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Optimal Placement of an Actuator for Suppressing Vibrations

Neeraj Singhal[1]*, Manu Sharma[2] and S. K. Mangal[1]

[1]Department of Mechanical Engineering, PEC University of Technology, Chandigarh - 160012, India; neerajatpec@rediffmail.com, skmangal_pec@rediffmail.com
[2]UIET, Panjab University, Chandigarh - 160014, India; manu@pu.ac.in

Abstract

Objective: To observe the effect of location of an actuator on the performance of a smart structure based on active vibration control scheme using an optimization criterion which is optimized by employing GA as a meta-heuristic optimization technique. **Methods/Analysis:** In this work a two degrees of freedom spring-mass-damper system has been considered with a hypothetical actuator placeable at three different locations. Equations of motion derived from free body diagrams are converted into a system of first order ordinary differential equations to get a state space model. Control gains for suppressing transient vibrations are obtained using Linear Quadratic optimal control. For optimal placement of hypothetical actuator, an optimization criterion (performance index) consisting of sum of transient displacements of both the masses is formulated. MATLAB optimization toolbox based Genetic Algorithm (GA) solver is employed to obtain optimal location of an actuator by minimizing the optimization criterion. **Findings:** It is found that optimal actuator location gives minimum value of performance index. By placing the actuator at 1st location performance index increases by 1.31 times and by placing at 2nd location it increases by 2.87 times the case when actuator is located at optimal location suggested by GA i.e. 3rd location. It is therefore concluded that an optimal location of an actuator plays a vital role in performance of an active vibration control scheme and GA can be easily employed for finding optimal actuator location. **Novelty/Improvements:** To optimally place an actuator in a typical actively controlled structure a strategy i.e. optimization criterion based on sum of transient vibrations of a smart structure has been suggested. GA can be used to solve this optimization problem.

Keywords: Active Vibration Control, Actuators, Genetic Algorithm, Optimal Placement, Optimization Criterion, Optimization Techniques, Smart Structure

1. Introduction

Excessive vibrations lead to malfunctioning and eventual failure of various structures, machines, automobiles, aircrafts, and civil structures etc[1-4]. It is important to keep vibrations within acceptable limits by using vibration control strategies. Vibrations can be controlled by using passive or active or semi active means. Passive Vibration Control (PVC) strategies are being used since time immemorial. In PVC, vibrations are controlled by changing mass or/and stiffness or/and damping of the structure. Since many centuries the basic strategy to control vibrations of a system is Passive Vibration Control (PVC). PVC uses three vibration controlling elements viz a resilient element (stiffness), mass and energy dissipating element (damper). In PVC, vibrations are controlled to obtain desired dynamic characteristics of a system either by increasing/decreasing the system stiffness or increasing/decreasing damping in the system or increasing/decreasing the system mass. Figure 1(a) shows that vibration of a mass 'm' can be passively controlled by suitably using constant stiffness spring 'K' and constant damping damper 'C'. In real world, a dynamic system has to operate over a wide frequency range, which is difficult to control using PVC techniques. To control low frequency vibrations, PVC increases overall weight of the system and hence is not suitable for applications where weight & space restrictions are present. This control technique

**Author for correspondence*

is most commonly used in automobiles, machine tools, building structures etc. In last two decades, lot of effort has been focussed on Active Vibration Control (AVC) strategies for vibration control of structures. AVC strategy has been developed as a substitute of PVC strategy mainly for low frequency vibrations and systems where stringent weight & space restrictions are present[5,6]. In AVC, vibration characteristics are altered by changing effective mass or/and effective stiffness or/and effective damping of structure by electrical means (external source of energy). An actively controlled system consists of sensors to sense vibrations, processor to manipulate sensor signal and an actuator to apply force/moment on the structure as shown in Figure 2. Sensor signal is manipulated in the proces-

