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Static Examination of Simply Supported Laminated Composite Beam with Varying Load Using Trigonometric Shear Deformation Theory

D.H. Tupe[1,*], A.G. Dahake[2] and G.R. Gandhe[3]

[1]Department of Civil Engineering, PES College of Engineering, Dr. BAMU, Aurangabad, MS 431005, India
[2]Department of Civil Engineering, G.H. Raisoni College of Engineering and Management, Wagholi, SPP University, Pune, MS 412207, India
[3]Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Dr. BAMU, Aurangabad, MS 431005, India

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***Author for correspondence:**
D.H. Tupe xxx Department of Civil Engineering, PES College of Engineering, Dr. BAMU, Aurangabad, MS 431005, India

Abstract

Objectives: To study the trigonometric shear deformation theory for the evolution of displacements and stresses of cross-ply simply supported laminated beam subjected to varying load. **Methodology:** A trigonometric shear deformation is used. The in-plain displacement field uses a sinusoidal function in terms of the thickness coordinate to include the shear deformation effect. The theory satisfies the shear stress free boundary conditions on the top and bottom surfaces of the plate. The present theory obviates the need of a shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. **Finding:** Stresses and displacements for orthotropic, single-layer, three-layer symmetric square cross-ply laminated beam subjected to varying load. **Novelty:** The numerical results of the present theory for displacement and stresses are compared with those of classical (ETB), first-order (FSDT), and higher-order shear deformation beam theories.

Keywords: Layerwise Composite Beam, Trigonometric Shear Deformation Theory, Varying Load, Simply Supported Beam.

1. Introduction

Nowadays, composite materials are widely used in worldwide. Composite material is a material made up with the help of two or more different properties of materials. The composite material gives more strength to structural components. The composite material having advantage in its life span. Generally, composite material having more uses in aircraft and ships. In previous works, we studied the isotropic beam with different loading condition as well as different supports. Thick beams and plates, either isotropic or anisotropic, basically form two- and three-dimensional problems of elasticity theory. Reduction of these problems to the corresponding one- and two-dimensional approximate problems for their analysis has always been the main objective. The shear deformations in beams and plates with the three-dimensional nature of these problems further intensified the research interest in their accurate analysis. The shear deformation effects are more pronounced in the thick beams when subjected to transverse loads than in the thin beams under similar loading. The shear deformation effects are more significant in the thick beams. These effects are neglected in Elementary Theory of Beam (ETB). In order to describe the correct bending behavior of thick beams including shear deformation effects and the associated cross sectional warping, shear deformation theories are required. This can be accomplished by selection of proper kinematic and constitutes models. The worked on a higher request discrete-layer hypothesis and stated that a limited component is displayed for foreseeing the damping of overlaid composite sandwich beams wherein quadratic and cubic terms for guess of the in-plane relocation in each discrete layer considered. Model frequencies and damping of sandwich composite beam are estimated and associated with anticipated qualities [1]. The displayed another reverse digression shears twisting hypothesis for the statics and free vibration and clasping investigation of covered composite and sandwich plates. Shear stresses are evaporated at top and base surfaces of the plate and shear amendment factors are never again required [2]. The bar assembles into three arrangements, which are Euler–Bernoulli column theory, first solicitation shear deformation speculation and higher solicitation shear twisting hypotheses [3]. The equations governing the dynamic response of laminated structures are derived by using Hamilton's principle. However, equations of equilibrium for buckling problems are given by employing the principle of virtual displacements. Moreover, using Navier's technique and solving the eigenvalue equations, analytical solutions based on the global–local higher-order theory used in this article are first presented in present study. At the same time, the effect of the order number of higher-order shear deformation as well as interlaminar continuity of transverse on the global response of both laminated beams and soft-core sandwiches has been also studied [4]. Previous studies [5] examined extended reason limits and higher solicitation shear distortion theories in the examination of secured composite beams and plates. The built-up a general definition for nothing and transient vibration investigations of composite overlaid beams with subjective lay-ups and limit conditions [6]. The shear cure factor displayed is difficult to decisively for overlaid composite beams, as it dependants on layer bearing, geometric parameters and point of confinement conditions [7]. The structure articulation to decide the compelling flexural modulus of a covered shaft is created and this powerful flexural modulus is applied to the bowing and free vibration reaction of by and large overlaid composite beams with different limit bolsters [8]. The blended limited component conditions which depend on a useful are gotten by utilizing Gateaux differential for overlaid beams [9]. The variationally relentless refined hyperbolic shear distortion theory for flexure and free vibration of thick isotropic beam. This theory considers transverse shear deformations impacts [10]. The refined beam theories are required to describe the correct thermal response of plates as well as shear deformation effects. Three variables are used in displacement field, to represent the effect of shear deformation [11]. It proposes a hyperbolic shear deformation for thick isotropic cantilever beam. A higher-order beam theory which takes into account shear curvature, transverse stresses and rotatory inertia is presented. The displacement field of the present theory was based on a two variable, in which the transverse displacement is partitioned into the bending and shear parts [12].

2. Methodology

Consider simply supported laminated composite beam with varying load as shown in Figure 1. The beam is made of many unidirectional plies stacked up in different orientations with respect to the x-axis. In the right-handed Cartesian coordinate system, the x-axis is coincident with the beam axis and the origin is on the mid-plane of the beam. The length, breadth, and height of the beam are represented by L, b, and h, respectively.

