An Explosive Instability of Kelvin-Helmholtz Flows in Ferrofluids: Effect of Periodic Tangential Magnetic Field With Rotation

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Abstract

Objectives: The work is to learn the impingement of tangential periodic magnetic field on the interface in the middle of two superposed ferrofluids with the rotation through a nonlinear perturbation analysis. **Methods or Statistical Analysis:** A nonlinear instability due to streaming superposed ferrofluids stressed by rotation and a periodic tangential magnetic field is examined by employing the mode of multiple scales. The fluids are taken to be incompressible. Using the boundary conditions, the solutions of the linearized equations of motion lead to the linear dispersion equation. **Findings:** From this work, it is clearly seen the unfolding of amplitude of waves was influenced by an equation of nonlinear Schrodinger. **Applications or Improvements:** The rotation and a periodic tangential magnetic field have enormous important effects over the stability of the system.

Keywords: Dispersion Relation and Schrodinger Equation, Ferrofluids, Linear Dispersion Relation, Method of Multiple Scales, Rotation

1. Introduction

Ferrofluids (called magnetic fluids) are stable colloidal cessation of magnetic nanoparticles. Brownian motion maintains the particles from settling and their surface is coated with a dispersant in order to stop them from sticking together. An applied magnetic field yields a new external parameter that can be stabilize or destabilize the fluid interface producing hydrodynamic instabilities. The liquid carrier in commercially produced ferrofluids are kerosene, toluene, water or one of several other fluids.

The ferromagnetic particles are typically 30–150 A in diameter and are magnetite, iron or cobalt. The electrical conductivity of a ferrofluid was in the same order as that of the liquid carrier, which is typically quite small. The Brownian motion also keeps the tiny magnets randomly oriented so the ferrofluid has no natural magnetic moment. Magnetic fluids are used in vacuum which seals around the rotating shafts and in loudspeakers to keep them cool and to damp vibrations. Experiments of magnetic fluids with rotation and magnetic field have shown in a wide range of interesting phenomena.

RTI happens when a weighty fluid is assisted by a light-weight fluid. Because of the wide applications of the RTI in planetary and steller atmospheres etc, several studies have been addressed. Kelvin-Helmholtz Instability (KHI) arises when fluids are in a relative motion.

The hydrodynamic stability for RTI, KHI and other has been reported through the pioneer book of 1-3 investigated the interfacial stability of a ferromagnetic fluid. An excellent book on the stability theory is given by⁴. They reported the analysis of RHI, KHI and capillary instability of perfect fluids. The linear KHI of flow in porous media is studied by⁵. He derived the instability conditions in KHI for Darcian and non-Darcian flows. Non linear stability of surface waves in magnetic fluids with tangential periodic magnetic field was analyzed by⁶. Nonlinear KHI of two miscible ferrofluids in porous media was investigated by². He analyzed linear as well as nonlinear aspects of stability theory of the interface in middle of two superposed magnetic fluids. Kelvin–Helmholtz instabilities in smoothed particle hydrodynamics have been treated extensively by⁸. KHI in a fluid layer bounded by a porous layer above and below by a rigid surface in the presence of magnetic field was discussed by⁹.

In¹⁰ experimented the stability analysis on Kelvin-Helmholtz flow of an electrified horizontal sheet through porous media investigated the instability of Kelvin-Helmholtz flows in magnetic fluids have examined the nonlinear KHI of superposed ferrofluids under rotation and magnetic field^{11–13}.

