# Adaptive Super-Resolution Image Reconstruction with Lorentzian Error Norm

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### Abstract

**Objectives:** To focus on an inverse problem of reconstructing a high resolution image from set of captured low resolution (LR) frames. **Methods/Statistical Analysis:** The captured LR images are blurred, warped, down-sampled, noisy, and contains complementary information. Super resolution reconstruction(SRR) is a computational technique to correct the degradation that the captured images normally suffer and this problem is ill-posed due to blur and noise present in the captured frames, and regularization is imperative to obtain a stable solution. **Findings:** The proposed approach is based on a *maximum-a-posteriori* (MAP) framework by minimizing a cost function. Persuaded by the performance of Lorentzian norm in reducing the outliers and regularization parameter ( $\lambda$ ) is obtained based on U-curve method, which significantly reduces the search interval, decreases the computation time, and step size is ( $\beta$ ) is calculated using successive over relaxation (SOR) technique. **Application/Improvements:** SRR problem is solved by locating search interval for optimal  $\lambda$  based on the U-curve method and demonstrated in test/colour images, and frames extracted from a video.

Keywords: Laplacian Regularization, Lorentzian Norm, Super-Resolution Reconstruction, U-Curve

## 1. Introduction

In imaging applications such as video surveillance, region of interest (ROI) processing, displaying of standard definition (SD) video onto a high definition (HD) display, remote sensing, and medical diagnosis it is always desirable to acquire an image/video with best possible resolution. Due to distortions introduced by the imaging sensor, zoom optics, and many physical constraints limit the captured image quality. By sensor manufacturing techniques, the spatial resolution of the images can be increased and has reached a limit. However, computational SRR has emerged as an alternative costeffective approach and unifies de-noising, de-blurring, and scaling-up tasks. The SRR problem was first proposed in frequency domain<sup>1</sup>, and these methods are theoretically simple, computationally efficient and have lessened applications due to their inability to accommodate prior knowledge. To overcome this drawback, many spatial domain approaches<sup>2-5</sup> have been proposed.

Multi-frame SRR problem consists of (i) registering a set of LR frames in a common coordinate system, (ii) solving the inverse problem of reconstruction. Many of the recently proposed algorithms for registration<sup>6</sup> exhibit various degrees of errors. The recent SRR algorithms depend on robust data fusion such as  $L_p$  norm,  $(1 \le p \le 2)^7$  for Gaussian and Poisson noise contributions. In the case of real images with unknown noise models,  $L_p$  norm,  $(1 \le p \le 2)$  may degrade the reconstructed image quality. To tackle this many approaches were proposed in the literature<sup>8-11</sup>. A SRR results depend on (i) accuracy of image registration, (ii) robustness of reconstruction and are interrelated.

In this paper, Lorentzian norm<sup>12,14,15</sup> is adopted, which is robust than  $L_p$  norm,  $(1 \le p \le 2)$  with superior outlier rejection capability as a data fidelity cost function and Laplacian regularization as a prior function. The role of regularization parameter ( $\lambda$ ) is decisive to control the trade-off between the data fidelity and prior term, and in many situations  $\lambda$  is calculated manually based on subjective measurements. In order to overcome such shortcomings, many approaches have been proposed in the literature, such as L-curve method<sup>16</sup>, generalized cross validation (GCV) method<sup>17</sup>, and Bayesian framework method<sup>18</sup>. U-curve method was proposed for discrete inverse problems<sup>19</sup> and extended to adaptive SRR problems<sup>20</sup>. In this paper, we deviate from the existing methods by adopting Lorentzian norm as a data fidelity cost function, regularization parameter ( $\lambda$ ) is selected using the U-curve method, and step size ( $\beta$ ) through simultaneous over relaxation (SOR) approach.

The remainder of the paper is organized as follows. Section II describes the forward data model. Section III illustrates robust reconstruction including selection of  $\lambda$  using U-curve method. Simulations on synthetic and real data sequences are demonstrated in Section IV, and Section V concludes this paper.