The displacement field for overlaid composite beam dependent on the trigonometric shear deformation theory (TSDT), higher-order shear deformation theory (HSDT), and hyperbolic shear deformation theory (HYSDT) can be given as follows

$$u^{(1)}(x,z)=u(x)-z\frac{dw}{dx}+\left[\frac{h}{\pi}+\sin\left(\frac{\pi z}{h}\right)\right]\phi(x),\ u^{(2)}(x,z)=0,\ u^{(3)}(x,z)=w(x)$$

$$u^{(1)}(x,z)=u(x)-z\frac{dw}{dx}+z\left[1-\left(\frac{4z^2}{3h^2}\right)\right]\phi(x),\ u^{(2)}(x,z)=0,\ u^{(3)}(x,z)=w(x)$$

$$u^{(1)}(x,z)=u(x)-z\frac{dw}{dx}+\left[h\sinh\left(\frac{z}{h}\right)-\frac{4}{3}\frac{z^2}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi(x),\ u^{(2)}(x,z)=0,\ u^{(3)}(x,z)=w(x) \quad (1)$$

where $u^{(1)}$ is the removal along x headings. $u^{(2)}$ is the uprooting along y headings, $u^{(3)}$ is the relocation along z bearings of a point in the beam. u is the removal in the x course and w is transverse dislodging in the y heading of a point on the bar in midplane.

The strain–displacement relations between strain–displacement corresponding to the displacement field are given by

$$\varepsilon_x^0=\frac{\partial u}{\partial x},\ \mathrm{k}_x^0=-\frac{\partial^2 w}{\partial x^2},\ \mathrm{k}_x^2=\frac{\partial \phi}{\partial x},\ \mathrm{k}_{xz}^2=\frac{h}{\pi}\phi \quad (2)$$

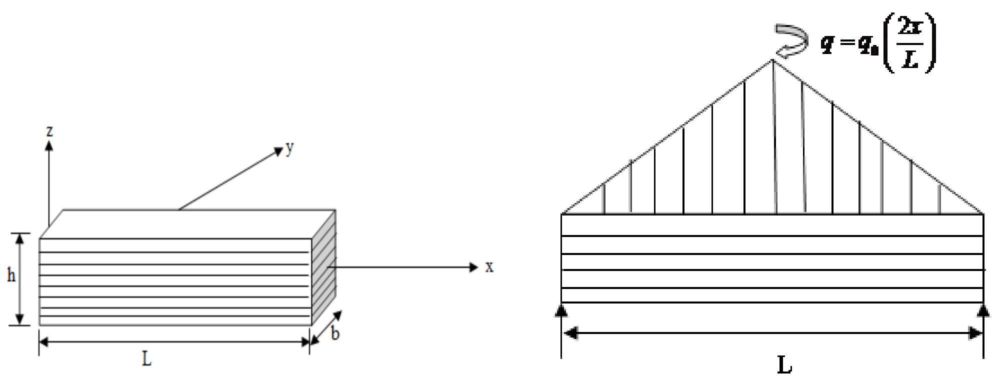


FIGURE 1. Geometry of simply supported laminated composite beam with varying load.

Using the trigonometric shear deformation theory, the constitution equations of the laminates are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ P_x \\ P_y \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} \\ B_{12} & B_{22} & B_{22} & D_{12} & D_{22} & D_{26} & F_{12} & F_{22} & F_{26} \\ B_{16} & B_{26} & B_{26} & D_{16} & D_{26} & D_{66} & F_{16} & F_{26} & F_{16} \\ E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & H_{16} \\ E_{12} & E_{22} & E_{26} & F_{12} & F_{22} & F_{26} & H_{12} & H_{22} & H_{26} \\ E_{16} & E_{26} & E_{66} & F_{16} & F_{26} & F_{66} & H_{16} & H_{26} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \\ k_x^2 \\ k_y^2 \\ k_{xy}^2 \end{Bmatrix} \tag{3}$$

where N_x, N_y, and N_{xy} are the in-plane forces, M_x, M_y, and M_{xy} the bending and twisting moments, P_x, P_y, and P_{xy} the refine bending and twisting moments, $\varepsilon_x^0, \varepsilon_y^0$, and ε_{xy}^0 the mid-plane strains, k_x^0, k_y^0, and k_{xy}^0 the bending and twisting curvatures k_x^2, k_y^2, and k_{xy}^2 the refines bending and twisting curvatures, $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}$ (i,j=1,2,6) are the stiffness coefficient. In the above theory, the constitutive equations of laminated composite beam which accounts for the Poisson effect are considered as follows. Assume N_y, N_{xy}, M_y, M_{xy}, P_y, and P_{xy} equal to zero while $\varepsilon_y^0, \varepsilon_{xy}^0, k_y^0, k_{xy}^0$, k_y^2 k_{xy}^2 are assumed to be non zero.