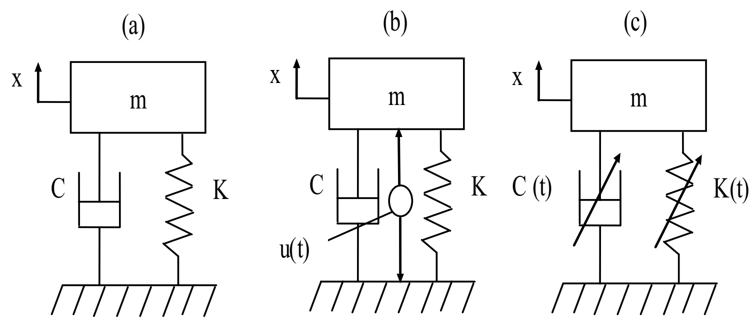

Figure 1. A Typical Primary Structure Equipped with Three Vibration Control Strategies a) Passive b) Active and c) Semi-Active Configuration.

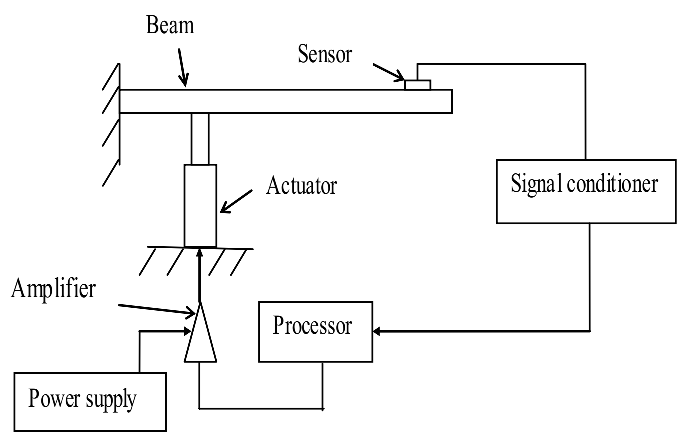

Figure 2. AVC Scheme Applied on a Cantilevered Beam.

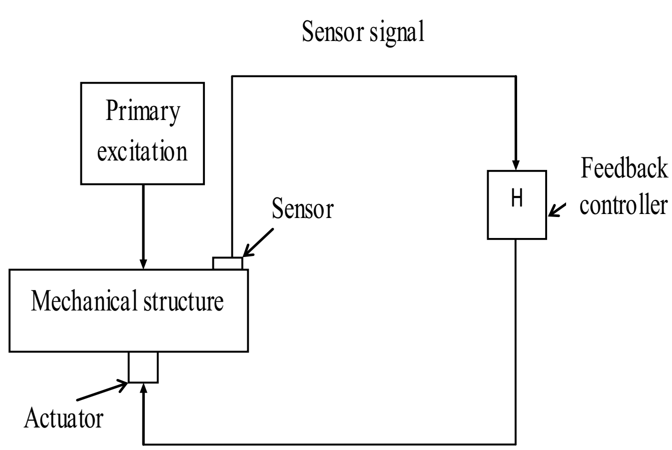

Figure 3. Feedback Control System.

sor using some control law. Figure 1(b) shows that in an AVC scheme vibrations of a mass 'm' can be controlled by using actuator force 'u(t)' in addition to passive elements. A structure equipped with some active vibration control scheme is called a smart structure[7-9]. Main disadvantage of this control technique is large capital cost and requirement of an external source of energy. This technique is suitable for aircrafts, helicopter, satellite, space structures, high span bridges, very large communication tower etc.

Semi Active Vibration Control (SAVC) strategy is based on some combination of above stated two control strategies. It offers adaptability of AVC without requiring large source of external power supply. Semi active vibration control system consists of a tuning controller with tuneable passive elements (spring and damper) as shown in Figure 1(c). Like PVC, this control technique is unsuitable for controlling low frequency vibrations and in applications where weight & space restrictions are present. This technique is mainly used in heavy vehicles, bridges, large building structures etc.[10]

