

Parametric Study of the Pressure Profile in a Nozzle as a Function of Compressible Flow Parameters

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Abstract

Background/Objectives: To take into an account the effects of compressibility when studying the flow of any fluid, it is necessary to define a series of additional parameters in the nozzle analysis, which is a complex job to do in a manual strategy, leaving as a suitable option the use of numerical methods programmed in a commercial software (such as MATLAB®). **Methods:** Variations were made in compressible flow parameters, such as initial pressure and temperature, and the Mach number, to study their influence over the pressure profile of the air inside a nozzle. **Findings:** The results show a low influence of the temperature over the pressure profile, a medium to high influence from its intake pressure, and a close relationship with the Mach number, getting even higher pressure drops when it only increases a bit. **Application:** To predict the behavior of the fluid flow through a nozzle, and to establish design and simulation criteria for these, when subjected to a particular application.

Keywords: Compressible Flow, Isentropic Process, Mach Number, Nozzle, Subsonic Flow

1. Introduction

Nozzles are devices designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it outlet (or inlet) an enclosed chamber or pipe¹; though these can be described as convergent (narrowing down from a wide diameter to a smaller diameter in the direction of the flow) or divergent (expanding from a smaller diameter to a larger one), their principle of operation is the same: the Bernoulli equation for fluid flow². As established in this simplification form Navier-Stokes law for fluid flow, when a fluid stream moves through a change of flow area, the static pressure changes, implying a conversion to kinetic energy (or the dynamic pressure). Due to this, the nozzles are widely used in applications where it is necessary to control flow rates, velocity and pressure of currents at the outlet, such as natural gas distribution systems and vehicle carburetors³⁻⁵.

For the study of the characteristics of the nozzles, analyzes based on the first law of thermodynamics are generally used, concerning which there is ample information in the literature⁶. However, the analysis carried out in these sources is based on the assumption that the fluid used is incompressible, which implies that its density remains constant over time. Although this is valid for a wide range of substances, there are applications in which this assumption becomes an appreciable source of errors, such as in aeronautics and space exploration, where high pressures induce changes in density of the fluid studied^{2,7,8}.

To take into account the effects of compressibility when studying the flow of any fluid, it is necessary to define a series of additional parameters with respect to the basic thermodynamic analysis; these parameters are related by theoretical and empirical expressions^{6,9-11}, which have been studied on recent years, and with the

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evolution of modern computers, these expressions can be easily programmed in a commercial package (such as MATLAB® and Octave). Based on the previous fact, the present investigation analyzes the influence of the parameters from the compressible flow equations on the pressure profile created inside the nozzle, in order to predict the behavior of the fluid moving through it, which can be used to establish design and simulation criteria for nozzles, when subjected to a particular application.

2. Methodology

This section presents a detailed description of the general purpose of the study, and the fundamental equations implemented for the realization of the investigation.

2.1 General Aim of the Study

The main purpose of the study developed is to analyze the influence of certain parameters of the compressible flow (initial conditions, pressure, temperature and Mach number) on the pressure distribution inside the nozzle. Given the fact equations that involve compressible flow are expressed implicitly, the commercial package MATLAB® was used to calculate the unknown variables and to plot the pressure distribution along the axis of the nozzle, for each change made to the initial parameters.

2.2 Fundamental Equations

Most problems in incompressible flow involve only two unknowns: pressure and velocity, which are typically found by solving the two equations that describe conservation of mass and linear momentum, with the fluid density presumed constant. In compressible flow, however, the gas density and temperature also become variables; this requires two more equations in order to solve compressible-flow problems: an equation of state for the gas and a conservation of energy equation, being the simple ideal gas law, as shown in equation (1), the appropriate state equation for the majority of gas-dynamic problems:

$$P = \rho \cdot R \cdot T \quad (1)$$

Where ρ denotes the gas density, R is the universal gas constant and T the temperature at the evaluated point. To study compressible flow applications, one of the most known approaches, consists in the *one-dimensional isentropic flow*; that is, the following conditions are met:

- Ratio of duct length to width (L/D) is \leq about 5 (to neglect friction and heat transfer).
- Flow is isentropic (i.e., a reversible adiabatic process).
- Ideal gas law.

With these conditions, the equations that are used to model compressible flow are defined in terms of *stagnation properties*, which are the properties in the point where the speed of the fluid is zero, and all of the kinetic energy has been converted to internal energy and is added to the local static enthalpy; given this fact, the stagnation properties act boundaries for the properties of a gas stream. Said stagnation properties can be related to the real properties in a specific point, through the use of two dimensional values: the isentropic coefficient γ , defined as the relationship between specific heats, as shown in equation (2),

$$\gamma = \frac{C_p}{C_v} \quad (2)$$

Where C_p and C_v are the specific heats from the fluid, and the second factor, is the *Mach number*, which relates the stream velocity and the velocity of sound in that substance, in the form given by equation (3),

$$M = \frac{v}{v_s} \quad (3)$$

Where v_s is the velocity of the sound in the fluid. Using equations (2) and (3), the expressions for the pressure and temperature as a function of Mach number, are formulated with the equations

$$\frac{T_t}{T} = \left(1 + \frac{\gamma - 1}{2} \cdot M^2 \right) \quad (4)$$

$$\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} \cdot M^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (5)$$

