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Application of algebra to geometry

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Abstract: The famous unsolved classical problem such as trisection of a general angle, doubling the cube, squaring the circle, to draw a regular septagon and to prove the parallel postulate of Euclidean geometry as a theorem are forgotten by the research community. These problems were shown mathematically and logically impossible to solve. In this work the author made a brief survey and attempted to establish the fifth Euclidean postulate.

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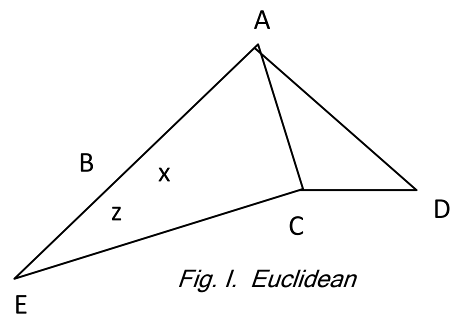

Fig. I. Euclidean

Introduction

The parallel postulate of Euclidean geometry is 2300 years old. Euclid proved all the proposition of his famous Elements based upon the following five postulate.

1. For every point P and for every point Q not equal to P there exists a unique line that passes through P and Q.
2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.
3. For every point O and every point A not equal to O there exists a circle with centre O and radius OA.
4. All right angles are congruent to each other.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

The first four postulates mentioned above are obvious. Euclid himself trotted his mind for a very long period to show a proof for his much deep effort and controversial parallel postulate. Since he was not able to locate a proof, he gave up his efforts and attempts and assumed it to be a true statement. After Euclid great mathematicians like Proclus, Possidoniuss, Wallis, Omarkhayyam, Saccheri, Lambert, Gauss, Bolyai, Lobachesky, Riemann and Beltrami attempted and slept on this controversial problem from 40 to 50 long years in order to prove this as a theorem. But their attempts were in vain. Their studies produced the following propositions which are equivalent to the parallel postulate.

1. Through a point not on a given line there passes not more than one parallel to the line
2. Two lines that are parallel to the same line are parallel to each other.
3. A line that meets one of two parallels also meets the other.
4. If two parallels are cut by a transversal, the alternate interior angles are equal.
5. There exists a triangle whose angle sum is a straight angle.
6. Parallel lines are equidistant from one another.
7. There exist two parallel lines whose distance apart never exceeds some finite value.
8. Similar triangles exist which are not congruent.
9. Any three points are collinear
10. Any three points are co-cyclic.
11. Through any point within any angle a line can be drawn which meets both sides of the angles.
12. There exists a quadrilateral whose angle sum is two straight angles.
13. Any two parallel lines can have a common perpendicular.
14, There exist a pair of straight lines everywhere equidistant from one another.
15. Two straight lines that intersect one another cannot be parallel to the third line.
16. There is no upper limit to the area of a triangle.
17. The sum of the angles is the same for every triangle.
18. There exists a quadrilateral of which all angles are right angles.
19. Phythacorean theorem.
20. There exists a pair of straight lines that are at constant distance from each other.
21. Given two parallel lines, any line that intersects one of them, also intersects the other.
22. If there is an acute angle such that a perpendicular drawn at every point on one side will meet the other also.

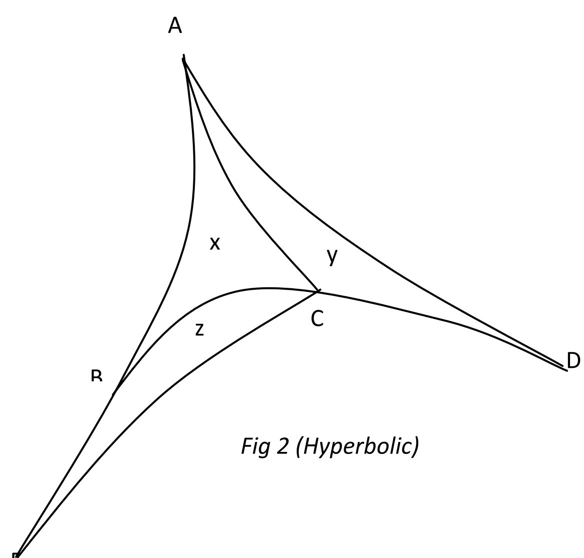

Fig 2 (Hyperbolic)

Besides this, Gauss-Bolyai-Lobachesky independently found a consistent model of non-Euclidean geometry which is known as hyperbolic geometry. Riemann formulated another non-Euclidean geometry which is known as elliptic geometry. The formulae of hyperbolic geometry are widely applied to study the properties of atomic objects in quantum physics. Albert Einstein assumed the basics of elliptic geometry to formulate the general theory of relativity. In these two fields of geometry the parallel postulate does not hold. It was Beltrami who had established that it is not merely difficult but impossible to deduce Euclid V from Euclid I, II, III and IV.

