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Wavelet Thresholding Algorithms for Image Denoising

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Abstract

This paper talks about the wavelet thresholding algorithm for image denoising. Any data, either in the form of signals, or images contains more noise than informations. To make sense out of it, it needs denoising. For that, this paper explains algorithm that makes active use of wavelet thresholding to achieve maximum denoising. For statistical analysis matlab software is used as it comes with wavelet thresholding application. This is then used to process standard lenna image to obtain haar wavelet transform for three levels of decomposition of image. On the contrary daubechies wavelet transform is also applied to the same sample image of lenna. Using Haar Wavelet for image compression has a little bifurcation in Retained Energy and Number of Zeros along x axis. On the other hand Daubechies Wavelet compression with global thresholding on decomposition level 4 for standard image of lenna yields different trend lines between Retained Energy and Number of Zeros. Its applications vastly covers all medias such as image, video, signals, etc. to achieve maximum information. With advances in image denoising, space can be utilized more appropriately as user can be able to save space in his personal devices like mobile phones, laptops, etc. With this user can be able to use or access that free space in order to upload more data, or use it for his computational use.

Keywords: Image Denoising, Thresholding, Wavelet Transform

1. Introduction

Digital Images are often corrupted with noise during acquisition, transmission, and retrieval from storage media. Many dots can be spotted in a Photograph taken with a digital camera under low lighting conditions. The denoising algorithm is used to remove such noise. Image-processing algorithms such as pattern recognition require a clean image to work effectively. Random and uncorrelated noise samples are not compressible which leads to the importance of denoising in image and video processing. If Y is the observed noisy image, X is the original image and N is the AWGN noise with variance σ2. The objective is to estimate X given Y. A best estimate can be written as the conditional mean $X = E[X \mid Y]$. The difficulty lies in determining the probability density function $\rho(x \mid y)$. The purpose of an image-denoising algorithm is to find a best estimate of X. Though many denoising algorithms have been published, there will always be scope for improvement and betterment.

Image noise is an important aspect in terms of processing the image. It ranges from sharp specks on a digital photograph taken in good light to optical and radio astronomical images that are almost entirely noisy, from which we can derive a small bit of information by sophisticated processing. Such high noise level in image will be unacceptable in a photograph since it would be impossible to even determine the subject in the image.

An another model of image noise is Gaussian which is additive in nature, independent at each pixel, and independent of the signal intensity, caused primarily by Johnson-Nyquist noise (thermal noise). As it comes from the reset noise of capacitors (kTC noise). In color cameras, which have more amplification in the blue channel rather

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than in green or red channel[6]. More noise can be found in blue channel. At higher exposures, image sensor noise is dominated by shot noise, which is not Gaussian and also dependent of signal intensity. In extreme noise cases, such as, astronomical images of distant objects, which are mostly noisy and using noise reduction to retrieve a little information buried in a lot of noise. Techniques are different, for seeking small regularities in massively random data.

2. Modelling Noise in Image

The signal to noise ratio (SNR) is a useful and universal way of comparing the relative amounts of signal and noise for any electronic system. High ratio yields very little visible noise whereas low ratio will give the opposite. Noise tells unwanted information in digital images. It is additive signal-dependent and signal independent random noise. Noise will produce undesirable effects such as artifacts, unrealistic edges, unseen lines, corners, blurred objects and disturbs background scenes. To minimize such effects, prior knowledge of noise models is essential for further processing. Digital noise[13] arises from a number of sources such as Charge Coupled Devices (CCD) and Complementary Metal Oxide Semiconductors (CMOS) sensors. For timely, complete and quantitative analysis of noise models, Point Spreading Function (PSF) and Modulation Transfer Functions (MTF) have been used. To design and characterize the noise models, we can use Histograms and Probability Density Function (PDF). Below few noise models are discussed with their types and categories in digital images[5].