In a typical AVC system, sensor signal is manipulated based on some control law. Control laws are either based on feedback control or on feedforward control depending upon the end application of a smart system. In feedback control, input to controller is error signal obtained from the sensor and accordingly control effort is applied by an actuator on the structure. Block diagram of feedback control system is shown in Figure 3. This control technique is used in those control systems where original excitation on the structure cannot be measured easily. This technique is effective in controlling free resonant response of a system. Control law based on feedforward control gives an improved performance over the control law based on feedback control. In feedforward control, disturbance which excites the system and error signal obtained from the sensor are used as inputs to the controller. An adaptive filter in the controller manipulates the signal that is correlated to primary disturbance and accordingly control effort is applied by an actuator to the structure. The block diagram of feedforward control system is shown in Figure 4. For implementation of feedforward control transfer function based on controller output to the error signal needs to be known. If this transfer function is not taken into consideration, it can lead to instability. Also, if control signal propagates back and is detected by the reference sensor, this can also lead to instability. This control law is effective for any frequency.

Performance of a smart structure depends upon sensors, actuators, control law and the processor. Performance of a smart structure is also dependent on judicious placement of sensors and actuators over a host structure. Designer of a smart structure is desired to place limited number of sensors and actuators over a host structure in an optimal manner. Piezoelectric materials are extensively being used in development of smart structures. There are number of options available to place a limited number of actuators/sensors over a structure. The actuators/sensors should be placed in such a way that leads to increase in performance of the system with better stability. It is usually better to place actuators/sensors considering the end application of the system. Locations of piezoelectric patches in the host structure at which system has highest performance are called optimal locations. These locations are determined by optimizing a designer defined performance index of the system which is also called optimization criterion. After finalizing the performance criterion, optimal locations of piezoelectric patches are determined based on some suitable search technique called as optimization technique.

Genetic algorithm has been used as an optimization technique to find optimal locations of two piezoelectric actuators over a simply supported plate taking population size of 30, mutation rate of 0.01, crossover rate of 0.6 and stopping criterion based on number of generations. Fitness function based on modal controllability/controllability Grammian has been maximized[11]. The locations of piezoelectric actuators and sensors for a cantilevered plate have been obtained using Genetic algorithm. The fitness function based on modal controllability and observability with spillover effects is maximized. The population size, mutation rate, crossover rate and stopping criterion have been kept in the reasonable range to avoid convergence at local optimum[12]. The locations of piezoelectric patches over a beam for different boundary conditions by maximizing dynamic deflection have been found using GA[13]. The technique has also been used for finding locations of piezoelectric actuators & sensors over a cantilevered plate by minimizing the performance index based on LQR[14]. Optimal locations of piezoelectric patches over an inflated toroidal shell have been determined using GA by selecting suitable values of various GA parameters. The performance index has been selected on the basis of controllability/observability with spillover effects[15]. In this work, a simple two degrees of freedom spring-mass-damper system has been considered. Genetic Algorithm (GA) has been employed to find optimal location of a hypothetical actuator for active control of vibrations. In this work, transient values of vibrations produced have been used to find optimal location of an actuator. Performance index has been created by adding all the transient vibrations of the system. Genetic algorithm has then been used to minimize the performance index. Following section discuss optimization criterion, Genetic Algorithm (GA) and effect of a hypothetical actuator on the performance of two degrees of freedom system.

2. Effect of Location of an Actuator on the Performance of a Two Degrees of Freedom System

To understand the basic principle of active vibration control and to analyze the effect of location of an actuator on performance of an AVC system, a simple two degrees of freedom mass-spring-damper system is considered. Two degrees of freedom system consists of two masses M_1& M_2 connected with three springs of stiffness K_1, K_2 & K_3 and three dampers having damping constants C_1, C_2, & C_3 as shown in Figure5. To excite these two degrees of freedom system an initial disturbance of 1mm is applied on mass M_1. An algorithm to obtain uncontrolled transient vibrations for this system is coded in MATLAB and response of system is plotted as shown in Figures 8 and 9. To control these vibrations, a hypothetical actuator capable of applying force 'f' is placed in this system. Three possible locations of actuator are: between left fixed end & first mass M_1(1st location), between masses M_1 & M_2(2nd location) and between mass M_2 & right fixed end of the system (3rd location). To obtain optimal location of an actuator an optimization criterion (performance index) based on sum of transient displacements of both masses over the entire span of time is developed in discrete time as under:

$$Minimize\ J = \sum_{k=k_0}^{k_f}\left(x_1 + x_2\right) \tag{1}$$

where k_0 is initial time instant, k_f is final time instant, x_1 & x_2 are transient displacements of mass M_1& mass M_2 respectively. MATLAB optimization toolbox based GA solver is used to solve optimization problem. In GA fitness function based on optimization criterion to be optimized is created in MATLAB. Actuator location is taken as optimization variable to find an optimal solution out of all possible combinations ($^3C_1$). In GA, the point at which value of fitness function (fitness value) is obtained is called individual. Individual represents vectorial form

