

# Design and Optimization of Cryogenic Regenerators: A Review

B. Jayaraman\*

School of Mechanical Engineering, SASTRA University, Thanjavur – 613401, Tamil Nadu, India;  
tabjayaraman@mech.sastra.edu

## Abstract

**Objectives:** The principles of cryogenic regenerator operation, its design and optimization procedures are reviewed in this paper. Recent and ongoing developments of these regenerators are highlighted. Due to the non linear behaviour of geometry and operating parameters with performance parameters, design and optimization of regenerator performance has not been possible in conventional mathematical terms. **Methods:** Methodologies of design to determine the volume having maximum heat transfer ability with minimum losses are listed. The optimization in terms of mathematical, analytical and experimental approaches with their inherent difficulties is discussed. **Findings:** The regenerator mesh with a lower hydraulic diameter in the cold region and a larger hydraulic diameter in hot region will lead to lower regenerator losses. Thus a regenerator consisting of layers of different matrix geometry would have a better performance than a regenerator with single mesh geometry. Regenerators that incorporate heat transfer components in a parallel orientation with respect to the oscillating flow theoretically provide a better performance than screen mesh regenerators. An optimum regenerator would have a continuous variation in its hydraulic diameter along the length of the tube. **Applications:** This review is expected to be a catalogue of principles needed for effective simulation, design, and optimization of regenerators for cryogenic refrigerators. The optimum regenerator has the potential to significantly improve the performance of cryogenic refrigerators and make them suitable for more applications.

**Keywords:** Cryogenic Regenerator, Design, Hybrid Regenerator, Optimization

## 1. Introduction

A regenerator is a thermal storage material that periodically absorbs energy when exposed to a hot fluid and releases it to the same fluid at a later stage in a cycle. Thermodynamic cycles include regenerative heat transfer via a regenerator, in order to attain low temperatures and high performance. The regenerator used in these cycles has a flow conduit that is filled with a porous matrix having high surface area and heat capacity leading to large heat transfers. The hot fluid entering at constant temperature, heats the matrix for half of the cycle, and cold fluid cools the matrix for the second half of cycle while

flowing in the reverse direction, thus periodically heating and cooling the matrix while reversing the flow. Since the first intuitive use of regenerator in the year 1816 by Robert Stirling in his hot air engine, regenerative heat exchangers, based on this principle have evolved to become one of the important components in achieving low temperatures in Cryogenic Refrigerators or Cryocoolers.<sup>1</sup>

Investigation of regenerators have been carried out, both on a theoretical and experimental basis, because of their significant impact on the performance of cryocoolers and due to their complicated internal flow phenomena associated with the fluid-solid interactions. Mathematical modeling of the regenerator and its solutions have aided

\*Author for correspondence

in the theoretical prediction of its performance. With the use of computers and Computational Fluid Dynamics (CFD), simulation of the regenerator model has improved the understanding of its functioning. This has led to a systematic search for materials, its shapes and positioning of matrix elements inside the regenerator. The individual and combined effects of the heat capacity of matrix, heat transfer between the working fluid and matrix and pressure drop of the flow determines the performance of the matrix.<sup>2,3</sup> Difficulty in theoretical simulation of regenerator and therefore its performance prediction has led to experimental analysis. Experimental analyses have been carried out to vary the operating parameters, material, geometry and volume of the regenerator so as to understand its working and thereby improve its performance.

Research on finding the right combination of volume or porosity, material, geometry and dimensions of the regenerator having a maximum thermal effectiveness with minimum losses including a low pressure drop has been a challenge for researchers. Due to the non linear behaviour of geometry and operating parameters with performance parameters, optimization of regenerator performance has not been possible in conventional mathematical terms. The difficulties inherent in regenerator design are highlighted in this review. The procedure adopted by researchers for optimization will be discussed.