2. Fluid-field Configuration

Let us imagine a heavy fluid (ρ_1) supported by a light fluid (ρ_{2}) under the gravity. The interface in between the two fluids is described as $z = \eta (x, t)$. From the undisturbed position, the elevation of the wave is measured and is denoted by η (*x*, *t*). When the interface is completely flat then $z = \eta (x, t) = 0$. A fluid with density (ρ_1) and permeability (μ_i) hold the region z < 0 whereas z > 0 is occupied by a fluid density (ρ_{2}) , magnetic permeability (μ_{2}) Here Cartesian coordinate system is chosen. It is assumed that X-axis rest in the unperturbed interface. Z-axis is taken perpendicular which is pointed from the lower level fluid to the upper fluid. The fluids are taken to be inviscid magnetic fluids moving with uniform velocity U_1 and U_2 in the x – direction and the fluid is assumed to rotate in the uniform angular velocity Ω about Z-axis. The physical quantities are undertake to differ with respect to x and zcoordinates except 't', since the disturbances are constricted to two dimensional plane waves. The system is emphasized by a tangential periodic magnetic field $H = \varepsilon^{\overline{2}} H_0 \cos \omega t \overline{i}$ in the direction of X-axis. Here ε denotes the small non-dimensional parameter, H_0 shows the magnetic field amplitude, ω marks the field frequency and along the direction of X-axis the unit vector is

Analysis begins with the basic equations, the Navier-Stokes equation:

$$\rho \frac{d\boldsymbol{q}}{dt} + 2\rho \left(\Omega \times q\right) = -\nabla p + (\hat{A} + \delta \hat{A})\boldsymbol{g} + \mu_0 \boldsymbol{M} \cdot \nabla \boldsymbol{H} \quad (1)$$

Here q = (u + U, v, w), $\Omega = (0, 0, \Omega)$, the final term $\mu_0 M \cdot \nabla H$ describes the magnetic body force on the fluid element, *M* is the magnetization vector and ∞_0 takes the value 1.

From a potential function, assume that an additional magnetic field 'h' is derived.

i.e.
$$\psi(x,z)$$
 exists with $h = -\varepsilon^{\overline{2}} \nabla \psi$.

The magnetic potential Ψ satisfies the equation:

$$\nabla^2 \psi^{(1)} = 0, \quad -\infty < z < \eta(x,t)$$
 (2)

$$\nabla^2 \psi^{(2)} = 0, \quad \eta(x,t) < z < \infty \tag{3}$$

The kinematic boundary condition is defined at z is equal to η (x, t).

$$\frac{\partial \eta}{\partial t} + (u^i + U^i)\frac{\partial \eta}{\partial x} - w^i = 0, i = 1, 2.$$
(4)

Here u represents the tangential velocity and w indicates the normal velocity. U^1, U^2 represents the speed of the fluid in region (1) and (2) respectively.

Non viscous fluid satisfies $\hat{n} \cdot \boldsymbol{q}^{(1)} = \hat{n} \cdot \boldsymbol{q}^{(2)}$ (5)

Continuity of normal and tangential components of the magnetic field at:

z = h(x, t), are

$$\mu_1 H_n^{(1)} = \mu_2 H_n^{(2)} \tag{6}$$

$$H_t^{(1)} = H_t^{(2)} \tag{7}$$

Normal stress is balanced on the system at the dividing surface.

$$[[-p + \frac{\mu}{2}(H_n^2 - H_t^2)]] = \frac{-T\eta_{xx}}{(1 + \eta_x^2)^{\frac{3}{2}}} \text{ at } z = h(x, t)$$
(8)

Where p indicates the magnetic strictive pressure, T shows the coefficient of the surface tension , [[]] signifying the jump in the amount across the regions 1 and 2.

3. Perturbation Methodology

By applying the mode of multiple scales, the perturbed equations are solved by taking ε which represents the steepness ratio of the wave. The values of t_n and x_n interpreted as:

$$t_n = t \frac{1}{\epsilon^{-n}}, \ x_n = x \frac{1}{\epsilon^{-n}} \quad (n = 0 \ to \ 2)$$
 (9)

taken as \overline{i} .

where $\varepsilon = \left(\frac{U}{U_m} - 1\right)^{\frac{1}{2}}$ and U_m denotes the marginal velocity.

In order to examine the transition of a carrier wave with wave number k, frequency ω , pretend the dependence of $\eta(x,t)$ to be:

$$\eta(x,t) = \sum_{n=1}^{3} \varepsilon^{n} \eta_{n}(x_{0}, x_{1}, x_{2}; t_{0}, t_{1}, t_{2}) + O(\varepsilon^{4})$$
(10)

and $\eta_1 = A(x_1, x_2; t_1, t_2) \exp i(kx_0 - \omega_0 t_0) + C.C$ (11)

Here *C*.*C* represents the complex conjugate of all previous terms and *A* marks amplitude of the propagating wave which is slowly varying function. *k* denotes the wave number, ω_0 marks the frequency of the disturbance.

