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Di-Mesonic Molecules Mass Spectra

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Abstract

Objectives: The mass spectra of mesonic and di-mesonic bound state in the nonrelativistic scheme have been calculated. The strong interaction in molecular binding states has been considered and study for the Cornell and Molecular potentials. The main objective of this work is to implement the operator and oscillator techniques for determining mass spectrum and eigenvalue of multi colored particle bound states. **Methods:** This work presented the Schrodinger equation with the oscillator method that based on the quadratic form of coordinate and momentum operators in quantum chromodynamics and quantum field theories by adding a free quantum harmonic oscillator potential partto the Hamiltonian of interactions. **Findings:** The mass spectra of hadronic molecule with two mesonic systems have calculated. The binding energy of di-mesonic states is found and it shown that had relation with the oscillator frequency and strong interactions parameters. The effect of tensor term in one pion exchange can be determined by this method. It is remarkably pointing the effect of the tensor term and its contribution in hadronic multi colored systems, but here we neglected any spin-spins interactions and defined the results in the ground state. **Improvements:** The proposed operator and harmonic oscillator techniques reduce the theoretically calculation in compared with other method like volitional and lattice.

Keywords: Colored Particle, Cornell Potential, Di-Mesonic Molecule, Hadronic Bound States, Mass Spectra

1. Introduction

In the present article, author focuses on the hadronic bound states like meson-mesonic (di-mesonic) $DD^*, D^*D^*, D^0\bar{D}^{*0}, \bar{B}B^*, KD, KD^*, BB^*$ [1] and all bound states that are a possible mesonic molecular systems or as a multi colored particle structures. These types of multi colored particles have been studied since 1977. Based on one pion exchange potential model during 1991-1994 multi colored molecular states with heavy-heavy, heavy-light colored particles flavour have been predicted and in the 200-2004 theoretical physicists exactly presented in various theoretical approaches the bound state of DD^* DD* and $\bar{B}B^*$ as a possible hadronic molecular structures. Hence, determination of the binding energy and the di-mesonic mass became the important issue in theoretical particle physics. Various authors determined the bound state characteristics by different method and for that they offered various types of hadronic molecular potentials. Evidenced in the 2004 showed that molecular binding states could not bind to each other just by colored particle exchanging, therefore, creation of hadronic molecule should be include other exchanging mechanism that it presented as one pion interchange, which it depends on the gluon interchange between colored particles in strong interaction inside the mesons. So in the hadronic molecular like structures: meson-mesonic (di-mesonic) system, each meson that involves gluon exchange can describe by phenomenological well-known potential model like Cornell and the two

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mesons binding state by Molecular potential type model. The Cornell and Molecular potentials represent the residual strong interaction at short distance and are included for long range behavior of the strong force. Hence, these dependencies in this study have been confirmed by using Cornell and Molecular potential that related to the confinement model of multi colored particle binding states in strong interaction principles as follow:

$$W_m(x) \cong -\frac{4}{3x}\alpha_s + \sigma x\,, \quad W_h(x) \cong -\frac{k_{mol}}{x} e^{-\frac{\eta^2 x^2}{2}} \tag{1}$$

where k_{mol} is the residual strength of the strong interaction coupling and η is the effective color screening parameters of confined gluon. As we know one pion exchange potential could be broken into central and spins parts interactions. But in our theoretically and analytically results and calculations that will present below, we did not include the one pion exchange potential and all spins interactions. For solving Schrodinger equation and define binding energy and mass spectrum and analyzing di-mesonic state as multi colored hadronic system, we have used operatormethod in the principles of harmonic oscillator representation.

2. Cornell-Molecular Potentialin OM and LQHO Model

To determine ground or excited state energies and mass spectrum of the multi colored particle systems, we have applied operator method[2] (OM) and Linear Quantum Harmonic Oscillator[3,4] (LQHO) to solve Schrödinger equation and analyze the bound state specifications. OM and LQHO model have demonstrated success explaining the strong interaction potential of multi colored particle bound states. Bound state characteristic of two or more linked colored or uncolored can be described by the LQHO vibrations. Two colored particle bound system with the rest masses m_1, m_2 and the mass in the bound state μ_1, μ_2 will have the new description of mass parameter that named after constituent mass $\mu = (\mu_1 + \mu_2)/\mu_1\mu_2$. Therefore, the Schrodinger equation with the potential energy based on the leaner quantum oscillator can define the physical characteristics of binding system and read

$$E\ \psi(x) = (\mathrm{M} - m_1 - m_2)\ \psi(x)$$

$$\left(\frac{\hat{p}_1^{\,2}}{2} + \frac{\hat{p}_2^{\,2}}{2} + \mu \cdot W(x)\right)\psi(r) = \mu E\ \psi(x) = (\mathrm{M} - m_1 - m_2)\ \psi(x) \tag{2}$$