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} & \overline{E}_{11} \\ \overline{B}_{11} & \overline{D}_{11} & \overline{F}_{11} \\ \overline{E}_{11} & \overline{F}_{11} & \overline{H}_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^0 \\ k_x^2 \end{Bmatrix}, \qquad \begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} & \overline{E}_{11} \\ \overline{B}_{11} & \overline{D}_{11} & \overline{F}_{11} \\ \overline{E}_{11} & \overline{F}_{11} & \overline{H}_{11} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ -\dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial \phi}{\partial x} \end{Bmatrix} \tag{4}$$

where

$$\begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} & \overline{E}_{11} \\ \overline{B}_{11} & \overline{D}_{11} & \overline{F}_{11} \\ \overline{E}_{11} & \overline{F}_{11} & \overline{H}_{11} \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} & E_{11} \\ B_{11} & D_{11} & F_{11} \\ E_{11} & F_{11} & H_{11} \end{bmatrix} - \begin{bmatrix} A_{12} & A_{16} & B_{12} & B_{16} & E_{12} & E_{16} \\ B_{12} & B_{16} & D_{12} & D_{16} & F_{12} & F_{16} \\ E_{12} & E_{16} & F_{12} & F_{16} & H_{12} & H_{16} \end{bmatrix} \begin{bmatrix} A_{22} & A_{26} & B_{22} & B_{26} & E_{22} & E_{26} \\ A_{26} & A_{66} & B_{26} & B_{66} & E_{26} & E_{66} \\ B_{22} & B_{26} & D_{22} & D_{26} & F_{22} & F_{26} \\ B_{26} & B_{66} & D_{26} & D_{66} & F_{26} & F_{66} \\ E_{22} & E_{26} & F_{22} & F_{26} & H_{22} & H_{26} \\ E_{26} & E_{66} & F_{26} & F_{66} & H_{26} & H_{66} \end{bmatrix}^{-1} \begin{bmatrix} A_{12} & B_{12} & E_{12} \\ A_{16} & B_{16} & E_{16} \\ B_{12} & D_{12} & F_{12} \\ B_{16} & D_{16} & F_{16} \\ E_{12} & F_{12} & H_{12} \\ E_{16} & F_{16} & H_{16} \end{bmatrix}$$

The laminated stiffness coefficients $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}$ (i,j=1,2,6) and the transverse shear stiffness F_{55}, which are capacity of overlay handle direction, material property, and stack succession, are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\ (1,\ z,\ z^2)\ dz, \quad (E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\ f(z)\ (1,\ z,\ f(z))\ dz,$$

$$G_{55} = \int_{-h/2}^{h/2} \overline{Q}_{55} \left[g\ (z) \right]^2\ dz, \qquad g\ (z)\ = 1 - f\ '(z) \tag{5}$$

The transformed reduced stiffness constants $\overline{Q}_{ij}$ (i,j=1,2,6) and $\overline{Q}_{55}$ are given by

$$\overline{Q}_{11} = Q_{11}\cos^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \quad (6a)$$

$$\overline{Q}_{12} = (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta+\cos^4\theta) \quad (6b)$$

$$\overline{Q}_{22} = Q_{11}\sin^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \quad (6c)$$

$$\overline{Q}_{16} = (Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta + (Q_{12}-Q_{22}+2Q_{66})\sin^3\theta\cos\theta \quad (6d)$$

$$\overline{Q}_{26} = (Q_{11}-Q_{12}-2Q_{66})\sin^3\theta\cos\theta + (Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta \quad (6e)$$

$$\overline{Q}_{66} = (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta\cos^4\theta) \quad (6f)$$

$$\overline{Q}_{55} = G_{13}\cos^2\theta + G_{23}\sin^2\theta \quad (6g)$$

where θ is the angle between the fiber direction and longitudinal axis of the beam and reduced stiffness constants Q_{11}, Q_{22}, Q_{12}, and Q_{66} can be obtained in terms of constant

$$Q_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}},\ Q_{12} = \frac{\upsilon_{12}E_2}{1-\upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21}E_1}{1-\upsilon_{12}\upsilon_{21}}, Q_{22} = \frac{E2}{1-\upsilon_{12}\upsilon_{21}},\ Q_{66} = G_{12}$$

All laminates made same orthotropic material, which properties are assumed

$$\frac{E_1}{E_2} = 25,\ E_2 = 1,\ \upsilon_{12} = 0.25,\ G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2$$

The force and the moment resultants are defined in the following form

$$N = \int_{-h/2}^{h/2} \sigma_x^k dz,\ M = \int_{-h/2}^{h/2} \sigma_x^k z\, dz, P = \int_{-h/2}^{h/2} \sigma_x^k f(z)\, dz,\ Q = \int_{-h/2}^{h/2} \tau_{zx}^k\, g(z)\, dz \quad (7)$$