"As long as algebra and geometry have been separated, their progress has been slow and their uses limited; but when these two sciences have been united, they have lent each mutual force, and have marched together towards perfection."-- Joseph-Louis Lagrange. Keeping this in mind, the author applies algebra to geometry and obtained the proof for the above mentioned theorem in abstract.

Construction

Let A, B and D are the given three points. Join A and B, B and D and D and A (Euclid's first postulate). Extend AB up to E (Euclid's second postulate). On BD choose a point C. Join A and C, E and C. This Euclidean construction as shown in Fig. 1 can easily be extended to hyperbolic and spherical spaces (vide Fig. 2- 3). In these figures, small letters x, y and z denote the sum of the interior angles in triangles ABC, ADC and BEC respectively. Also, let a and b respectively refer to the sum of the interior angles in triangles ABD and ACE.

Result

Assuming that the angles BCD or ABE are straight angles and addling we get that

$x + y = 180^0+a$ (1)

$x + z = 180^0 + b$ (2)

(1)-(2) gives, $y-z= a-b$ (3)

Squaring (3) $y^2+z^2 -2yz = a^2+b^2-2ab$ (3a)

Squaring (1) $x^2+y^2+ 2xy = 180^{02} +a^2+ 360^0 a$ (1a)

Squaring (2) $180^{02} +b^2+ 360^0b= x^2+z^2 2xz$ (2a)

Adding the above three eqns. $a^2-y^2 +180^0(a-b)+z(x +y)-ab-xy=0$

Putting (1) in the third factor $a^2-y^2+180^0(a-b)+ z (180^0+a) -ab.xy=0$

$a^2+a (180^0+z-b)+180^0(z-b)-y(x+y)=0$

Using (1) in the last factor, $a^2+a((180^0+z-b)+180^0(z-b)-y (180^0+a)=0$

i.e $a^2+a ((180^0+z-b-y)+ 180^0 (z-b-y)=0$ (4)

Equation (4) is quadratic in a.

$$a= \frac{y+b-180^0-z \pm [(180^0+z-b-y)^2- 720^0(z-b-y)]^{1/2}}{2}$$

Applying (3) in RHS $a=\frac{(a-180^0)\pm [(180^0-a)^2 +720^0a)]^{1/2}}{2}$

i.e $a= \frac{a-180^0\pm [(180^0+a)^2]^{1/2}}{2}$

i.e. $a= \frac{a-180^0 \pm (180^0+a)}{2}$ (4a)

Using positive value in (4a), $a=a$ (5)

Choosing negative value in (4a) $a= -180^0$ (6a)

In geometry it is well known that - theta is the vertically opposite angle. Since vertically opposite angles are equal, we have $a=180^0$ i.e., the sum of the interior angles of triangle ABD is equal two right angles (7)

Discussion

Euclid had to assume the parallel postulate to show that the sum of the interior angles of a triangle is equal to two right angles. There are other proofs such as a) Thiabut proof, b) Thomas Heath proof c) Paper folding proof and d) Phythacorean proof.

Algebraic proof based up on constant hypothesis.

The constant hypothesis implies that the sum of the interior angles of each and every triangle is a constant. This hypothesis is equivalent to 17th proposition mentioned above. So the constant hypothesis assumes the parallel postulate. TILL THIS DATE, THIS WENT UNNOTICED. But our algebraic proof not at all like this and equation (7) is consistent. Earlier, Kalimuthu (2009) and Sivasubramanian (2009) found several such results on this topic. If it is not so, it makes a curious thinking on the laws of quadratic equations. But equation (7) is an acid test for the research community. Because the mere validity and applications of non-Euclidean geometry in theoretical and experimental physics is well known. Equation (7) forces one to analysis this. A brief history of scientific research reveals that controversial results are followed by new branches of science. If further probes are to be focused on this track, more important results can be explored.

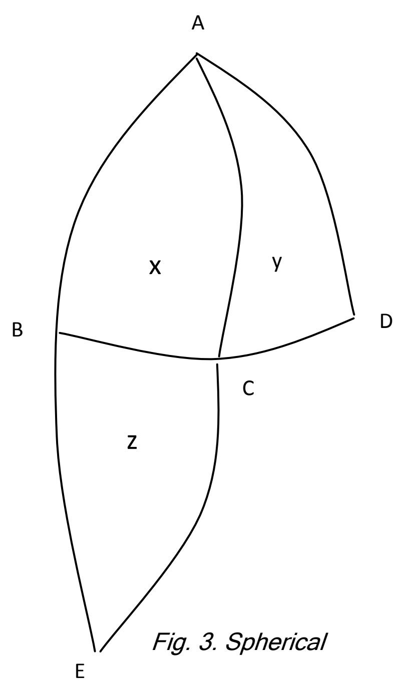

Fig. 3. Spherical

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References

1. Kalimuthu S (2009) The parallel postulate -return of the roaring lion. *Indian J. Sci. Technol.* 2(4), 16-22.
2. Sivasubramanian M (2009) A phenomena in geometric analysis. *Indian J. Sci.Technol.* 2(4), 16-22.