Expand the physical quantities q and ψ as,

$$\boldsymbol{q}(x,z,t) = \sum_{n=1}^{3} \varepsilon^{n} \, \boldsymbol{q}_{n}(x_{0},x_{1},x_{2},z\,;\,t_{0},t_{1},t_{2}) + O(\varepsilon^{4}) \quad (12)$$

$$\psi(x,z,t) = \sum_{n=1}^{3} \varepsilon^{n} \psi_{n}(x_{0},x_{1},x_{2},z;t_{0},t_{1},t_{2}) + O(\varepsilon^{4}) \quad (13)$$

The scale x_0 stands for the wavelength; the scale t_0 marks the frequency of the wave. The scales of the phase, amplitude are denoted by t_1 and t_2 respectively whereas spatial modulations of the phase is denoted by x_1 and the amplitude is noted by x_2

Utility the series of Maclaurins at z = 0 for the quantities, the boundary conditions are evaluated. Replacing the above equations (9) – (13) into a series (1) – (8) and comparing the particulars of same powers of ε , ε^2 , ε^3 etc, to reach 3 consequent equations.

4. Linear Dispersion Relation

By using first order equations, we arrive at the linear dispersion relation:

$$\rho_{2}(U_{2}k-\omega_{0})^{2} \frac{1}{\left(1-\frac{4\Omega^{2}}{(U_{2}k-\omega_{0})^{2}}\right)^{-\frac{1}{2}}} + \rho_{1}(U_{1}k-\omega_{0})^{2} \frac{1}{\left(1-\frac{4\Omega^{2}}{(U_{1}k-\omega_{0})^{2}}\right)^{-\frac{1}{2}}}$$

$$= (\rho_1 - \rho_2)gk - \alpha^2 k^2 + Tk^3$$
(14)

Where
$$\alpha^2 = \frac{H_0^2 \cos^2 \omega t (1-\mu)^2 \mu_2}{(1+\mu)}$$
 and $\mu = \frac{\mu_1}{\mu_2}$

The dispersion relation (14) coincides with the result of a linear perturbation¹ in the absence of both rotation, magnetic field for RTI. In addition, if there is no field frequency, then to learn the amplitude modulation of the progressive waves, we assume $\omega_0^2 > 0$. The system is stable if $(\rho_1 - \rho_2)g - \alpha^2 k + Tk^2 > 0$.

5. Second Order Problem

The solution of the linear order is substituted into the equations of second order. Solve them by assuming,

$$\eta_2 = \wedge A^2 \exp(2i\theta) + C.C$$
(15)
Where $\wedge = \frac{Nr}{Dr}$,

$$Nr = -H_{0}^{2}\cos^{2}\omega t k^{2}\mu_{2}(1-\mu)\left(\frac{(1-\mu)}{(1+\mu)}\right) + \frac{2}{k^{2}}\left[\rho_{2}\gamma_{2}\beta_{2}\left(\omega_{0}-U_{2}k\right)^{2}-\rho_{1}\gamma_{1}\beta_{1}\left(\omega_{0}-U_{1}k\right)^{2}\right] - \frac{2\Omega^{2}}{k^{2}}\left(\rho_{2}\gamma_{2}\beta_{2}-\rho_{1}\gamma_{1}\beta_{1}\right) + \frac{1}{k^{2}}\left[\rho_{1}\beta_{1}^{2}\left(\omega_{0}-U_{1}k\right)^{2}-\rho_{2}\beta_{2}^{2}\left(\omega_{0}-U_{2}k\right)^{2}\right] + \frac{4\Omega^{2}}{k^{2}}\left(\rho_{2}\beta_{2}^{2}-\rho_{1}\beta_{1}^{2}\right)$$
(16)

$$Dr = -\frac{2}{k^2} \left[\rho_2 \gamma_2 \left(\omega_0 - U_2 k \right)^2 + \rho_1 \gamma_1 \left(\omega_0 - U_1 k \right)^2 \right] + 2k H_0^2 \cos^2 \omega t \, \frac{(1-\mu)^2 \mu_2}{(1+\mu)} + \frac{2\Omega^2}{k^2} \left(\rho_2 \gamma_2 + \rho_1 \gamma_1 \right) - (\rho_2 - \rho_1) g + 4k^2 T$$