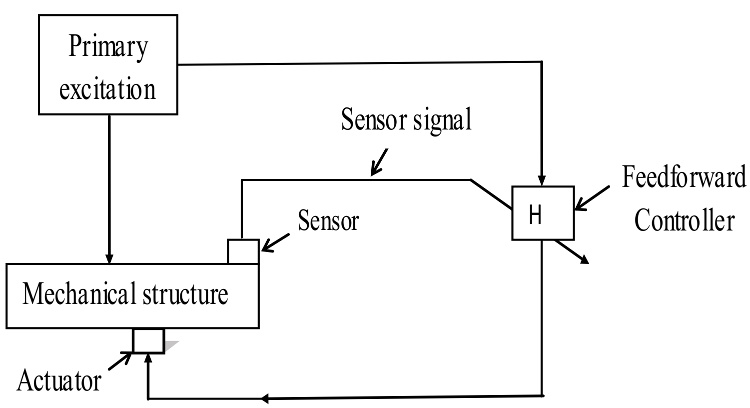

Figure 4. Feedforward Control System.

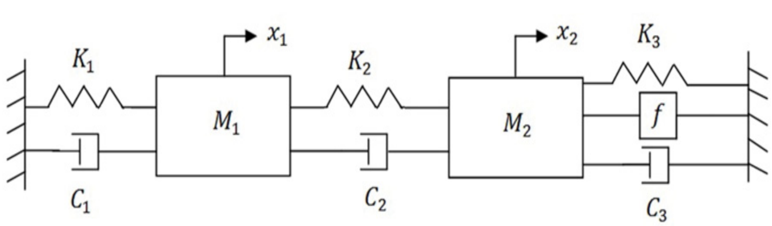

Figure 5. Schematic Diagram of Two Degrees of Freedom mass-spring-damper System.