## 2. Regenerator Design

Static regenerators are constructed by packing geometries into a cylindrical housing. Regenerator design calculates the volume of this packing required to absorb and release heat energy for reducing the temperature of working fluid as it leaves the regenerator. The design parameters are length, matrix material and geometry, frontal area and porosity. The material selected should have a heat capacity higher than the working fluid for effective heat transfer. Simultaneously the design should also minimize the losses and pressure drop. Besides the matrix material, the operational parameters dependent on regenerator performance are the frequency of oscillation of working fluid, phase difference and initial pressure in the system.

Regenerator design becomes more complicated as its operating temperature reduces to the cryogenic region. These complications arise from the fact that the thermal properties vary over the temperature range of the operation of regenerator. The influence of each loss mechanism differs as a function of temperature. At low temperatures,

the heat exchange loss is very large, while pressure drop in this region can be neglected. The heat transfer effectiveness is a function of fluid properties, solid material properties, and the flow geometry and changes in these parameters affect the regenerator performance. The conflicting design requirements, and their solution, for a regenerator are shown in Table 1.

**Table 1.** Requirements of a good Regenerator

Requirements	Solution
Maximum heat capacity	Large solid matrix
Minimum flow losses	Small porous matrix
Minimum dead space	Small dense matrix
Maximum heat transfer	Finely divided matrix
Minimum contamination	No obstruction

To satisfy the conflicting solutions for a given application, permutations of regenerator material, geometries and working fluids have been used. The material and working fluid properties vary with temperature and therefore affect heat transfer, thereby affecting the regenerator effectiveness significantly. Effectiveness is defined as the ratio of heat transfer surface area to the fluid pressure drop. A major design consideration is to select the mesh size of a regenerator with high effectiveness at minimum cost.<sup>1,4</sup>

Geometric factors of the mesh with high effectiveness are always a compromise of either a mesh with high heat transfer area or one having a low pressure drop. Geometrical configurations and materials that are appropriate for regenerator operation throughout the temperature range of the working fluid are discussed in the following sections.

### 2.1 Geometry

The porosity requirement dictates the size of regenerator. The area of the regenerator matrix has to be large to improve its heat transfer rate and thereby reducing the temperature difference between gas and matrix. However, a large regenerator may also have a large void volume. The large void volume can be eliminated by using a fine structure resulting in a higher frictional pressure drop between gas and matrix. The overall size of the regenerator does not necessarily influence the heat conduction or pressure drop in isolation. A short and wide regenerator leads to a low pressure drop, whereas a long and narrow regenerator reduces the heat conduction rates.<sup>5,6</sup> For a constant volume, a low aspect ratio leads to a short and

wide regenerator. In such a case, the velocity of fluid in the regenerator is low, giving a low pressure drop and a low heat transfer coefficient. This gives rise to a large temperature difference between the gas and matrix, consequently increasing the heat exchange losses. On the other hand, if the regenerator diameter is small and its length is long, the flow velocity will be high, leading to a large pressure drop.

In wire mesh screen geometry, the influence of hydraulic diameter by both wire diameter and porosity are similar. A lower hydraulic diameter means a finer mesh and the fine structure causes a large pressure drop but high heat transfer. For large hydraulic diameter, the flow is less obstructed so the pressure drop is low. The first to report that an optimum regenerator structure would be having a varying hydraulic diameter was Andeen.<sup>2</sup>

Since the first intuitive use of a bundle of thin wires as a regenerator by Stirling, different geometrical configuration has been analyzed to improve the performance of Cryocoolers.<sup>1</sup> Woven screen materials, spherical balls, powders, random fibers, sintered glass, perforated plates, foam metal, dimpled ribbon *etc.*, are the commonly used regenerator geometries. To date, most regenerators have been assembled from materials originally created and manufactured for other purposes. Woven wire mesh screens and lead balls are widely used for a myriad of other things and, as a result of mass production, reasonably cheap. Different sizes of meshes and balls have been used to control the porosity. While these materials have relatively large surface areas, their pressure drop characteristics are not ideal. They are not engineered specifically for cryocooler application.<sup>7</sup>