So, the bounding state Hamiltonian[5,6] by adding a free quantum harmonic oscillator potential part $\left(W(\hat{x}, E) - \frac{\omega^2 \hat{x}^2}{2}\right)$ read

$$\left(-\frac{1}{2}\hat{P}^2 + \mu W(x) \pm \frac{\omega x^2}{2}\right)\psi_{n\ell}(x) = \mu E\, \psi_{n\ell}(x) \tag{3}$$

where ω is oscillator frequency parameter of a free LQHO. The Hamiltonian of the simple linear quantum harmonic oscillator involves with the operator by the creation (â+) and annihilation (â-) operators in space time coordinate ($d = 4$, $\hbar$=c=1)and it can present as the quadratic form of coordinate ($\hat{x}$) and momentum ($\hat{p}$) operators as follows[5,6]:

$$\widehat{H}=\omega\left(\widehat{a}^{+}\widehat{a}^{-}\right)+\frac{\omega}{2},\quad \widehat{H}=\omega\left(\widehat{a}^{-}\widehat{a}^{+}\right)-\frac{\omega}{2},$$
$$\left(\widehat{a}^{-}\widehat{a}^{+}\right)=\frac{m\omega}{2}\widehat{x}^{2}+\frac{1}{2m\omega}\widehat{p}^{2}+\frac{i}{2}[\widehat{p},\widehat{x}],$$
$$\left[\widehat{a}^{-},\widehat{a}^{+}\right]=\widehat{a}^{-}\widehat{a}^{+}-\widehat{a}^{+}\widehat{a}^{-}=d \tag{4}$$

Now, Schrodinger equation could be solved with harmonic oscillator potential by plugging the quadratic form of coordinate and momentum operators and define the mass spectrum and ground state energy of di-meson molecular state based on the new coordinate space ($\widehat{x}=q^{2n}$) determination. Hence, the radial Schrödinger equation with potential oscillator with the quadratic form of coordinate and momentum operators reads[7,8]

$$H_{total}\psi(r)=E\psi(r),$$
$$H_{total}=H_0+H_{E_0}+H_I,$$
$$H_0=\omega\left(\widehat{a}^{+}\widehat{a}^{-}\right) \tag{5}$$

Where H_0 is free oscillator Hamiltonian, $H_{E_0}=E_0$ is ground state energy and H_I is the interaction Hamiltonian. The new parameters x, q, ρ being offered to describe the Hamiltonian in quadratic form from Equation (1) with the new dimension (d) of the auxiliary space, so the Hamiltonian and the wave function of strong interaction will depend on them and one can present these relations as follow:

$$x=q^{2n},$$
$$d=4\rho\ell+2\rho+1,$$
$$\psi_{n\ell}(x)=q^{2n\ell}\psi_{n\ell}(q),$$
$$\widehat{\mathrm{P}}_{\ell}=\frac{1}{2}\left(\frac{\partial^2}{\partial q^2}+\frac{d+4\ell-1}{q}\frac{\partial}{\partial q}\right) \tag{6}$$

Hence, the radial Schrodinger equation with these new parameters for mesonic and di-mesonic bounding states read[9-12]

$$\left(\widehat{P}_{\ell}-\mu_m\left(\frac{4}{3}q^{-2\rho}\alpha_s+\sigma q^{2\rho}-4q^{2\rho}E_m\right)\right)\psi_m(q)=0,$$
$$\left(\widehat{P}_{\ell}-\mu_h\left(k_{mol}q^{2(1-\rho)}e^{-\frac{c^2q^{4\rho}}{2}}-4q^{2\rho}E_h\right)\right)\psi_h(q)=0 \tag{7}$$

Where the coordinate operators ($\widehat{x},\widehat{q}$) and the ω oscillator frequency parameter for mesonic and di-mesonic systems can be determined by the quadratic term $:\widehat{q}^2:$ that don't include in the interaction Hamiltonian parts for $n=1$:

$$\widehat{q}^2=\frac{d}{2\omega}+:\widehat{q}^2:,\quad \widehat{p}^2=\frac{d\omega}{2}+:\widehat{p}^2: \tag{8}$$

so one can rewrite (8) as below [2]:

$$\widehat{x}=\widehat{q}^{2n}=\frac{\Gamma(2\rho\ell+\rho+1+n)}{\omega^n},\quad \widehat{p}^2=(4\rho\ell+\rho+1)\omega. \tag{9}$$