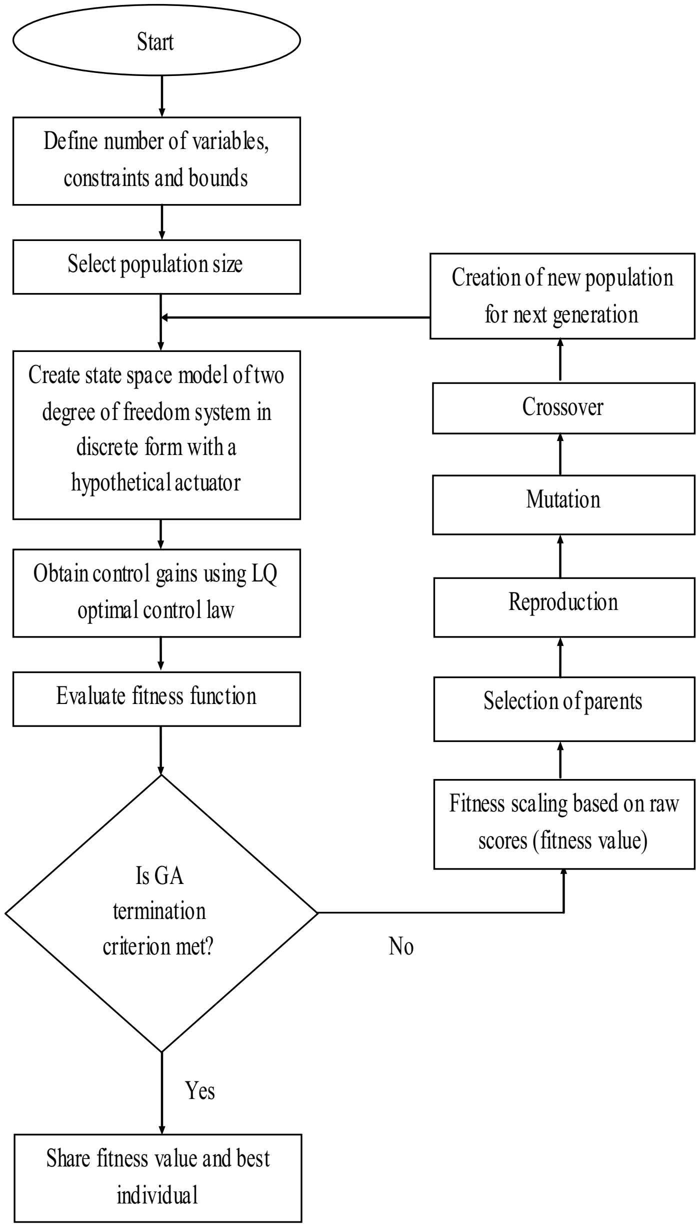

Figure 6. Flow Chart to Solve Optimization Problem.

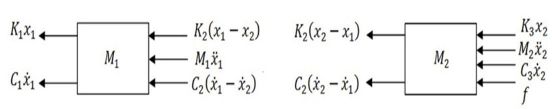

Figure 7. Free Body Diagrams of Masses M_1 and M_2.

of variables affecting the fitness function. Vector entries of individual (variables affecting a fitness function) are called chromosomes. GA solver is binary coded in which optimization problem is represented by strings of binary numbers 0 and 1. In binary coded GA, information of each chromosome is contained in different sections of string. Detailed algorithm to solve optimization problem using GA is shown in Figure 6. GA terminates after meeting the stopping criteria which is based on combination of a well-defined number of generations, stall generations, fitness limit and function tolerance. Various parameters affecting the optimization problem are defined before execution of GA as:

Population size = 10
Crossover fraction = 0.8
Elite children = 2
Mutation function = adaptive feasible
Crossover function = scattered
Number of generation = 50
Number of stall generation = 25
Fitness limit = - infinite
Tolerance function = 10^{-6}

Free body diagrams of both masses while placing the hypothetical actuator at 3rd location are shown in Figure 7. Based on free body diagrams of both masses, equations of motion of masses M_1 and M_2 are written as:

$$M_1\ddot{x}_1 + K_2(x_1 - x_2) + C_2(\dot{x}_1 - \dot{x}_2) + K_1x_1 + C_1\dot{x}_1 = 0$$
$$M_2\ddot{x}_2 + C_3\dot{x}_2 + K_3x_2 + f + K_2(x_2 - x_1) + C_2(\dot{x}_2 - \dot{x}_1) = 0 \quad (2)$$

These second order ordinary differential equations are converted into a first order state-space model by taking:

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = x_4 \quad (3)$$

now equations (2) are rewritten as:

$$M_1x_3 + K_2(x_1 - x_2) + C_2(x_3 - x_4) + K_1x_1 + C_1x_3 =$$
$$M_2x_4 + C_3x_4 + K_3x_2 + f + K_2(x_2 - x_1) + C_2(x_4 - x_3) = \quad (4)$$

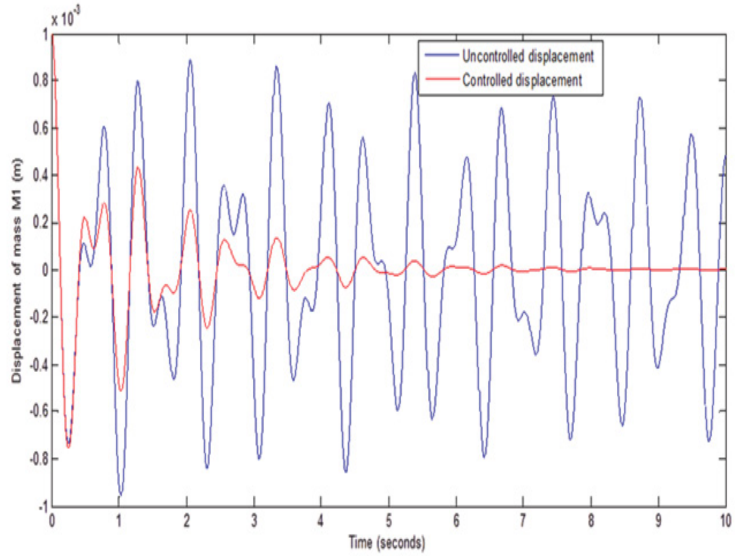

Figure 8. Controlled and Uncontrolled Time Response of Mass M_1 when Actuator is located at Optimal Location.