2.1 Gaussian Noise Model

It is an electronic noise generated in amplifiers or detectors. Natural sources such as thermal vibration of atoms and discrete nature of radiation of warm objects[5] results in generation of Gaussian Noise. It generally disturbs the gray value in digital images. Hense, Gaussian Noise model is essentially designed and characterized by its Probability Density Function or normalized histogram with respect to gray value. This is given as

$$p(g) = \sqrt{1/\left(2*\pi*\sigma^2\right)*\epsilon}^{(g-\mu)^2/2*\sigma^2}$$

Where g = gray scale, s = standard deviation and μ= mean. Gaussian Noise mathematical model represents the correct approximation of real world scenarios. In this noise model, the mean value is set to zero, variance is set to 0.1, and gray scale is set to 256 in terms of its PDF, which is shown in Figure 1.

3. Image Segmentation

Image segmentation[12] involves separating an image into regions (or their contours) corresponding to objects. We normally try to segment regions by identifying common properties. Or we identify contours by identifying differences between regions (edges).

Thresholding is the simplest method of image segmentation. From a grayscale image, we can create binary images using thresholding. It is achieved by turning pixels below a set threshold to zero and remaining pixels are given the value one. If g(x, y) is a threshold version of f(x, y) at some global threshold T,

$$g(x,y) = f(x,y) \qquad \text{if } T \geq 0$$

$$= 0, \qquad \text{otherwise}$$

According to simplest thresholding method, each pixel in an image is replaced with a black pixel if the intensity of an image is below some constant T. Whereas into a white pixel if the image intensity is above constant T. In order to automate thresholding completely, it re required for the computer to automatically select the threshold T. Sezgin and Sankur (2004) categorized the thresholding methods into the following six groups based upon the information the algorithm manipulates[11]. They are Histogram shape-based methods, Clustering-based methods, Entropy-based methods, Object Attribute-based methods, Spatial methods. Object Attribute-based method search a measure of similarity between the gray-level and the binarized images. Some local methods are also present which adapt the threshold value on each pixel to the local image characteristics. In these methods, a different T is selected for each pixel in the image.

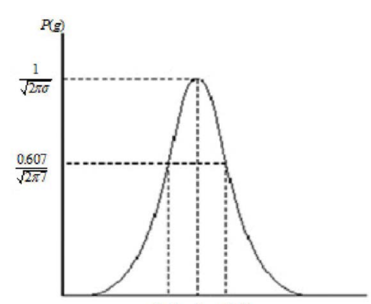


Figure 1. Hard and Soft Wavelet Thresholds.

4. Automatic Thresholding

It is a great way to extract useful information encoded into pixels while minimizing the background noise. This is accomplished by utilizing a feedback loop to optimize the threshold value before converting the original gray-scale image to binary image. The vision is to separate the image into two parts namely background and foreground. It can done by firstly selecting initial threshold value, the mean 8-bit value of the original image. Than divide the original image into two portions that are pixel values which are less than or equal to the threshold, background. And pixel values which are greater than the threshold, foreground. Average mean values of the two images obtained (background, foreground) is calculated. We will then calculate the new threshold value by taking the average of the two means. If the difference between the previous threshold value and the new threshold value are below a specified limit, than this automatic thresholding fails. Otherwise new threshold value is applied to the original image.

5. Problems in Thresholding

Thresholding techniques face many problems rather challenges which can be overcome with improved methods, the first problem is that we consider only the intensity and relationship between pixels is either ignored or appropriate attention is not paid towards it, and secondly there is no guarantee that the pixels identified by the threshold process are contagious.

6. Limits and Thresholding Selection

A larger limit will allow a greater difference between successive threshold values. This will result in quicker execution but with a less clear boundary between background and foreground. Starting threshold is often chosen by taking the mean value of the gray-scale image. The starting threshold value can be picked using histogram in which the two separate peaks of the image histogram and calculating the average pixel value of those points. This approach allows the algorithm to converge faster and also allow much smaller limit to be chosen.

7. Method Limitations

Automatic thresholding will work at par when a good background to foreground contrast ratio exists. This means that the picture must be taken in good lighting conditions with minimal glare on the image. This equal randomness will make the normalized Gaussian noise curve to look like bell shape. The PDF of this noise model shows that seventy to ninety percent noisy pixel values of degraded image in between μ - s and μ + s. The shape of normalized histogram is almost similar in spectral domain.