Where N and Q are the force resultant; M and P are the moment's resultants. The principle of virtual work is used to obtain the governing equations and boundary conditions associated with the present theory. The principle of virtual work is given as

$$b\int_0^L \int_{-h/2}^{h/2} \left(\sigma_x^k \delta\varepsilon_x + \tau_{zx}^k \delta\gamma_{zx}\right) dz\, dx - \int_0^L q\, (\delta w)\, dx = 0 \quad (8)$$

where δ is the variational administrator. Coordination by parts and gathering the coefficients of δu, δw, and $\delta\phi$, one can acquire the administering conditions and limit states of the pillar related with the present hypothesis utilizing major lemma of analytics of varieties. The variationally reliable administering conditions of the present hypothesis regarding power and minute resultants are as per the following

$$\frac{dN}{dx} = 0 \ :\ \delta u,\ \frac{d^2M}{dx^2} + q = 0 \ :\ \delta w,\ \frac{dP}{dx} = 0 \ :\ \delta\phi$$

$$\frac{dN}{dx} = 0 \ : For\ N_x = \delta u,\ \ -\overline{A}_{11}\frac{d^2u}{dx^2} + \overline{B}_{11}\frac{d^3w}{dx^3} + \overline{E}_{11}\frac{d^2\phi}{dx^2} = 0 \quad (9a)$$

$$\frac{d^2M}{dx^2}+q=0 \; For \; M_x=\delta w, \; -\overline{B}_{11}\frac{d^3u}{dx^3}+\overline{D}_{11}\frac{d^4w}{dx^4}+\overline{F}_{11}\frac{d^3\phi}{dx^3}=q \tag{9b}$$

$$\frac{dP}{dx}=0 \; : For \; P_x=\delta\phi, \; -\overline{E}_{11}\frac{d^2u}{dx^2}+\overline{F}_{11}\frac{d^3w}{dx^3}-\overline{H}_{11}\frac{d^2\phi}{dx^2}+G=0 \tag{9c}$$

for symmetrical angle ply $\overline{E}_{11}$ and $\overline{\mathrm{B}}_{11}$ is zero.

$$\overline{D}_{11}\frac{d^4w}{dx^4}+\overline{F}_{11}\frac{d^3\phi}{dx^3}=q, \; \overline{F}_{11}\frac{d^3w}{dx^3}-\overline{H}_{11}\frac{d^2\phi}{dx^2}+G=0 \tag{9d}$$

Associated boundary condition is as follows
Along edges $x = 0$ and $x = L$

$$\overline{B}_{11}\frac{d^2u}{dx^2}-\overline{D}_{11}\frac{d^3w}{dx^3}+\overline{F}_{11}\frac{d^2\phi}{dx^2}=0 \text{ or } w \text{ is prescribed, } \overline{B}_{11}\frac{du}{dx}-\overline{D}_{11}\frac{d^2w}{dx^2}+\overline{F}_{11}\frac{d\phi}{dx}=0 \text{ or } \frac{dw}{dx} \text{ is prescribed}$$

$$\overline{B}_{11}\frac{du}{dx}-\overline{D}_{11}\frac{d^2w}{dx^2}+\overline{F}_{11}\frac{d\phi}{dx}=0 \text{ or } \frac{dw}{dx} \text{ is prescribed, } \overline{E}_{11}\frac{du}{dx}-\overline{F}_{11}\frac{d^2w}{dx^2}+\overline{H}_{11}\frac{d\phi}{dx}=0 \text{ or } \phi \text{ is prescribed}$$

$$\overline{E}_{11}\frac{du}{dx}-\overline{F}_{11}\frac{d^2w}{dx^2}+\overline{H}_{11}\frac{d\phi}{dx}=0 \text{ or } \phi \text{ is prescribed, } \overline{A}_{11}\frac{du}{dx}-\overline{B}_{11}\frac{d^2w}{dx^2}+\overline{E}_{11}\frac{d\phi}{dx}=0 \text{ or } u \text{ is prescribed}$$

Example: Simply supported beam with varying load $q=q_0\left(\frac{2x}{L}\right)$
Non-dimensional transverse displacement $\overline{w}$

$$\overline{w}=\left[100\;\frac{E_2}{\overline{D}_{11}}\;\frac{1}{\Omega}\;\left\{\frac{1}{60}\frac{x^5}{L^5}-\frac{1}{18}\frac{x^3}{L^3}+\frac{7}{1800}\frac{x}{L}\right\}\;\right]+\left\{\left[100\frac{E_2}{\overline{D}_{11}}\frac{h^2}{L^2}\frac{2}{G_{55}}\;\frac{1}{\lambda^2}\;\left(\frac{\overline{H}_{11}}{\Omega}\lambda^2-G_{55}\;\right)\right]\left[\frac{1}{\lambda^2}\frac{h^2}{L^2}\frac{\mathrm{Sinh}\,\lambda x}{L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}\frac{x}{L}-\frac{1}{6}\frac{x^3}{L^3}+\frac{1}{6}\frac{x}{L}\right]\right\}$$

$$\text{where } \Omega=\left[1+\left(\frac{\pi}{\lambda L}\right)^2\right]$$

Non-dimensional axial displacement $\overline{u}$

$$\overline{u}=\left\{\left[-\frac{z}{h}\;\frac{L^3}{h^3}\frac{1}{\overline{D}_{11}}\frac{1}{\Omega}\left\{\frac{1}{12}\frac{x^4}{L^4}-\frac{1}{6}\frac{x^2}{L^2}+\frac{7}{180}\right\}\;\right]+\left[-\frac{z}{h}\frac{2}{G_{55}}\frac{1}{\overline{D}_{11}}\frac{1}{\lambda^2}\frac{L}{h}\;\left(\frac{\overline{H}_{11}}{\Omega}\lambda^2-G_{55}\;\right)\left(\left(\left[\frac{1}{\lambda}\frac{h}{L}\frac{\mathrm{Cosh}\,\lambda x}{\mathrm{Cosh}\,\lambda L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}-\frac{1}{2}\frac{x^2}{L^2}+\frac{1}{6}\right]\right)\right)\right]\right\}$$
$$+\left[\frac{1}{\pi}\frac{\overline{F}_{11}}{G_{55}}\frac{2}{\overline{D}_{11}}\frac{L}{h}\;\frac{1}{\Omega}\;\mathrm{Sin}\left(\frac{\pi z}{h}\right)\left(\frac{1}{\lambda}\frac{h}{L}\frac{\mathrm{Cosh}\,\lambda x}{\mathrm{Cosh}\,\lambda L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}-\frac{1}{2}\frac{x^2}{L^2}+\frac{1}{6}\right)\right]$$