Where

$$\beta_{1} = k \left(1 - \frac{4\Omega^{2}}{(U_{1}k - \omega_{0})^{2}} \right)^{-\frac{1}{2}}$$
$$\beta_{2} = k \left(1 - \frac{4\Omega^{2}}{(U_{2}k - \omega_{0})^{2}} \right)^{-\frac{1}{2}}$$

(17)

$$\gamma_{1} = k \left(1 - \frac{\Omega^{2}}{(U_{1}k - \omega_{0})^{2}} \right)^{-\frac{1}{2}}$$
$$\gamma_{2} = k \left(1 - \frac{\Omega^{2}}{(U_{2}k - \omega_{0})^{2}} \right)^{-\frac{1}{2}}$$

6. Third Order Problem

The dispersion relation of third order is:

$$\begin{split} &\frac{\partial^{2}A}{\partial x_{1}^{2}} \left\{ \frac{\rho_{s}\omega_{0}}{k^{3}} \left(\frac{\beta_{s}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{2}} \right) + \frac{\rho_{1}\omega_{0}}{k^{3}} \left(\frac{\beta_{s}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{1}} \right) + \frac{\rho_{1}\omega_{1}\beta_{1}}{k^{2}} \left(\omega_{0} - U_{2}k \right) + \frac{\rho_{1}U_{1}\beta_{1}}{k^{3}} \left(\omega_{0} - U_{1}k \right) - \frac{\rho_{1}U_{2}}{k^{2}} \left(\frac{\beta_{1}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{2}} \right) \\ &- \frac{\rho_{1}U_{1}}{k^{2}} \left(\frac{\beta_{1}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{1}} \right) + \frac{4\Omega^{2}\rho_{2}}{k^{2}} \left(U_{2}\beta_{2} - \frac{\beta_{2}\omega_{0}}{k} - \frac{s_{1}}{2\beta_{2}} \right) \\ &- \frac{4\Omega^{2}\rho_{1}}{k^{3}} \left(\omega_{0} - U_{2}k \right)^{2} + \frac{\rho_{1}\omega_{0}}{k^{4}} \left(\alpha_{0} - U_{2}k \right)^{2} + \frac{\omega_{0}\rho_{2}}{k^{4}} \left(\frac{\beta_{2}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{2}} \right) \\ &- \frac{4\Omega^{2}\rho_{2}}{k^{4}} \left(\omega_{0} - U_{2}k \right)^{2} + \frac{\rho_{1}\omega_{0}\rho_{2}}{k^{4}} \left(\frac{\beta_{2}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{2}} \right) \\ &- \frac{4\Omega^{2}\rho_{1}}{k^{3}} \left(\omega_{0} - U_{2}k \right)^{2} \left(\frac{\beta_{1}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{1}} \right) \\ &- \frac{4U^{2}\rho_{1}}{k^{3}} \left(\omega_{0} - U_{1}k \right) \left(\frac{\beta_{1}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{1}} \right) \\ &- \frac{4U^{2}\rho_{2}}{k^{2}} \left(\omega_{0} - U_{1}k \right) \left(\frac{\beta_{1}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{1}} \right) \\ &- \frac{4U^{2}\rho_{2}}{k^{3}} \left(\omega_{0} - U_{2}k \right) \left(\frac{\beta_{1}\omega_{0}}{k} + \frac{s_{1}}{2\beta_{1}} \right) \\ &- \frac{4U^{2}\rho_{2}}{k^{3}} \left(\omega_{0} - U_{2}k \right) + \frac{\rho_{1}\beta_{1}}{k^{3}} \left(\omega_{0} - U_{1}k \right) + \frac{2\rho_{2}\omega_{0}}{k^{2}} \left(\beta_{2} + \frac{s_{4}}{2\beta_{2}} \right) \\ &+ \frac{2\rho_{1}\omega_{0}}{k^{3}} \left(\omega_{0} - U_{2}k \right) \left(\frac{\beta_{1}\omega_{0}}{k^{3}} - \frac{s_{1}}{2\beta_{1}} \right) \\ &- \frac{2\Omega^{2}\rho_{2}s_{4}}{k^{3}} \left(\omega_{0} - U_{2}k \right) + \frac{\rho_{1}\beta_{1}}{k^{3}} \left(\omega_{0} - U_{1}k \right) + \frac{\rho_{2}\beta_{2}\omega_{0}}{k^{3}} \left(\beta_{2} + \frac{s_{4}}{2\beta_{2}} \right) \\ &+ \frac{2\Omega^{2}}{k^{3}} \left(\omega_{0} - U_{2}k \right) \left[\frac{\beta_{2}\omega_{0}}{k^{3}} + \frac{s_{1}}{2\beta_{2}} \right] \\ &- \frac{2\Omega^{2}\rho_{2}s_{4}}{k^{3}} \left(\omega_{0} - U_{2}k \right) + \frac{\rho_{1}\beta_{1}}{k^{3}} \left(\omega_{0} - U_{2}k \right) \left[U_{2}\beta_{2} - \frac{\beta_{2}\omega_{0}}{k^{3}} \left(\beta_{1} + \frac{s_{1}}{2\beta_{1}} \right) \\ \\ &- \frac{2\Omega^{2}\rho_{2}s_{4}}{k^{3}} \left(\omega_{0} - U_{2}k \right) \left[\frac{\beta_{2}}{k^{3}} \left(\frac{\beta_{2}}{k^{3}} + \frac{s_{1}}{2\beta_{2}} \right] \\ \\ &- \frac{2\Omega^{2}\rho_{2}s_{4}}{k^{3}} \left(\frac{\beta_{2}}{k^{3}} - \frac{\beta_{2}\beta_{1}}{k^{3}} \right) \\ \\ &- \frac{2\Omega^{2}\rho_{2}\rho_{5}}}{k^{3}} \left(\frac{\beta_{2}}{k^{3}} + \frac{s_{1}}{2\beta_{2}} \right) \\ \\ &$$