8. Wavelet Thresholding

Wavelet thresholding is simple non-linear technique, easily understandable, which operates on one wavelet coefficient at a time. In its basic form, each coefficient of threshold is compared against the threshold. If it is found that the coefficient is smaller than the threshold, set it to zero. Whereas if the coefficient is larger, it is kept and modified for further iterations. Replacing all the small noisy coefficients by zero and doing inverse wavelet transform on the result may lead to reconstruction with the essential signal characteristics and with less noise. Wavelet transform has three major steps namely, A linear discrete wavelet transform, Nonlinear thresholding step and A linear inverse wavelet transform. Let us consider a signal {Sab, a, b = 1,2,...N} denotes the N X N matrix of the original image to be recovered and N is some integer power of 2. During transmission the signal is corrupted by independent and identically distributed (i.i.d) zero mean. White Gaussian Noise, Gab with standard deviation i.e. Gab ~ M(0, 2) as Nab = Sab + Gab. Let us consider Nab as noisy signal. Now we want to find an approximation Sab. The goal is to estimate the signal Sab from noisy observations Nab, such that Mean Squared Error (MSE) is minimum. I.e,

$$\left\| X - \bar{X} \right\|^2 = \frac{1}{M} \sum_{a=0}^{N-1} \left(Xa - \bar{X}a \right)^2 \qquad \text{Equation 1}$$

Let D and D^{-1} be the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its Inverse DWT respectively. Then the equation (1) can be written as , $d_{ab} = c_{ab} + \varepsilon_{ab}$ where d = D_N, c = D_S, ϵ = D_G. Since D is orthogonal transform, ε_b is also an i.i.d

Gaussian random variable with ab $\varepsilon \approx (0, \sigma^2)$. Now T(.) be the wavelet thresholding function then the wavelet thresholding based denoising scheme can be written as $X = D^{-1}\{T(D_N)\}$. Wavelet transform of noisy signal should be taken first and then thresholding function is applied on it. Finally, the inverse wavelet transformation is applied to the output to obtain the estimate S. We have two thresholding which can be used, namely, hard thresholding and soft thresholding. The hard thresholding is described as

$$f_h(S) = S \qquad \text{if } S \geq \lambda$$

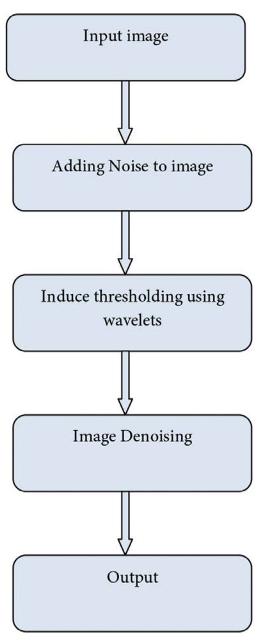
$$= 0 \qquad \text{otherwise}$$

The hard thresholding function chooses all wavelet coefficients that are greater than the given threshold λ and sets the other wavelet coefficients to zero. Signal energy and the noise variance(σ^2) are responsible for deciding the threshold λ The soft thresholding function on the other hand has a little different approach than the hard thresholding function. It shrinks the wavelet coefficients by λ towards zero,

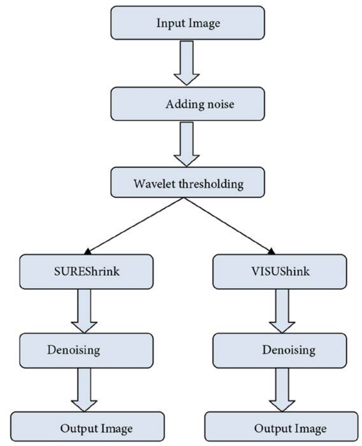
$$f(S) = S - \lambda \qquad \text{if } S \geq \lambda$$

$$= 0 \qquad \text{if } S < \lambda$$

$$= S + \lambda \qquad \text{if } S \leq -\lambda$$



Flowchart 1.



Flowchart 2.

We prefer soft thresholding rule over hard thresholding, as soft thresholding method yields more visually pleasant images over hard thresholding. The algorithm used in denoising can be shown using following flowchart 1;

Noise is responsible for corrupting an image during acquisition or transmission. The goal is to minimize or remove the noise, while retaining the important signal features as much as possible. The wavelet decomposition[10] of an image is done as follows: In the initial levels of decomposition, the image is split into 4 subbands, namely the HH, HL, LL, LH subbands. The HH subband gives the diagonal details of the image. The HL subband gives the horizontal features. The LL subband is the low resolution residual consisting of low frequency components and it also gets further split at higher levels of decomposition. The LH subband represent vertical structure. For denoising image the basic procedure remains same irrespective of the methodology used, which is as follows; Firstly, calculate the DWT of the image. Than threshold the wavelet coefficients, it may be universal or subband adaptive. And lastly, compare the IDWT to get the denoised estimate.

