branching nature of lightning are well modeled using the proposed model. Moreover, a new charge distribution model for branching downward leader is introduced. The effect of branch fading is taken into consideration. The evaluation of the proposed model exhibits that the proposed model simulates the lightning close to the reality. The protection system of a practical substation against direct lightning is investigated by means of mention model name model. The effect of substation instruments is analyzed. The main conclusions of this study are, 1- The presence of substation instruments may surely affect the lightning protection system performance. 2- Lack of consideration of substation instruments decreases the

reliability of the results obtained by the simulation methods, 3- The investigation of lightning system protection should be performed statistically, 4- The proposed model give the investigators a good sight on the performance of the lightning protection system performance, 5- The proposed model simulates the tortuosity, branching nature, faded branches, and stochastic nature of lightning. As a final Results, the simulation of lightning should be performed statistically and the space of under study should be modeled in detail.

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