The oscillator frequency parameter (ω) form (4) quadratic oscillator condition of each system can be defined. Based on analytically method that can define in the references, the interaction of multi colored particle in the mesonic and di-mesonic systems by OM and LQHOmethod give us the energy engine value and mass spectrum of bound states, that can read for mesonic system in ($\ell = 0$) ground state[7,8]

$$M_m = \mu_q + \mu_{\bar{q}} + \mu_m \frac{dE_m}{d\mu_m} + E_m,$$
$$E_m = \frac{\Gamma(2+\rho)}{8\rho^2\Gamma(3\rho)}\omega^{2\rho} - 4\alpha_s \frac{\Gamma(2\rho)}{3\Gamma(3\rho)}\omega^{\rho} + \sigma\frac{\Gamma(4\rho)}{\Gamma(3\rho)}\omega^{-\rho},$$
$$\mu_q = \left(m_q^2 - 2\mu_m^2 \frac{dE_m}{d\mu_m}\right)^{1/2}, \quad \mu_{\bar{q}} = \left(m_{\bar{q}}^2 - 2\mu_m^2 \frac{dE_m}{d\mu_m}\right)^{1/2}, \quad \mu_m = \frac{\mu_q \cdot \mu_{\bar{q}}}{\mu_q + \mu_{\bar{q}}}. \tag{10}$$

and for the di-mesonic system read

$$M_h = \mu_{m_1} + \mu_{m_2} + \mu_h \frac{dE_h}{d\mu_h} + E_h,$$
$$E_h = \frac{\Gamma(2+\rho)}{8\rho^2\Gamma(3\rho)}\omega^{2\rho} - \frac{\Gamma(2\rho)}{\Gamma(3\rho)}\omega^{\rho} e^{-\frac{\eta^2 d}{4\omega^{2\rho}}},$$
$$\mu_{m_1} = \left(m_{m_1}^2 - 2\mu_h^2 \frac{dE_h}{d\mu_h}\right)^{1/2}, \quad \mu_{m_2} = \left(m_{m_2}^2 - 2\mu_h^2 \frac{dE_h}{d\mu_h}\right)^{1/2},$$
$$\mu_h = \frac{\mu_{m_1} \cdot \mu_{m_2}}{\mu_{m_1} + \mu_{m_2}}. \tag{11}$$

Hence, the spectrums of two colored particles theoretically have been determined with the different type of potential in strong interactions a long time ago. But these mesonic states continue to provide very important exotic multi colored states that binding together and create new hadronic states that are largely predicted for the hadronic molecules are very valuable. Nowadays, these theoretically predicted states have been discovered in LHCs. LQHO model is clearly one of the best ways to predict these states and determine physical parameters like mass and energy. Base on life time and predicted creation of multi colored states in this article we theoretically presented possible creation and shown calculations of mass spectrum including the u, d, ands quarks inside mesonic and hadronic molecules. The exotic hadronic states include multi colored particles states such as two, three, four and etc. colored particle states are studied in this article as di- mesonic molecules in a LQHO model. In the LQHO Model extended for colored particles interactions, the colored and anti-colored particles inside a hadron are under the action of a strong interaction and mean field potential of the form Cornell, Yukawa and etc. We have computed the mass of the multi colored particle states by considering them as di-mesonic state. The mesonic states have been created as that due to the asymptotic behavior of the confined one gluon exchange between two or more colored particles. Thedi-mesonic or hadronic molecule masses have been created from the molecular like potential in chemical molecules but with the strong-molecular interaction as we describe above. We determined colored particle binding states as mesonic and as molecular states in a systematic way, including both potentials in strong and in hadronic molecular interactions. Using these potentials, we calculate mesonic and molecular correlation functions and perform quantum chromodynamics rule and OM plus LQHO model to obtain mass spectrum of the mesonic and hadronic molecular states. Masses of multi colored particles for these various masses for u, d, c quarks are extracted which are in good agreement with observed masses of the meso-hadron molecules like DD^*, BB, BB^*, etc. To study the exotic hadronic molecules, we can construct for all colored particle with specific flavors including different type structures. We calculated the two colored particles (mesonic state) and with four colored particle (hadronic molecule or di-mesonic state based on Equation (7). In this section the major features of the presented model has been summarized, which can be used to determine ground state energy and mass of di-mesonic system.

3. Conclusion

In this work, author has calculated the mass spectrum of mesonic and di-mesonic ground state in the OM and LQHO model. The inter-mesonic interaction considered as the combination of colored and hadronic molecule potentials. For the mass spectrum of the bound systems the strong interaction of colored particles and their properties in long and short distance structure of hadronic molecule have been studied. The theoretical properties of flavored di-mesons are presented with the coupled potentials. The mass of colored particle lie in the range of 0.2 GeV for light mesons to 4.85GeVfor heavy mesons. They are in close agreement with the values commonly presented in the other theoretical analysis. We present our calculation with the combination of Cornell and Molecular potentials that have been provided the experimental values of the properties of multi colored states. The LQHO approach employed here is found to be successful in the study of mesons and di-mesons as molecular states. It can be extended for the study of four, five and more than five colored systems and also seems to be success to predict the various properties of hadronic molecules. The bound state the results and parameters have been obtained here can be useful in the study of the resonance and decay. The masses of mesonic and di-mesonic states in semi non-relativistic approach have been computed. The interdi-mesonic interaction has been taken as a Van Der Waals force. The masses of colored particle and mesons are taken from Particle Data Group. Mesonic and hadronic molecular parameters have been taken from previous works. So, it is interesting to discuss the obtained results with some exotic candidates and also predict new hadron molecules states.

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