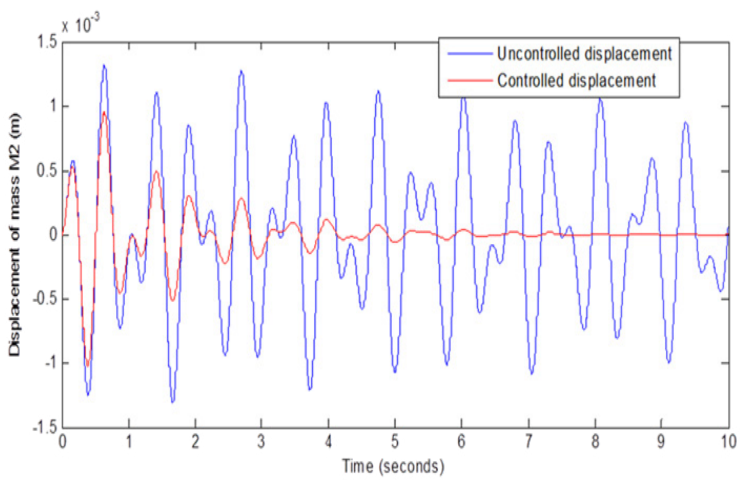

Figure 9 Controlled and Uncontrolled Time Response of Mass M_2 When Actuator is Located at Optimal Location.

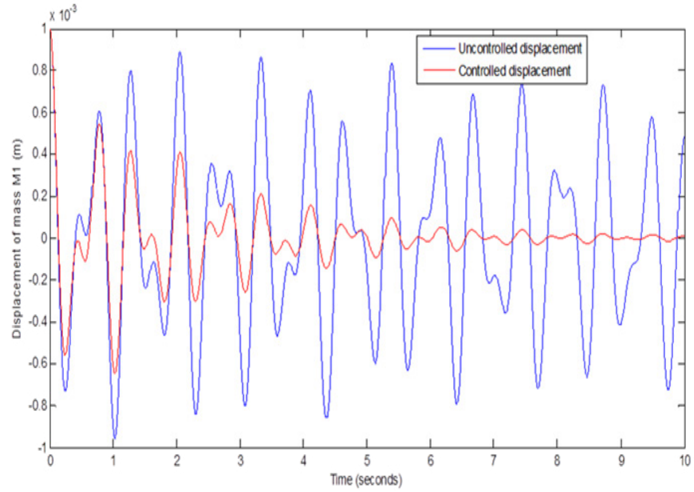

Figure 10 Controlled and Uncontrolled time Response of Mass M_1 when Actuator is Located at 1^{st} Location.

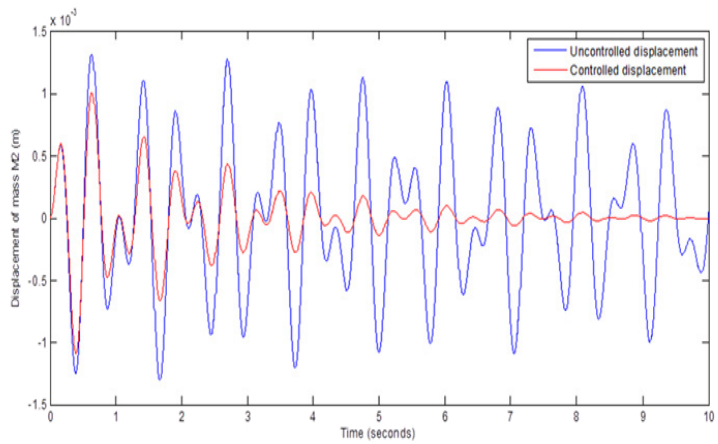

Figure 11. Controlled and Uncontrolled Time Response of Mass M_2 when Actuator is Located at 1^{st} Location.