$$\begin{split} & -\frac{\partial A}{\partial t_{i}} \left\{ \frac{\rho_{i}\beta_{i}!}{k^{2}} (\omega_{b} - U_{i}k) + \frac{\rho_{i}\beta_{i}!}{k^{2}} (\omega_{b} - U_{i}k) \right\} + A^{2}\overline{A} \left\{ \frac{H^{2}_{i}\cos^{2} \sigma t k^{2}(1-\mu)}{2(1+\mu)^{2}} (\mu_{i}-1) \left[k(\mu+3) + (1-\mu)6\Lambda + 4\Lambda \mu \right] - \\ & -\frac{H^{2}_{i}\cos^{2} \sigma t k^{2}(1-\mu)}{2(1+\mu)^{2}} (\mu_{i}-1) \left[k(\mu+3) + (1-\mu)6\Lambda - 4\Lambda \right] \\ & +4k^{2}(\mu_{i}-1)H^{2}_{0}\cos^{2} \sigma t \left\{ \frac{1-\mu}{1+\mu} \right\}^{2} (\Lambda+k) - 4k^{2}(\mu_{i}-1)H^{2}_{0}\cos^{2} \sigma t \left\{ \frac{1-\mu}{1+\mu} \right\}^{3} (\Lambda-k) \\ & -\frac{2\rho_{i}\beta_{i}\gamma_{i}}{k^{2}} (\omega_{b} - U_{i}k)^{2} (\beta_{i}+\Lambda) - \frac{2\rho_{i}\beta_{i}\gamma_{i}}{k^{2}} (\omega_{b} - U_{i}k)^{2} (\beta_{i}-\Lambda) - \frac{4\rho_{i}\gamma_{i}^{2}}{k^{2}} (\omega_{b} - U_{i}k)^{2} (\beta_{i}+\Lambda) \\ & -\frac{4\rho_{i}\gamma_{i}^{2}}{k^{2}} (\omega_{b} - U_{i}k)^{2} (\beta_{i}-\Lambda) + \frac{2\rho_{i}\beta_{i}\gamma_{i}}{k^{2}} (\omega_{b} - U_{i}k)^{2} (\Lambda+\beta_{i}) + \frac{2\rho_{i}\gamma_{i}}{k^{2}} (\omega_{b} - U_{i}k)^{2} (\beta_{i}-\Lambda) \\ & -\frac{4\rho_{i}\gamma_{i}^{2}}{k^{2}} (\omega_{b} - U_{i}k) (\beta_{i}-\Lambda) + \frac{2\rho_{i}\beta_{i}\gamma_{i}}{(\omega_{b}-U_{i}k)^{2}} (\omega_{b}-U_{i}k) (\beta_{i}-\Lambda)) \\ & -\frac{4\rho_{i}\gamma_{i}}{k^{2}} (\omega_{b} - U_{i}k) (\rho_{i}-\Lambda) + \rho_{i}\gamma_{i}^{2} (\omega_{b}-U_{i}k) (\beta_{i}-\Lambda)) \\ & -\frac{4\rho_{i}\gamma_{i}^{2}} (\omega_{b}-U_{i}k) (\rho_{i}+\Lambda) - \rho_{i}\gamma_{i}^{2} (\beta_{b}-U_{i}k) (\beta_{i}-\Lambda)) \\ & -\frac{4\Omega_{i}^{2}}{k^{2}} (\rho_{i}\gamma_{i}^{2} (\omega_{b}-U_{i}k) (\beta_{i}-\Lambda) - \rho_{i}\gamma_{i}^{2} (\omega_{b}-U_{i}k) (\beta_{i}-\Lambda)) \\ & -\frac{4\Omega_{i}^{2}}{k^{2}} (\rho_{i}\gamma_{i}^{2} (\omega_{b}-U_{i}k) (\beta_{b}-\Lambda) - \rho_{i}\gamma_{i}^{2} (\omega_{b}-U_{i}k) (\beta_{b}-\Lambda)) \\ & (\mu_{i}-1)H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right)^{-} (\mu_{i}-1)H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right) ((\mu_{i}-1)(\Lambda+k) + (1-\mu_{i})(k-\Lambda)) \right \\ & (\mu_{i}-1)H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right)^{-} (\mu_{i}-1) + 4H^{2}_{b}\cos^{2} \sigma t k^{2} \Lambda \left(\frac{1-\mu}{1+\mu} \right) (\mu_{i}-\mu_{i}) \\ & +H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right)^{2} (2\mu_{i}+2\mu_{i}-1) + 4H^{2}_{b}\cos^{2} \sigma t k^{2} \Lambda \left(\frac{1-\mu}{1+\mu} \right) (\mu_{i}-\mu_{i}) \\ & +(\mu_{i}-1)H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right)^{2} (\mu_{i}+(\lambda-\lambda)-k) - 4H^{2}_{b}\cos^{2} \sigma t k^{2} \Lambda \left(\frac{1-\mu}{1+\mu} \right) (\mu_{i}+\mu_{i}) \\ & -4H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right)^{2} (\mu_{i}+(\lambda-\lambda)-k) - 4H^{2}_{b}\cos^{2} \sigma t k^{2} \Lambda \left(\frac{1-\mu}{1+\mu} \right) \\ & -4H^{2}_{b}\cos^{2} \sigma t k^{2} \left(\frac{1-\mu}{1+\mu} \right)$$