### 2. Forward Data Model

The first step in SRR is to formulate an observation model to replicate the physical process of imaging conditions including various degradation factors and depict the original HR image with recorded LR frames. The degradation process includes geometric transformation  $(M_k)$ , blurring  $(B_k)$ , sub-sampling  $(D_k)$  and AWGN  $(n_k)$  term. The forward model is given as,

$$y_{1} = D_{1}B_{1}M_{1}x + n_{1}$$
(1)  

$$y_{2} = D_{2}B_{2}M_{2}x + n_{2}$$
  
:

$$y_k = D_k B_k M_k x + n_k$$

where, k is the number of LR images (images are lexicographically ordered) and equation (1) can be written as,

$$\underline{Y}_{k}^{1} = D_{k}B_{k}M_{k}\underline{X} + n_{k}$$
<sup>(2)</sup>

In imaging applications (digital photography and surveillance),  $B_k$  combines both camera blur and atmospheric turbulence and  $B_k^{cam}$  is dominant. As the frames are down-sampled and blurred by same amount, i.e., $\forall D_k = D, \forall B_k = B$ , and forward model can be rewritten as,

$$\underline{Y}_{k} = DBM_{k}\underline{X} + \underline{n}_{k} \tag{3}$$

The problem tackled in this paper is to estimate the HR image  $\underline{X}$  (matrix dimensions are listed in<sup>7</sup>) from  $\underline{Y}_k$ . It is assumed that the system is square and optical blur functions are uniform, already known or estimated. The down sampling is implemented using an average strategy. To have a more realistic model for  $M_k$ , consisting of both translation ( $\Delta x$ ,  $\Delta y$ ) and rotation ( $\theta$ ), Taylor series

approximation method<sup>13</sup> is used to estimate the motion parameters. The contributions in this paper are listed below:

- To reduce the outliers, adopted Lorentzian error norm with Laplacian regularization as a prior. U-curve method provides a suitable search interval for λ to increase the computational efficiency of SRR problems.
- The proposed approach is tested with static monochrome/colour images and frames extracted from the video sequences.

### 3. Robust Reconstruction

The statistical problem of estimating unknown  $\underline{X}$  is not exclusively based on  $\underline{Y}_k$  and depends on some prior information about the noise and motion models that maps the HR scene to the recorded LR images. The estimator performance degrades due to outliers and mismatch of prior information with the measurement data. Finding the inverse solution to the ill-posed problem is unstable due to minute changes in  $\underline{Y}_k$  resulting enormous change in  $\underline{\widehat{X}}$ .

### 3.1 Error Norm and Regularization

In M-estimators, SRR can be considered as the following minimization problem:

$$\widehat{\underline{X}} = ArgMin\left\{\sum_{k=1}^{N} \rho(\underline{Y}_{k}, DBM_{k}\underline{X})\right\}$$
(4)

where  $\rho$  measures the distance between the model and actual measurements. The LS approach for equation (4) is given as,

$$\widehat{\underline{X}} = ArgMin\left\{\sum_{k=1}^{N} || DBM_k \underline{X} - \underline{Y}_k ||_2^2\right\}$$
(5)

where,  $||DBM_{k}\underline{X} - \underline{Y}_{k}||_{2}^{2}$  is data fidelity term. Regularization compensates for the missing measurement information with prior knowledge, constraints the solution space, provides a stable solution by means of prior information, removes artifacts from  $\underline{\widehat{X}}$ , improves rate of convergence and is implemented as a penalty function in minimization of the cost function, and equation (5) can be written as,

$$\widehat{\underline{X}} = ArgMin\left\{\sum_{k=1}^{N} \rho(\underline{Y}_{k}, DBM_{k}\underline{X}) + \lambda \cdot \gamma(\underline{X})\right\}$$
(6)

 $\gamma(X)$  is the prior term and  $\lambda$  is a scalar quantity, weights data fidelity term against the regularization cost function. Higher values of  $\lambda$  brings over smoothing i.e., reconstructed signal will become blurry and small  $\lambda$  reduces noise ineffectively.

L<sub>p</sub> (1≤p≤ 2) norm is used as robust estimators to reduce the outliers. L<sub>2</sub> norm is broadly sensitive to outliers because the influence function escalates linearly without having an upper bound. Also, L<sub>2</sub> norm infers that the extra resolution content is equally distributed in all  $\underline{Y}_k$  and the resulting  $\underline{X}$  is an average of the contributions from  $\underline{Y}_k$ . In case of L<sub>1</sub> norm, the pixel-wise median minimizes the cost function and it is robust against outliers. Here the influence function is bounded, and it characterizes the bias. From the robust statistics theory, Lorentzian norm is adopted due to its outlier rejection capability, and is given below.