3. Result and Discussion

The objective of the study, was to analyze the influence of the parameters from the compressible flow equations on the pressure profile created inside the nozzle, to predict the behavior of the fluid moving through it, by the use of equations (1) through (5). Due to the fact these equations can't be solved in an explicit form, these were compiled on the package MATLAB® and formulated in a Graphical User Interface (GUI), for a more friendly appearance. In the

present study, the initial temperature, pressure, and Mach number were varied, in order to observe the pressure distribution that occurs inside the nozzle; as initial conditions for the analysis, we have an initial pressure and temperature of 200 kPa and 700 K, respectively, and for the Mach number, a value of 0.1, which corresponds to a subsonic application, was used with air as the fluid inside the nozzle ($\gamma = 1.4$).

3.1 Influence of the Parameters over the Pressure Profile

The results obtained for a variation of 50 kPa around the standard conditions are shown in Figure 1. From it, it is possible to observe that although the general trend remains constant, the magnitude of the pressure in each test it varies in the same ratio as the increment, which implies that the losses throughout the system are low, which coincides with the initial assumption that neglects the viscous dissipation of the equations of the energy balance. For the temperature, Figure 2 shows the behavior when it's varied from 250–350 K; however, the variation of temperature doesn't affect considerably the pressure profile, which it can be explained by the considerations of the model, which insulates the nozzle to reduce the heat losses, as the ideal case¹²⁻¹³.

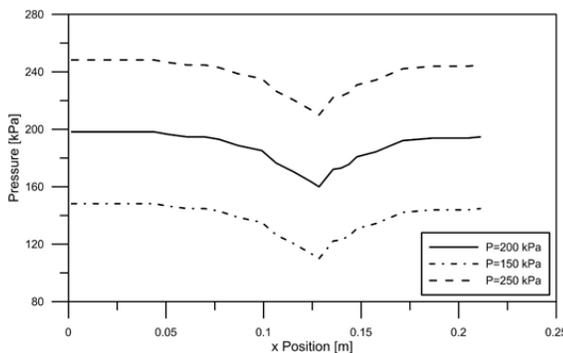


Figure 1. Pressure profile at a different initial pressure values.

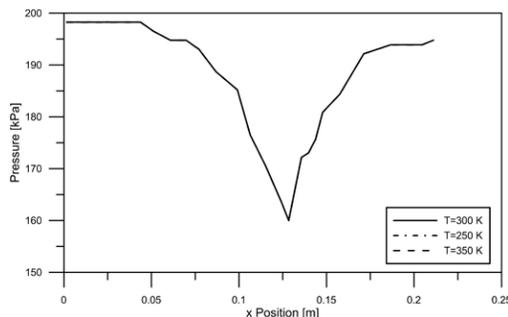


Figure 2. Pressure profile at a different initial temperature values.

From the studied parameters, the major variation was obtained in the Mach number; given the fact this number relates closely with the speed of the stream, lower values implies a slow movement by the stream. Therefore, when the Mach number falls close to zero, the fluid doesn't have enough energy to propitiate the static pressure-dynamic pressure conversion, which explains the quasi-linear pressure profile obtained in Figure 3. However, the increase in Mach number raises considerably the pressure drop in the nozzle, which is accompanied with a sudden increase in its velocity, reaching even the 60% of the velocity of sound in the neck of the nozzle.

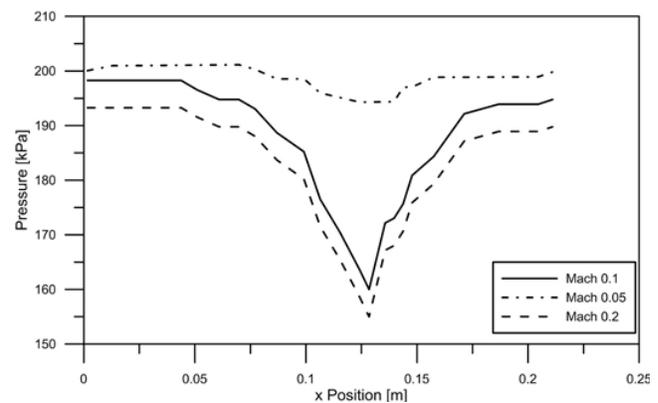


Figure 3. Pressure profile at a different initial Mach values.

4. Conclusions

As a conclusion, the investigation demonstrated the effectiveness of the numerical method in the implicit solution of equations that govern the compressible flow problems, with the implementation of a software package, which represents a method easier to formulate, program and solve than the analytic solution, and represents a time and cost saving. From the parametric study, it was observed, that the Mach number has the greater role in the definition of the nozzle characteristics, such the pressure drop and so because this term can define when the transfer of energy (from static pressure to dynamic pressure) can take place, when subsonic flows are studied. Besides that, it was also found that the temperature has little to no relevance over the pressure profile, mainly because the heat transfer isn't a suitable method to transfer energy when an adiabatic nozzle is used, which is the common case in the isentropic flow.

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