9. Thresholding Techniques

There are many thresholding techniques available for thresholding, some of which are briefly discussed here. *VISUshrink*[8] is the thresholding which is applied by the universal threshold proposed by Donoho[1] and Johnstone[2]. It is given by $\sigma\sqrt{2 * log * M}$, where σ is the noise variance and M is the number of pixels in the image. *SUREShrink*[9] is another thresholding technique, SURE (Stein's unbiased Risk Estimator) is a method for estimating the loss $\|\mu - \mu\|^2$ in an unbiased fashion. Steins result[] can be applied to get an unbiased estimate of the risk

$E\left\|\mu^{(t)}(x)-\mu\right\|^2$. SURE is represented by the following equation: $SURE\left(t;x\right)=d-2\cdot\#\{i:\left|x_i\right|<T\}+\sum_{(i=1)}^{d} min(|x_i|,t)^2$, where x be multivariate normal observations with mean vector μ[4].

10. Implementation of Algorithm

Implementation of algorithm described below will help in achieving better image processing. Our goal is to ensure that we collect information or data from the image with pinpoint accuracy. As astronomical and distant cosmic imagery is mostly full of noise and distortion in signals. As space is expanding there is no near future to when this randomness will condense. With proper technique and algorithm, patterns in cosmic imagery can be identified and distinguished apart from the regular randomness. As shown in the flowchart below, Input is taken from the images or data collected by astronomical lenses or sensors. This input image is added with noise, and Wavelet thresholding is applied to it. SUREShrink and VisuShrink is implemented on the sample. Both SUREShrink and VisuShrink samples get denoised and the output is generated.

11. RESULTS

The Daubechies wavelets[7] are based on the work of Ingrid Daubechies. They are a family of orthogonal wavelets that defines a discrete wavelet transform. Results of Daubechies (db) db2, db4, db8 for soft and hard thresholding in different denoising scheme are shown in Table 1. They are characterised by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal multiresolution analysis. Four signals that are standard in wavelet literature are used to investigate the effect of thresholding selection. Blocks, Bumps, Heavy Sine and Doppler signal results are shown in Table 1. Soft thresholding of Blocks and Bumps show similar results whereas there is a little deviation in db8 of Heavy Sine as compared with Block and Bumps. Doppler's soft thresholding in db2, db4, and db8 shows much deviation than Bumps and Blocks. On the other hand hard thresholding of Bumps and Heavy Sine is similar whereas Blocks and Doppler both are distinct among all four signals. Lena's image after adding noise and after denoising is shown in figure 3 and figure 4 respectively. Input Image is also shown in figure 2.

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13. Conclusion

It is concluded that for image compression, Haar Wavelet is better and more efficient than Daubechies because of named reasons, firstly, Haar wavelet working in additive computational process which leads to less utilization of computational space and memory. Secondly, this property can be utilized in improving computational space and memory management. Daubechies incorporates matrix multiplication computational technique which leads to

Figure 2. Original Image

Figure 3. After Adding Noise

Figure 4. After Denoising

Table 1. Hard and Soft Thresholding

Blocks			Bumps		Heavy Sine		Doppler	
	SOFT	HARD	SOFT	HEAD	SOFT	HEAT	SOFT	HEAT
Daube Chies 2	1.6	1.2	1.6	1.4	1.6	1.4	1.6	1.6
Daube Chies 4	1.6	1.2	1.6	1.4	1.6	1.4	2	1.6
Daube Chies 8	1.8	1.2	1.8	1.4	1.6	1.4	2.2	1.6

more utilization of computational space and memory. Using Haar 2D Wavelet gives complexity $O(4N^2 \mathrm{Log}_2 N)$[14] whereas 2 X 1D wavelet method has complexity $O(16/3N^2)$[15], one can with appropriate methodology, reduce this complexity further to $O(14/3N^2)$.

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