$$\widehat{\underline{X}} = ArgMin\left\{\sum_{k=1}^{N} \rho_{LOR}(\underline{Y}_{k}, DBM_{k}\underline{X}) + \lambda \cdot \gamma(\underline{X})\right\}$$
(7)
(8)

$$\rho_{LOR}(x,T) = \log\left[1 + \frac{1}{2}\left(\frac{x}{T}\right)^2\right]$$
(8)

where, T is Lorentzian constant parameter which is soft threshold value. The Lorentzian norm assigns zero or calculated weight to outliers depending on their magnitude. In equation (7), the Tikhonov regularization function ( $\gamma(\underline{X})$ ) is replaced with 2D Laplacian kernel ( $\Gamma$ ), and the cost function to the inverse problem i.e., equation (7) can be written as,

$$\widehat{\underline{X}} = ArgMin\left\{\sum_{k=1}^{N} \rho_{LOR} (DBM_k \underline{X} - \underline{Y}_k) + \lambda \cdot (\Gamma \underline{X})^2\right\}$$

By steepest descent method, the solution to the cost function is computed by differentiating equation (9) with respect to  $\underline{X}$  and HR image is iteratively estimated.

$$\underline{\widehat{X}}_{n+1} = \underline{\widehat{X}}_n - \beta_n \left[ \sum_{k=1}^N M_k^T B^T D^T \cdot \rho'_{LOR} (DBM_k \underline{\widehat{X}}_n - \underline{Y}_k) + \left( \lambda \cdot (\Gamma^T \Gamma) \underline{\widehat{X}}_n \right) \right]$$

(10)

(9)

 $\beta_n$  is the n<sup>th</sup> iteration step size, plays an important role in the convergence of steepest descent method. If the step size is too large, divergence will occur, and smaller step

size results a slower rate of convergence. For Lorentzian error norm, simultaneous over-relaxation (SOR) method<sup>21</sup> provides a better constant step size.

The iteration is terminated when,

$$\frac{\|X_{n+1} - X_n\|^2}{\|X_n\|^2} \le d$$
(11)

where, d is the specified error.

# 3.2 Estimation of λ based on U-Curve Method

The SRR problem depends on trade-off between the data fidelity term and prior term, and is totally controlled by the  $\lambda$ . The model in this paper is similar to that of traditional Tikhonov regularization with Laplacian prior and additive white Gaussian noise (AWGN). The cost function in equation (7) can be rewritten as Tikhonov regularization by considering the matrices DBM<sub>k</sub> as A and is given below.

$$\widehat{\underline{X}} = ArgMin\left\{ \| y - A\widehat{\underline{X}} \|^2 + \lambda \cdot \| \Gamma \widehat{\underline{X}} \|^2 \right\}$$
(12)

By applying singular value decomposition (SVD) least squares (LS) method to A, i.e.,

$$A = U \begin{bmatrix} \sum & 0 \\ 0 & 0 \end{bmatrix} V$$

where U is left SVD of A, V is right SVD of A and is  $\Sigma$  is singular values of A and the details are in<sup>20</sup>.

$$R(\lambda) = ||y - A\widehat{\underline{X}}||^2$$
<sup>(13)</sup>

$$P(\lambda) = \|\Gamma \widehat{\underline{X}}\|^2 \tag{14}$$

$$U(\lambda) = \frac{1}{R(\lambda)} + \frac{1}{P(\lambda)}, \lambda > 0$$
<sup>(15)</sup>

The U-curve is the plot of  $U(\lambda)$  and provides an interval where the optimal  $\lambda$  exists, and reduces the manual search and increases the computational time.  $\lambda$  is selected at curvature of the U-curve for a local maximum close to the left part of the U-curve, and an optimal  $\lambda$  is obtained.

# 4. Experimental Results and Discussions

The proposed approach is validated using

MATLAB by employing three different data sets and the results are presented. Mean square error  $(MSE = \frac{1}{N}\sum_{\alpha} (\underline{x} - \hat{\underline{x}})^2)$  is used as an objective measurement. To appraise the performance of the proposed approach, the results are compared with interpolation method and  $L_1$  norm with Laplacian regularization<sup>14</sup> ( $L_1LR$ ) with  $\beta$ = 0.05. In numerical experiments,  $B_{cam}$  is Gaussian kernel of size 5x5 with variance of 0.5 is used.

### 4.1 Synthetic Experiment

In the first set of experiment, standard monochrome images are used and are shown in Figure 1. The second set of experiment is performed on colour images (car image of size 896x592x3, downloaded from website and acknowledged) and is shown in Figure 2. From these HR images ( $\underline{X}$ ), created a sequence of LR images ( $\underline{Y}_k$ ). In these experiments,  $\underline{X}$  blurred by H<sub>cam</sub>, down-sampled by a factor of 2 in horizontal and vertical directions, warped (randomly in  $\Delta x$ ,  $\Delta y$ ,  $\theta$ ), AWGN is added, and  $\underline{Y}_k$  with k = 1...6 are generated.



