

Economic order quantity for multiple items in resource constraints

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Abstract

In this paper we present a model and solution methodology for production and inventory management problem that involve space constraint. The model formulation is quite general, allowing organization to handle a variety of multi-item decision such as determining order quantities. With the growing interest in dealing with the volatile demand fluctuations, there is a renewed interest to enhance responsiveness in capacity planning and in particular, to address the dynamic changes in product mix. We propose a new representation for the capacity utilization. With a more refined representation, overlapping and non-overlapping, capability regions can be identified and their inter-relationships with respect to capacity can then be applied for improving the match between the capacity and production orders. Thus, better planning and capacity utilization can be achieved. An example is presented to demonstrate the proposed approach and summarizes the work on a user interface for an MRP II planning system which has been developed using artificial intelligence techniques as a support tool for the final user. Experiences with real applications show that the success of such a system depends on the flexibility and facility to acquire the knowledge through a correct model and the facilities for displaying results.

Keywords: Material requirement planning, capacity planning, Economic order quantity (EOQ).

Introduction

The objective of capacity planning is to ensure the right match between the available capacity in specific work centers and the capacity required to achieve planned production. This is crucial in realizing the potential benefits of manufacturing planning and control systems. If capacity planning overestimates the available capacity, WIP levels will escalate, and late deliveries will ensue. Conversely, an underestimation of the available capacity may lead to the underutilization of resources and lost sales. The success of the matching between the required and the available capacity depends on their correct identification (Coleman *et al.*, 1991; Drexel & Kimms, 1997; Enns, 1999; Wolsey, 2002; Choi & Enns, 2004).

A typical hierarchy of capacity planning usually consists of three phases. In the first phase, the overall planning of resources is performed followed by a rough-cut capacity planning to validate the particular master production schedule. In the second phase, the evaluation of capacity plans is based on detailed material requirements. In order to economize enormous amount of wealth held in material stocked as inventory in any industries or distributing agencies in the world the research analysis is going on the different areas of research viz. the design of a multilevel system; lot sizing in assembly system; the coordination of machine loading in manufacturing system and production and shipping schedules in a requirement planning system; warehouse retailer system etc. (Wemmerlov, 1981; Khare & Sadiwala, 1987; Webster & Francis, 1989; Bector *et al.*, 1992). In this paper, we formulate multi-product inventory under space constraint in order to maximize the availability of items for production.

Model formulation

If there is no interaction between the different items, and if we are not calculating the economic order quantity jointly (in terms of rupees) for the multiple items as one group, then it is difficult to fulfill the inventory requirement as desired by system. The model formulation is quite general, allowing organization to handle a variety of multi-item decision, such as determining order quantities. In production environment, there may be limited workspace for manufacturing different product families.

EOQs for the individual items will be calculated by the usual EOQ formula.

$$EOQ = \sqrt{2 Co. A / Cc}$$

Ordering cost = Co

Carrying cost = Cc

Annual requirement = A

The maximum space available is 8,500 Sq.ft; determine the optimal order quantities for the three items

Item	1	2	3
Price per unit (in Rs.)	38	71	30
Carrying cost (Rs./unit/year)	10	20	8
Ordering cost (Rs./order)	280	375	295
Annual requirement (No. of units)	6400	9050	5000
Space required (Sq. ft./unit of item)	4	8	13

First let us find the EOQ's without the space constraint and see if space is really a constraint.

$$EOQ = \sqrt{2 Co. A / Cc}$$

$$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10} = 598.66 \text{ Units.}$$

$$Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20} = 582.55 \text{ Units}$$

$Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10} = 543.13 \text{ Units}$
 If all the deliveries coincide at the same time, the space requirement, would we
 $(598.66 \times 4) + (582.55 \times 8) + (543.13 \times 13) = 14,116 \text{ Sq. ft.}$
 This requirement is much above the maximum available area, which is only 8500 Sq. ft.
 For solving this problem let us now bring in the Lagrange Multiplier Procedure

$Q_i \text{ opt} = \sqrt{2A_i \cdot C_{oi} / (C_{ci} + 2K_i \cdot \lambda)}$
 $i = 1, 2, 3, \dots, n.$
 λ Value should be such that it satisfies the equation.
 n
 $\sum_{i=1}^n K_i Q_i = K$
 $i=1$

Since the $Q_{i \text{ opt}}$ and λ values are interdependent, a trial and error procedure is usually carried out assuming different values of λ in order to satisfy the above given constraint equation (Fig.1-5).