Non-dimensional axial stresses $\overline{\sigma_x}$

$$\overline{\sigma}_x=\left\{\left[-\frac{z}{h}\;\frac{L^2}{h^2}\frac{1}{\overline{D}_{11}}\frac{1}{\Omega}\left\{\frac{1}{3}\frac{x^3}{L^3}\;-\;\frac{1}{3}\frac{x}{L}\right\}\;\right]-\left[\frac{z}{h}\frac{2}{G_{55}}\frac{1}{\overline{D}_{11}}\frac{1}{\lambda^2}\left(\frac{\overline{H}_{11}}{\Omega}\lambda^2-G_{55}\;\right)\left(\left(\left[\frac{\mathrm{Sinh}\,\lambda x}{\mathrm{Cosh}\,\lambda L}-\frac{x}{L}\right]\right)\right)\right]\right\}$$
$$+\left[\frac{1}{\pi}\mathrm{Sin}\left(\frac{\pi z}{h}\right)\frac{\overline{F}_{11}}{G_{55}}\frac{2}{\overline{D}_{11}}\frac{1}{\Omega}\;\left(\left[\frac{\mathrm{Sinh}\,\lambda x}{\mathrm{Cosh}\,\lambda L}-\frac{x}{L}\right]\right)\right]$$

Non-dimensional transverse shear stresses $\overline{\tau_{zx}^{EE}}$ using equilibrium equation

$$\overline{\tau}_{zx}^{EE} = \left[\frac{1}{8}\frac{1}{\overline{D}_{11}}\frac{L}{h}\frac{1}{\Omega}\left(\frac{x^2}{L^2}-\frac{1}{3}\right)\left[\frac{4z^2}{h^2}-1\right] + \left\{\frac{1}{G_{55}}\frac{1}{\overline{D}_{11}}\frac{1}{\lambda^2 L}\frac{1}{4}\left(\frac{\overline{H}_{11}}{\Omega}\lambda^2\text{-}G_{55}\right)\left(\frac{\lambda L \operatorname{Cosh}\lambda x}{\operatorname{Cosh}\lambda L}-1\right)\left[\frac{4z^2}{h^2}-1\right]\right\}\right]$$

$$+\left\{\frac{2}{G_{55}}\frac{F_{11}}{\overline{D}_{11}}\frac{h}{L}\frac{1}{\Omega}\frac{h^2}{\pi^2}\left(\operatorname{Cos}\frac{\pi z}{h}-1\right)\left(\frac{\lambda L \operatorname{Cosh}\lambda x}{\operatorname{Cosh}\lambda L}-1\right)\right\}$$

Non-dimensional transverse shear stresses $\overline{\tau}_{zx}^{CR}$ using constitutive relationship

$$\overline{\tau}_{zx}^{CR} = \left[-\frac{L}{h}\frac{2}{\Omega}\frac{\overline{F}_{11}}{\overline{D}_{11}}\operatorname{Cos}\left(\frac{\pi z}{h}\right)\left(\left[\frac{1}{\lambda}\frac{h}{L}\frac{\operatorname{Cosh}\lambda x}{\operatorname{Sinh}\lambda L}-\frac{1}{\lambda^2}\frac{h^2}{L^2}-\frac{1}{2}\frac{x^2}{L^2}+\frac{1}{6}\right]\right)\right]$$

3. Numerical Result and Discussion

$$\overline{u}\left(0,\frac{h}{2}\right)=\frac{buE_2}{q_0 h},\ \overline{w}\left(\frac{L}{2},0\right)=\frac{100wh^3E_2}{q_0L^4},\ \overline{\sigma}_x\left(0,\text{-}\frac{h}{2}\right)=\frac{b\sigma_x}{q_0},\ \overline{\tau}_{zx}(0,0)=\frac{b\tau_{zx}}{q_0},\ E_2=1$$