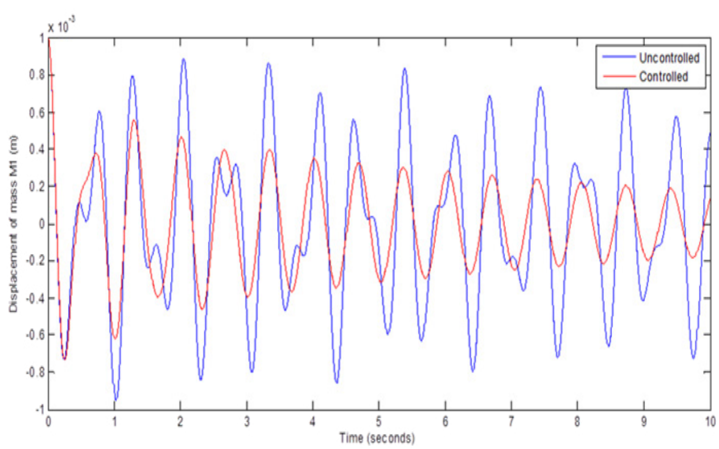

Figure 12. Controlled and Uncontrolled Time Response of Mass M_1 when Actuator is Located at 2^{nd} Location.

after rearranging the terms equations (4) come out as:

$$\dot{x}_3 = -\frac{(K_1+K_2)}{M_1}x_1 + \frac{K_2}{M_1}x_2 - \frac{(C_1+C_2)}{M_1}x_3 + \frac{C_2}{M_1}x_4$$
$$\dot{x}_4 = \frac{K_2}{M_2}x_1 - \frac{(K_2+K_3)}{M_2}x_2 + \frac{C_2}{M_2}x_3 - \frac{(C_2+C_3)}{M_2}x_4 - \frac{f}{M_2} \tag{5}$$

equations (3) and (5) are expressed in matrix form as:

$$\{\dot{x}\}_{4\times1} = [G]_{4\times4}\{x\}_{4\times1} + \{H\}_{4\times1}\{f\}_{1\times1} \tag{6}$$

where

$$x = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}$$

$$[G] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{(K_1+K_2)}{M_1} & \frac{K_2}{M_1} & -\frac{(C_1+C_2)}{M_1} & \frac{C_2}{M_1} \\ \frac{K_2}{M_2} & -\frac{(K_2+K_3)}{M_2} & \frac{C_2}{M_2} & -\frac{(C_2+C_3)}{M_2} \end{bmatrix} [H] = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{-1}{M_2} \end{Bmatrix}$$

Control law for this system is expressed as:

$$f_{1\times1} = -\{K\}_{1\times4}\{x\}_{4\times1} \tag{7}$$

where $\{K\}_{1\times4}$ is vector of control gains and can be easily obtained using Linear Quadratic optimal control i.e. DLQR command in MATLAB. State matrix G and control vector H have been discretized into matrix A and control vector B with a sampling time of 0.001 seconds. State-space equations are solved in discrete form using following algebraic equation:

$$x_{4\times1}(k+1) = A_{4\times4}x_{4\times1}(k) + B_{4\times1}f(k) \tag{8}$$

where 'k' is sample number. In this work, we have assumed $M_1 = 2M_2 = 10$ kg, $K_1 = 2K_2 = 3K_3 = 1000$ N/m, $C_1 = 0.3$ Nsec/m, $C_2 = 0.25$ Nsec/m, $C_3 = 0.20$ Nsec/m. Genetic algorithm as detailed in Figure 6 when applied to this problem suggests optimal location of an actuator at 3rd location. Using DLQR command in MATLAB and taking weight matrix Q = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1] and control weight factor R=0.01, control gains are derived as:

$$K = [12.3341 \quad -3.4247 \quad -1.1017 \quad -11.5433] \tag{9}$$

Taking initial condition at time = 0 second as $x^T = [0.001\ 0\ 0\ 0]$, the controlled and uncontrolled response of both the masses are shown in Figures 8 and 9. Placing an actuator at optimal location control system has successfully suppressed vibrations of both the masses effectively. The effects of placing a hypothetical actuator at a location other than optimal are also be observed. When actuator is placed at 1st location, control vector H comes out as

$$[H] = \begin{Bmatrix} 0 \\ 0 \\ \frac{1}{M_1} \\ 0 \end{Bmatrix} \tag{10}$$

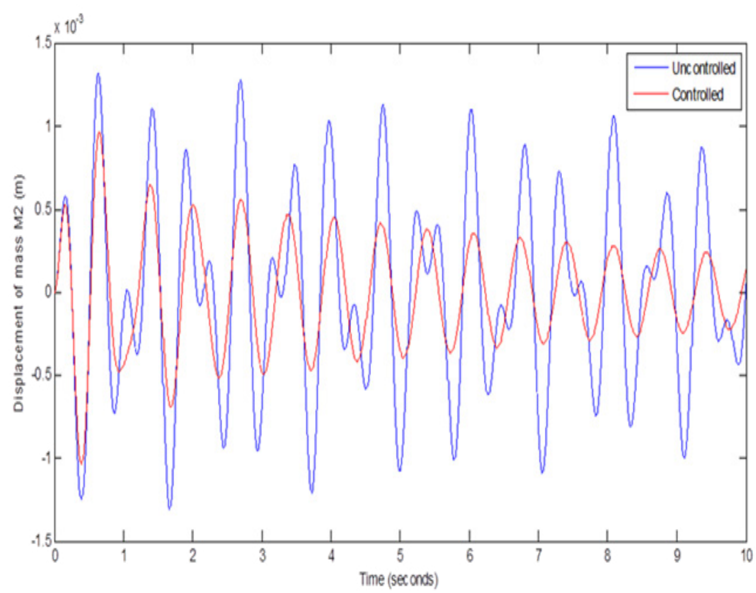

Figure 13. Controlled and Uncontrolled time Response of Mass M_2 when Actuator is Located at 2nd Location.

Control gains are derived as:

$$K = [\ 7.2806 \quad -13.6271 \quad 15.8683 \quad -0.3105] \tag{11}$$

Table 1. Value of performance index with optimal actuator location and randomly selected actuator locations

Location of Actuator	Value of Performance Index
1st: between left fixed end and mass M_1	2.24
2nd location: between masses M_1 and M_2	4.90
Optimal location (3rd): between mass M_2 and right fixed end	1.71

Controlled and uncontrolled time response of both the masses is shown in Figures 10 and 11. When actuator is placed at 2nd location, control vector H comes out as:

$$[H] = \begin{Bmatrix} 0 \\ 0 \\ \frac{-1}{M_1} \\ \frac{1}{M_2} \end{Bmatrix} \tag{12}$$

Control gains are derived as:

$$K = [\ -7.4276 \quad -3.5422 \quad -0.1913 \quad 11.1165] \tag{13}$$

Controlled and uncontrolled time response of both the masses is shown in Figures 12 and 13. Vibration suppression of both the masses is better when hypothetical actuator is placed at GA suggested optimal location (3rd location) than when the actuator is placed either at 1st or 2nd location. Value of performance index by placing the actuator at GA suggested optimal location i.e. 3rd location and random locations i.e. 1st and 2nd locations are tabulated in Table 1. Optimal actuator location gives minimum value of performance index. By placing the actuator at 1st location performance index increases by 1.31 times and by placing at 2nd location it increases by 2.87 times the case when actuator is located at optimal location suggested by GA i.e. 3rd location. It is therefore concluded that an optimal location of an actuator plays a vital role in performance of an active vibration control scheme and GA can be easily employed for finding optimal actuator location.

3. Conclusions

In this work, a novel strategy has been suggested to optimally place an actuator in a typical actively controlled structure. This work, concludes that sum of transient vibrations of a smart structure can be effectively taken as performance index for optimal placement of an actuator. GA can be used to solve this optimization problem. Vibration suppression is best when actuator is placed as per the strategy suggested in this work.

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