(1) If we start with a value of $\lambda=3$ in appropriate units, then

$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10 + (2 \times 4 \times 3)} = 325 \text{ Units.}$
 $Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20 + (2 \times 8 \times 3)} = 316 \text{ Units}$
 $Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10 + (2 \times 13 \times 3)} = 185 \text{ Units}$

Therefore, with this trial, the space requirement is
 $(325 \times 4) + (316 \times 8) + (185 \times 13) = 6249 \text{ Sq. ft.}$
 This is much lower than the available space, 8500 Sq. ft.
 (2) If the value of $\lambda= 2$, then

$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10 + (2 \times 4 \times 2)} = 371 \text{ Units.}$
 $Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20 + (2 \times 8 \times 2)} = 361 \text{ Units}$
 $Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10 + (2 \times 13 \times 2)} = 221 \text{ Units}$
 Therefore, the space requirement is
 $(371 \times 4) + (361 \times 8) + (221 \times 13) = 7245 \text{ Sq. ft.}$

Which is still much lower than the available space, 8500 Sq. ft.

(3) If the value of $\lambda= 1$, then

$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10 + (2 \times 4 \times 1)} = 446 \text{ Units.}$
 $Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20 + (2 \times 8 \times 1)} = 434 \text{ Units}$
 $Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10 + (2 \times 13 \times 1)} = 295 \text{ Units}$

Therefore, the space requirement is
 $(446 \times 4) + (434 \times 8) + (295 \times 13) = 9091 \text{ Sq. ft.}$
 This is slightly more than the maximum available space, 8500 Sq. ft.

(4) If the value of $\lambda= 1.1$, then

$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10 + (2 \times 4 \times 1.1)} = 437 \text{ Units.}$

$Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20 + (2 \times 8 \times 1.1)} = 425 \text{ Units}$

$Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10 + (2 \times 13 \times 1.1)} = 284 \text{ Units}$

Therefore, the space requirement is
 $(437 \times 4) + (425 \times 8) + (284 \times 13) = 8840 \text{ Sq. ft.}$

(5) If the value of $\lambda= 1.2$, then

$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10 + (2 \times 4 \times 1.2)} = 428 \text{ Units.}$

$Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20 + (2 \times 8 \times 1.2)} = 416 \text{ Units}$

$Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10 + (2 \times 13 \times 1.2)} = 274 \text{ Units}$

Therefore, the space requirement is

$(428 \times 4) + (416 \times 8) + (274 \times 13) = 8602 \text{ Sq. ft.}$

If the value of $\lambda= 1.25$, then

$Q1 \text{ opt} = \sqrt{2 \times 6400 \times 280 / 10 + (2 \times 4 \times 1.25)} = 423 \text{ Units.}$

$Q2 \text{ opt} = \sqrt{2 \times 9050 \times 375 / 20 + (2 \times 8 \times 1.25)} = 412 \text{ Units}$

$Q3 \text{ opt} = \sqrt{2 \times 5000 \times 295 / 10 + (2 \times 13 \times 1.25)} = 270 \text{ Units}$

Therefore, the space requirement is
 $(423 \times 4) + (412 \times 8) + (270 \times 13) = 8498 \text{ Sq. ft.}$
 This is almost close to the maximum space available 8500 Sq. ft.

Therefore, the value of λ is 1.25 and the optimal order quantities for the three items are:

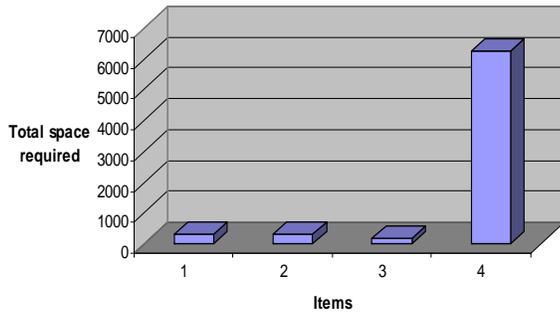
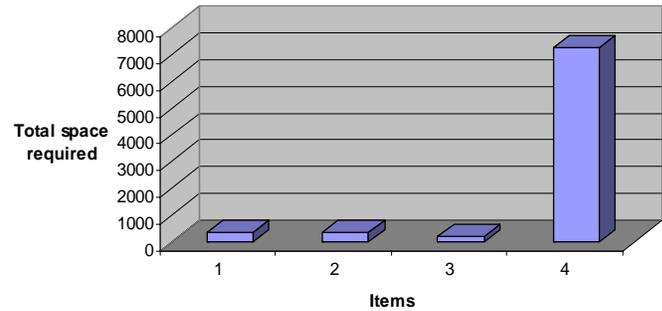
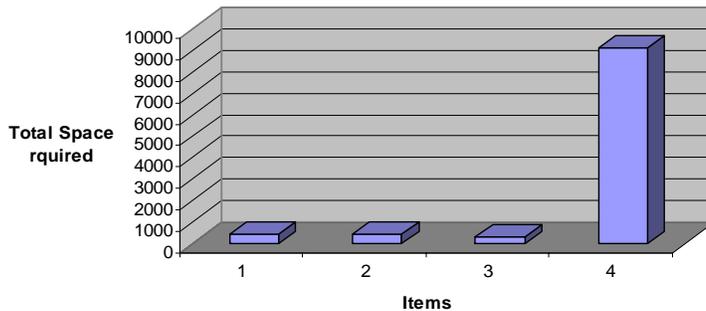
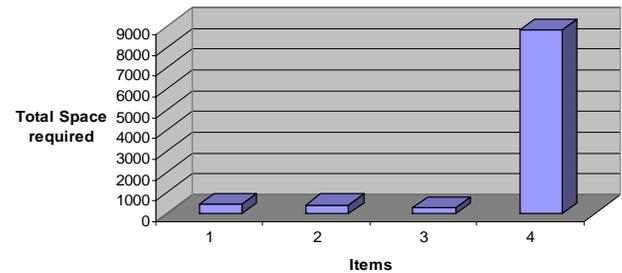
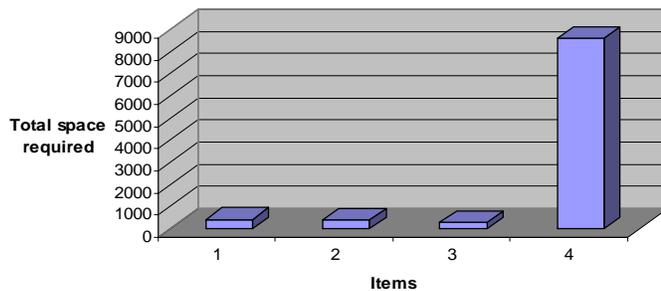
$Q1 \text{ opt} = 424, Q2 \text{ opt} = 412, Q3 \text{ opt} = 270$

Based on above resulted these items can be ordered in following quantities ($Q1 \text{ opt} = 424, Q2 \text{ opt} = 412, Q3 \text{ opt} = 270$) for better utilization of available space (i.e 8500 Sq. ft.).

Conclusion

In this paper we have analyzed a model for multi-tier inventory systems, a more general class of inventory systems than a forward-reserve or multi-mode inventory system under space constraint. Multi-tier inventory systems can be commonly found in warehouses; systems can provide guidance for warehouse personnel based on the results of our case study.

This study help to identify ways to reduce inventory, based on total space available. Appropriately managing the quantities in which they are required to store there. The most important area we have identified for future research is therefore to develop methods for multi-tier inventory systems under space constraint and so on. We believe it will be valuable to study multi-tier inventory systems where the inventory cost is major concern.

Fig. 1. Items vs total space required ($\lambda=3$)Fig. 2. Items vs total space required ($\lambda=2$)Fig. 3. Items vs total space required ($\lambda=1$)Fig. 4. Items vs total space required ($\lambda=1.1$)Fig. 5. Items vs total space required ($\lambda=1.2$)

Solving such problems manually, shall be very intensive and impractical for a normal sized warehouse. For this reason, heuristics on data for multiple items under different constraint can be developed for future research. All the heuristics can be easily codeable on software that to be used for warehouses. In future work, tighter bounds will be determined and heuristics will be developed for inventory systems.

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