The removals and stresses are determined for essentially bolstered covered composite bar for fluctuating stacking. The relationship of the maximum transverse deflection $(\overline{w})$ to aspect ratio (AS) is shown for different angle ply in Table 1a and 1b. As would be expected Euler–Bernoulli (ETB) underestimates the beam deflections and gives poor estimates for relatively low value of aspect ratio (AS). The relationship of the maximum axial deflection $(\overline{u})$ to aspect ratio (AS) is shown for different angle ply in Table 1c and 1d. The axial deflection increases when the aspect ratio (AS) increases. The distribution bending stress $(\overline{\sigma})$ of by the five theories is represented in Table 1e and 1f for aspect ratio 4 and 10. The stresses are maximum at top and zero at centre. The distribution of transverse shear stresses according to constitutional relationship and equilibrium equations are presented in Table 1g, 1h, 1i, and 1j. The angle proportion aspect ratio (AS) are considered as 4 and 10. Figure 2a–d shows the transverse deflection, bending stresses by means of condition of harmony and constitutive relationship. As the aspect ratio (AS) increases the transverse deflection becomes constant as shown in Figure 2a. Figure 2b and c shows the through thickness distribution of axial displacement which is maximum at the top and bottom surface of the beam. From Figure 2d, it is seen that as viewpoint proportion expands the estimations of transverse removal got steady, in-plane dislodging and in-plane ordinary burdens are most extreme at top and base surface of the bar and those are zero at nonpartisan hub. The transverse shear pressure is zero at top and base surfaces of the beam while most extreme at nonpartisan pivot. Figure 2 represents non-dimensional (a) transverse deflection, (b) axial deflection for AS 4 for three-layer symmetric (0°/90°/0°), (c) axial deflection for AS 10 for three-layer symmetric (0°/90°/0°), (d) bending stresses for AS 4 for three-layer symmetric (0°/90°/0°).

TABLE 1. Examination of non-dimensional (a) transverse displacement $(\bar{w})$ for single, three and four layer laminated beam for aspect ratio (AS) 4, (b) transverse displacement $(\bar{w})$ for single, three and four layer laminated beam for aspect ratio (AS) 10, (c) axial displacement $(\bar{u})$ for single, three and four layer laminated beam for aspect ratio (AS) 4, (d) axial displacement $(\bar{u})$ for single, three and four layer laminated beam for aspect ratio (AS) 10, (e) bending stresses $(\bar{\sigma})$ for single, three and four layer laminated beam for aspect ratio (AS) 4, (f) bending stresses $(\bar{\sigma})$ for single, three and four layer laminated beam for aspect ratio (AS) 10, (g) transverse shear stress $\left(\bar{\tau}_{zx}^{CR}\right)$ for single, three and four layer laminated beam for aspect ratio (AS) 4, (h) transverse shear stress $\left(\bar{\tau}_{zx}^{CR}\right)$ for single, three and four layer laminated beam for aspect ratio (AS) 10, (i) transverse shear stress $\left(\bar{\tau}_{zx}^{EE}\right)$ for single, three and four layer laminated beam for aspect ratio (AS) 4, (j) transverse shear stress $\left(\bar{\tau}_{zx}^{EE}\right)$ for single, three and four layer laminated beam for aspect ratio (AS) 10

a.

AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
4	Present TSDT	2.679	20.934	2.678	14.091	2.706	10.497
	HYSDT [10]	3.094	21.913	3.083	15.219	3.116	11.701
	HSDT [7]	3.104	21.928	3.096	14.613	3.118	11.666
	FSDT [5]	2.498	20.281	2.990	11.376	3.386	6.574
	ETB [3]	0.625	15.631	0.647	8.261	0.710	3.899

b.

AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
10	Present TSDT	0.945	16.457	0.966	7.859	1.025	4.944
	HYSDT [10]	1.011	16.620	1.028	7.976	1.088	5.140
	HSDT [7]	1.013	16.619	1.032	7.942	1.089	5.126
	FSDT [5]	0.923	16.343	1.021	8.751	1.137	4.324
	ETB [3]	0.625	15.631	0.647	8.261	0.710	3.899

c.

AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
4	Present TSDT	1.964	20.457	1.917	15.263	1.832	10.836
	HYSDT [10]	2.121	20.790	2.071	16.066	1.995	11.509
	HSDT [7]	2.126	20.817	2.074	16.229	1.991	11.422
	FSDT [5]	0.682	17.072	0.706	9.023	0.776	4.259
	ETB [3]	0.682	17.072	0.706	9.023	0.776	4.259

d.

AS	Theory/ angle ply	0°	90°	0°/90°/0⁰	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
10	Present TSDT	13.790	274.889	14.020	133.925	14.718	82.945
	HYSDT [10]	14.184	275.823	14.377	135.084	15.098	84.675
	HSDT [7]	14.204	275.841	14.407	136.354	15.103	84.359
	FSDT [5]	10.665	266.752	11.046	140.992	12.127	65.548
	ETB [3]	10.665	266.752	11.046	140.992	12.127	66.548

e.

AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
4	Present TSDT	0.972	13.277	0.9606	8.146	0.947	5.493
	HYSDT [10]	1.033	13.404	1.0215	8.483	1.011	5.741
	HSDT [7]	1.034	13.414	1.0216	8.512	1.010	5.721
	FSDT [5]	0.750	18.757	0.7767	9.914	0.852	4.679
	ETB [3]	0.750	18.757	0.7767	9.914	0.852	4.679

f.

AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
10	Present TSDT	3.453	76.197	3.542	35.048	3.790	21.163
	HYSDT [10]	3.512	76.353	3.590	35.076	3.842	21.431
	HSDT [7]	3.516	76.348	3.599	35.414	3.845	21.365
	FSDT [5]	3.000	75.030	3.107	39.657	3.411	18.718
	ETB [3]	3.000	75.030	3.107	39.657	3.411	18.718

g.

AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
4	Present TSDT	1.015	1.0306	1.011	1.237	0.9981	1.194
	HYSDT [10]	1.047	1.0662	1.039	1.229	1.0297	1.214
	HSDT [7]	1.049	1.0660	1.043	1.237	1.0292	1.210
	FSDT [5]	0.558	13.972	0.579	7.393	0.6351	3.490
	ETB [3]	–	–	–	–	–	–

h.

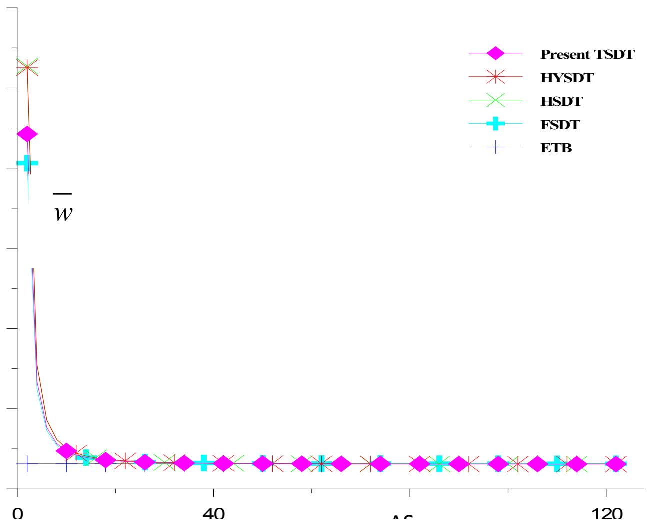
AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
10	Present TSDT	2.540	2.5766	2.529	3.093	2.495	2.986
	HYSDT [10]	2.620	2.6655	2.598	3.075	2.575	3.035
	HSDT [7]	2.625	2.6651	2.612	3.093	2.573	3.026
	FSDT [5]	8.727	2.1801	9.050	115.52	9.918	54.545
	ETB [3]	–	–	–	–	–	–

i.

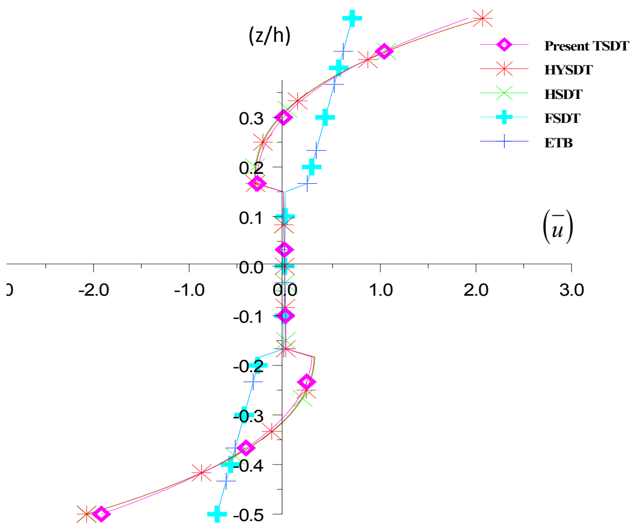
AS	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
4	Present TSDT	1.4000	5.408	1.3873	5.6030	1.3714	4.7293
	HYSDT [10]	1.2580	4.991	1.2417	5.1752	1.2264	4.3649
	HSDT [7]	1.2570	4.990	1.2405	5.1629	1.2251	4.3524
	FSDT [5]	0.0800	2.000	0.8285	1.0575	0.0909	0.4991
	ETB [3]	0.0800	2.000	0.8285	1.0575	0.0909	0.4991

j.

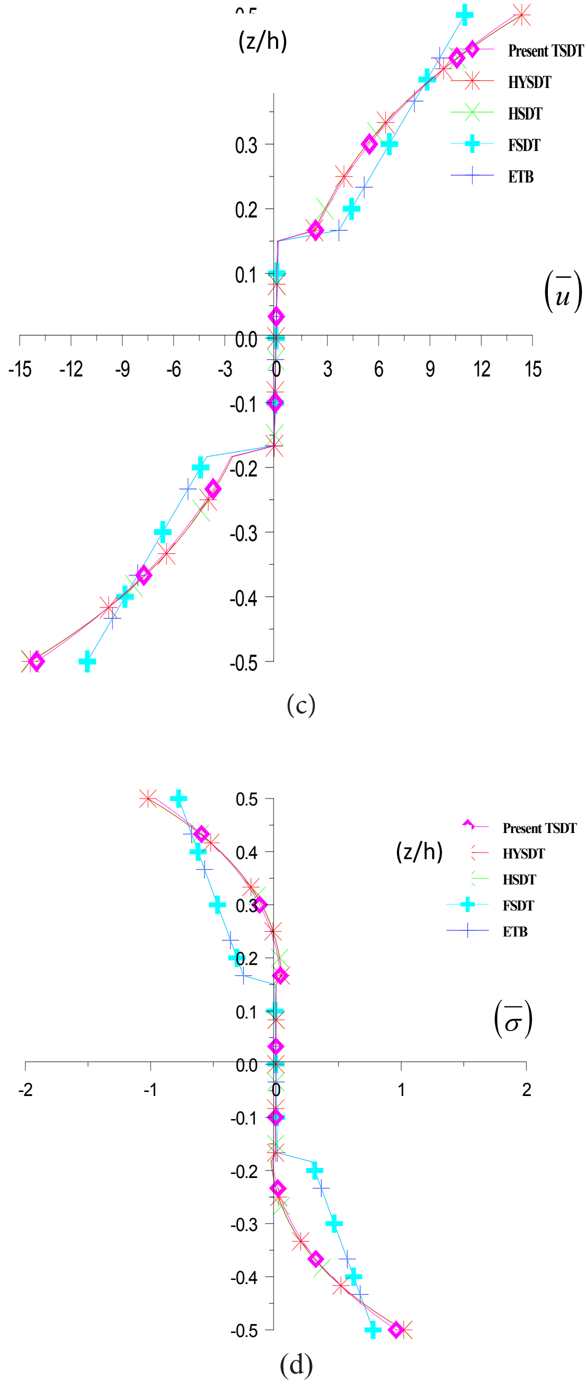
L/h	Theory/ angle ply	0°	90°	0°/90°/0°	90°/0°/90°	0°/90°/90°/0°	90°/0°/0°/90°
10	Present TSDT	1.5189	8.404	1.5103	6.884	1.5068	5.4755
	HYSDT [10]	1.6213	8.612	1.6080	7.1569	1.6069	5.7913
	HSDT [7]	1.6201	8.600	1.5955	7.1454	1.5954	5.7799
	FSDT [5]	0.2000	5.002	0.2071	2.6438	0.2274	1.2478
	ETB [3]	0.2000	5.002	0.2071	2.6438	0.2274	1.2478