7. Nonlinear Schrodinger **Equation and Stability Conditions**

Let the coefficients of the terms $\frac{\partial^2 A}{\partial x_1^2}$, $\frac{\partial^2 A}{\partial x_1 \partial t_1}$, $\frac{\partial^2 A}{\partial t_1^2}$, $\frac{\partial A}{\partial x_2}$, $\frac{\partial A}{\partial t_2}$ in Equation (18) are represented by N_{lk}^* , N_{2k}^* , N_{3k}^{*} , N_{4k}^{*} and N_{5k}^{*} respectively.

Third order dispersion relation presumes the form:

$$i\frac{\partial A}{\partial \tau} + \beta_{1k}^* \frac{\partial^2 A}{\partial \xi^2} + \beta_k^* \frac{\partial A}{\partial \xi} + Q_k \left| A \right|^2 A = 0$$
(19)

Which is the well-known second order nonlinear Schrodinger equation.

Where:

$$\beta_{1k}^* = \frac{i}{N_{5k}} [N_{1k} - C_{gk} N_{2k} + C_{gk}^2 N_{3k}]$$

$$\beta_k^* = i \left(\frac{N_{4k}}{N_{5k}} - C_{gk} \right)$$

$$Q_k = \frac{i}{N_{5k}} Q_{1k} \text{ Where } Q_{1k} = \frac{\partial D}{\partial |A|^2} \text{ and } C_{gk} = -\frac{Dk}{D\omega_0}$$
with

$$Dk = \frac{\rho_2}{2} \Big[(U_2 k - \omega_0)^4 - 4\Omega^2 (U_2 k - \omega_0)^2 \Big]^{-\frac{1}{2}} \\ \Big[4U_2 (U_2 k - \omega_0)^3 - 8\Omega^2 U_2 (U_2 k - \omega_0) \Big] \\ + \frac{\rho_1}{2} \Big[(U_1 k - \omega_0)^4 - 4\Omega^2 (U_1 k - \omega_0)^2 \Big]^{-\frac{1}{2}} \\ \Big[4U_1 (U_1 k - \omega_0)^3 - 8\Omega^2 U_1 (U_1 k - \omega_0) \Big] \\ - (\rho_1 - \rho_2)g + 2H_0^2 \cos^2 \omega tk \frac{(1 - \mu)^2 \mu_2}{(1 + \mu)} - 3Tk^2 \\ D\omega_0 = \frac{2\rho_2}{\Big((U_2 k - \omega_0)^2 - 4\Omega^2 \Big)^{\frac{1}{2}}} \Big[2\Omega^2 - (U_2 k - \omega_0)^2 \Big] \\ + \frac{2\rho_1}{\Big((U_1 k - \omega_0)^2 - 4\Omega^2 \Big)^{\frac{1}{2}}} \Big[2\Omega^2 - (U_1 k - \omega_0)^2 \Big]$$

Which are obtained from linear dispersion equation.

The stability criterion is analyzed, using⁶ by considering the following time varying solution.

$$A = m e^{iQ_k m^2 \tau}, m \text{ is a constant.}$$
(20)

Perturb the Equation (20) by stating:

$$A = \left[m + D_{1k}(\xi, \tau) + iE_{1k}(\xi, \tau)\right] e^{iQ_k m^2 \tau}$$
(21)

Where D_{1k} and E_{1k} are real functions.

Using Equation (21) in Equation (19) and neglecting the terms which are nonlinear in D_{1k} and E_{1k} .

$$-\frac{\partial E_{1k}}{\partial \tau} + \beta_{1k}^* \frac{\partial^2 D_{1k}}{\partial \xi^2} + \beta_k^* \frac{\partial D_{1k}}{\partial \xi} = 0 \text{ and}$$
(22)

$$\frac{\partial D_{1k}}{\partial \tau} + \beta_{1k}^* \frac{\partial^2 E_{1k}}{\partial \xi^2} + \beta_k^* \frac{\partial E_{1k}}{\partial \xi} = 0$$
(23)

Writing

$$D_{1k}(\xi,\tau) = D^* e^{(ik_2\xi + \omega_{2k}\tau)} + C.C$$
(24)

and
$$E_{1k}(\xi,\tau) = E^* e^{(ik_2\xi + \omega_{2k}\tau)} + C.C$$
 (25)

where D^* and E^* are constant values, k_2 denotes the wave number and ω_{2k} indicates the disturbance frequency. By using (24) and (25), ω_{2k} and k_2 desire the dispersion relation.

$$\omega_{2k}^{2} + \left[\beta_{1k}^{*2}k_{2}^{4} - k_{2}^{2}\beta_{k}^{*2} - 2ik_{2}^{3}\beta_{k}^{*}\beta_{1k}^{*}\right] = 0$$
(26)

where the wave number k_2 may assume real and positive values.