Figure 1. (a) Cameraman, size 512x512x1, (b) Aerial, size 256x256x1, (c) Boat, size 512x512x1, (d) Lena, size 512x512x1, (e) Peppers, size 512x512x1, (f) Finstones, size 512x512x1.



Figure 2. Original car image of size 896x592x3.

Steps involved in the SRR are the estimation of motion parameters ( $\Delta x$ ,  $\Delta y$ ,  $\theta$ ) from ( $\underline{Y}_k$ ), projection of LR images onto a common HR grid, and robust reconstruction. The reconstructed image (cameraman) and the corresponding ROI are shown in Figure 3 and Figure 4.



**Figure 3.** (a) HR image of size  $512 \times 512 \times 1$ , (b) One of LR image of size  $256 \times 256 \times 1$ , reconstructed by (c) bi-cubic interpolation, (d) L<sub>1</sub>norm with Laplacian regularization method, (e) Proposed method.



**Figure 4.** ROI of (a) HR image, (b) LR image, reconstructed by (c) bi-cubic interpolation, (d)  $L_1$  norm with Laplacian regularization method, (e) Proposed method.

The MSE plot for all the test images is shown in Figure 5. From this plot, it is evident that the proposed approach performs better in comparison with interpolation and  $L_1$  norm with Laplacian regularization method.



Figure 5. Mean square error.

For clarity, a section of the colour image (\_\_), LR image ( $\underline{Y}_1$ ) and reconstructed images ( $\underline{\hat{X}}$ ) are shown in Figure 6. The ROI of the colour image i.e., the number plate is shown in the Figure 7. From the reconstructed images, it is evident that the proposed method rejects outliers and reduces discontinuities because of the influence function. The outliers fetch a constant weight of one in L<sub>1</sub> norm, and zero/one in Lorentzian norm. Hence, Lorentzian norm produces sharper boundaries than the L<sub>1</sub> norm. The reconstructed images by the proposed approach sustain edges/finer details and thereby increase spatial resolution.



**Figure 6.** ROI of (a) HR image, (b) one of LR image, reconstructed by (c) bi-cubic interpolation, (d)  $L_1$  norm with Laplacian regularization method, (e) Proposed method.



**Figure 7.** ROI of (a) HR image, (b) One of LR image, reconstructed by (c) bi-cubic interpolation, (d)  $L_1$  norm with Laplacian regularization method, (e) Proposed method.

### 4.2 Real Experiment

In the second set of experiment, to demonstrate the potential of the proposed method under realistic imaging conditions, applied to a real video sequence (viplanedeparture.avi, duration 11.2 seconds, 30 frames per second, 24 bits per pixel, 337 frames, size 360 x240x3, copied from MATLAB and acknowledged), consisting of multiple independently moving objects for frame freezing and ROI enhancing applications. Here,  $B_{cam}$  (motion blur is not considered) is assumed as 5x5 with  $\sigma$ = 0.7 The reconstructed image (only a part of the frame is shown) and ROI with an up-sampling factor of 2 is also shown in the bottom right hand corner of Figure 8.



**Figure 8.** ROI of (a) Original frame, reconstructed by (b) bi-cubic interpolation, (c) L1 norm with Laplacian regularization method, (d) Proposed method.

### **4.3 Applicability Issues**

The proposed method has limitation in registering multiple moving objects in a scene. This restraint the applicability in real world scenarios, and conditions where simultaneous changes in scale, pan/tilt occurs. Since the human visual system (HVS) is more sensitive to luminance part, in colour image/video processing, only the luminance channel is considered for registration, reconstruction, and finally colour channels are added. In this paper, the correlation between the chrominance channels and compression effects are not considered, which results colour, blocking artifacts in the reconstructed image and needs to be addressed.

# 5. Conclusion

Computational efficiency of the super resolution reconstruction problem based on MAP framework is improved by locating an interval for  $\lambda$  based on the U-curve method, and  $\beta$  is calculated using SOR technique. The proposed approach is tested with monochrome test images and extended to colour images, and frames extracted from the video. The experimental results of the proposed approach provide an enhanced result with less artifacts, blur, noise, discontinuities in the reconstructed image/video frames and increase the spatial resolution.

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