(a)



(b)



AQ2 **FIGURE 2. A.** Non-dimensional transverse deflection. b. Non-dimensional axial deflection for AS 4 for three-layer symmetric (0°/90°/0°). c. Non-dimensional axial deflection for AS 10 for three-layer symmetric (0°/90°/0°). d. Non-dimensional bending stresses for AS 4 for three-layer symmetric (0°/90°/0°)

4. Conclusions

This paper builds up the static firmness grid for essentially upheld composite overlaid beams dependent on the trigonometric shear distortion hypothesis. The static solidness network is inferred by straightforwardly illuminating overseeing differential condition of movement of the beam. The use of static strength system to get shirking and stresses of fundamentally maintained beam with different aspect ratio. The numerical outcomes got from the present technique show great concurrence with the accessible arrangements in the writing.

References

1. Plagianakos TS, Saravanos DA. Higher order layerwise mechanics and finite element for the damped dynamic characteristics of sandwich composite beams. *International Journal of Solids and Structures*. 2004, (41), 6853–6861. https://doi.org/10.1002/nme.2465.
2. Thai CV, Ferreira AJM, Bordas SPA, Rabczuk T, Nguyen-Xuan H. Isometric analysis of laminated composite and sandwich plates using a new inverse trigonometric shear deformation theory. *European Journal of Mechanics A/Solids*. 2014, (43), 89–108. https://doi.org/10.1016/j.euromechsol.2013.09.001
3. Reddy JN. Mechanics of laminated composite plates and shells: theory and analysis. CRC Press. 2004.
4. Zhen Wu, Wanji Chen. An assessment of several displacement based theories for the vibration and stability analysis of laminated composite and sandwich beams. *Composite Structures*. 2008, (84), 337–349. https://doi.org/10.1016/j.compstruct.2007.10.005
5. Ferreira A, Roque C, Martins P. Radial basis functions and higher order shear deformation theories in the analysis of laminated composite beams and plates. *Composite Structures*. 2004, (66), 287–293. https://doi.org/10.1016/j.compstruct.2004.04.050
6. Qu Yegao Qu, Long Xinhua, Li Hongguang, Meng Guang. A variational formulation for dynamic analysis of composite laminated beams based on a general higher order shear deformation theory. *Composite Structures*. 2013, 102, 175–192. DOI: 10.1016/j.compstruct.2013.02.032.
7. Bannerjee J, Williams F. Exact dynamic stiffness matrix for composite Timoshenko beams with application. *Journal of Sound and Vibration*. 1996, (194), 573–585. https://doi.org/10.1006/jsvi.1996.0378
8. Boay Chai., Wee Yap. Coupling effects in bending, buckling and free vibration of generally laminated composite beams. *Composite Science and Technology*. 2008, (68), 1664–1670. https://doi.org/10.1016/j.compscitech.2008.02.014
9. Ozutok A, Madenci E. Static analysis of laminated composite beam based on higher order shear deformation theory by using mixed-type finite element method. *Composite Structures*. 2017, (130), 234–243. https://doi.org/10.1016/j.ijmecsci.2017.06.013
10. Ghugal YM, Sharma R. A Hyperbolic shear deformation theory for flexure and vibration of thick isotropic beams. *International Journal of Computational Methods*. 2009, (6), 585–604. https://doi.org/10.1142/S0219876209002017
11. Gandhe GR, Salve SB, Pankade PM, Taur PG, Tupe DH. Trigonometric shear deformation theory for thermal flexural analysis of isotropic plate. Proceeding manufacturing, India. 2018, 499–504. https://doi.org/10.1016/j.promfg.2018.02.074
12. Gandhe GR, Pankade PM, Tupe DH, Taur PG. Higher order computational method for static flexural analysis of thick method. Proceeding Manufacturing, India. 2018, 493–498. https://doi.org/10.1016/j.promfg.2018.02.073