which implies

$$\omega_{2k} = \pm (J_R + iJ_I)$$

where $J_R = \left(\frac{G_R + \sqrt{(G_R^2 + G_I^2)}}{2}\right)^{\frac{1}{2}}$ and (27)

$$J_{I} = \frac{G_{I}}{|G_{I}|} \left(\frac{-G_{R} + \sqrt{(G_{R}^{2} + G_{I}^{2})}}{2}\right)^{\frac{1}{2}}$$
(28)

$$G_{R} = C_{1}^{2}k_{2}^{2} - D_{1}^{2}k_{2}^{2} - \beta_{1k}^{*2}k_{2}^{4} - 2\beta_{1k}^{*}D_{1}k_{2}^{3}$$
$$G_{I} = 2C_{1}D_{1}k_{2}^{2} + 2C_{1}\beta_{1k}^{*}k_{2}^{3}$$
$$D_{1} = \left(\frac{B_{1}}{A_{2}} - C_{gk}\right)$$

$$\begin{split} C_{1} &= \frac{1}{\rho_{2} \beta_{2} \left(U_{2} k - \omega_{0} \right) + \rho_{1} \beta_{1} \left(U_{1} k - \omega_{0} \right)} [\left(U_{2} k - \omega_{0} \right) \\ &\rho_{2} \beta_{2} \omega_{0} + \left(U_{1} k - \omega_{0} \right) \rho_{1} \beta_{1} \omega_{0} - \left(U_{1} k - \omega_{0} \right) \rho_{1} \beta_{1} U_{1} k \\ &- \left(U_{2} k - \omega_{0} \right) \rho_{2} \beta_{2} U_{2} k] \\ A_{2} &= \frac{\rho_{1} \beta_{1}}{k^{2}} \left(U_{1} k - \omega_{0} \right) + \frac{\rho_{2} \beta_{2}}{k^{2}} \left(U_{2} k - \omega_{0} \right) \\ B_{1} &= \left(1 - \mu_{2} \right) H_{0}^{2} \cos^{2} \omega t \left(\frac{\left(1 - \mu \right)}{\left(1 + \mu \right)} \right) \\ &- \left(1 - \mu_{1} \right) H_{0}^{2} \cos^{2} \omega t \left(\frac{\left(1 - \mu \right)}{\left(1 + \mu \right)} \right) \\ &+ 8 \rho_{2} \Omega^{2} \frac{\beta_{2}}{k^{3}} + 8 \rho_{1} \Omega^{2} \frac{\beta_{1}}{k^{3}} \\ &- \frac{1}{k^{3}} \left(\left(U_{2} k - \omega_{0} \right)^{2} \beta_{2} \rho_{2} + \left(U_{1} k - \omega_{0} \right)^{2} \beta_{1} \rho_{1} \right) \\ &+ \frac{1}{k^{2}} \left(\rho_{2} U_{2} \beta_{2} \left(U_{2} k - \omega_{0} \right) + \rho_{1} U_{1} \beta_{1} \left(U_{1} k - \omega_{0} \right) \right) - 2TK \end{split}$$

8. Discussion and Conclusion

Linear dispersion relation (14) coincides with Rosensweig in the absence of rotation, the horizontal magnetic field. It also coincides with Chandrasekhar in the non-appearance of rotation, magnetic field.

It is clearly seen from the above values that the system is found to be unstable when:

$$J_{R} = \left(\frac{G_{R} + \sqrt{(G_{R}^{2} + G_{I}^{2})}}{2}\right)^{\frac{1}{2}} > 0 \quad ,$$

$$J_{R} = \left(\frac{G_{R} + \sqrt{(G_{R}^{2} + G_{I}^{2})}}{2}\right)^{\frac{1}{2}} < 0 \text{ and}$$

$$\left(U_{1}k - \omega_{0}\right)^{2} > 4\Omega^{2} , \left(U_{2}k - \omega_{0}\right)^{2} > 4\Omega^{2} .$$

Hence it is concluded that rotation and a periodic tangential magnetic field have great significant effects over the stability of the system.

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