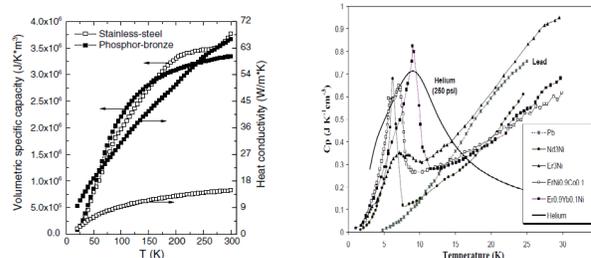
Regenerators that incorporate heat transfer components in a parallel orientation with respect to the oscillating flow theoretically provide a better performance than screen mesh regenerators. Using gas gaps between parallel plates made of conventional materials and rare earths produces significantly better performance below 50 K.<sup>8,9</sup> Etched foil regenerators can be treated as “parallel plate” configurations and in turn, are expected to offer a superior relationship between heat transfer and pressure drop.<sup>10,11</sup> The flow channels are created through a photochemical etching process that generates slots in disks. The disks are then stacked in the regenerator aligning the slots to produce parallel channels in the axial direction. Dimpled metal foil and perforated etched foils have been engineered specifically as regenerator materials. The dimensions of these regenerators including size of

flow passages, porosity, and direction of the flow path can be appropriately engineered.<sup>11</sup> Usage of parallel wires and concentric tubes also in the low temperature region of regenerator has been reported.<sup>12</sup> Efforts are on to achieve in practice the improvements in cryocooler performance predicted by regenerator theory.<sup>11</sup>

Different materials become attractive in different temperature zones within the regenerator due to the varying properties of materials from 300 K to the cold temperature. Hence, the regenerator mesh material properties viz. specific heat capacity, radial heat conductivity, density *etc.* are a direct function of the cooling power of Cryocoolers.<sup>13</sup>

## 2.2 Material

The regenerator material is selected in such a way that its heat capacity must be larger than that of working gas. Based on the temperature range in which the regenerators operate, its design and analysis can be divided into three main groups. The first group is the efforts to find the optimum regenerator from 300 K to 30 K and the second is the research below 30 K. The first group from 300 K to 30 K are the stainless steel and copper alloy materials which are ductile and can be woven into screen meshes. Lead and lead-antimony spheres are used in the range between 100 K and 30 K because of their higher heat capacity compared to mesh screens. The specific heat of copper or lead decreases rapidly in the low temperature region and is close to the value of the working fluid, helium gas, leading to increased losses. Helium gas specific heat is around  $0.2 \text{ J/cm}^3 \text{ K}$  depending on its pressure up to 15 K and then increases with a sharp peak below 10 K as shown in Figure 1. At temperatures below 20 K, the solid properties have a significant influence on the overall regenerator performance making it impossible for copper or lead to obtain refrigeration capacity at 4 K.



**Figure 1.** Specific heat of (a) Stainless Steel, Phosphor bronze and (b) magnetic materials and helium at 17 bar.<sup>15</sup>

This barrier has been broken by the use of magnetic materials. Rare-earth compounds like Lanthanide materials with low magnetic ordering temperatures at less than 10 K could be used as cryogenic regenerators.<sup>14</sup> These materials have large specific heats below 10 K caused by the magnetic specific heat anomaly during magnetic phase transition.

Erbium, Neodymium and other rare earth alloys have been developed as regenerator materials.<sup>15</sup> Research on regenerator materials below 30 K is experimental in nature as theories predicting the transition temperature and heat capacity of an alloy or inter-metallic compound are lacking and materials are discovered by intuition or trial and error. Therefore, extensive studies on search of materials or combination of them with higher performance have been carried out.<sup>16</sup> Research on the theoretical and experimental analyses is reviewed in the following section.

### 2.3 Analysis of Regenerator

Design of regenerators is to determine the volume required for regeneration having maximum heat transfer ability with minimum losses. It is also done by predicting and analyzing their performance with mathematical models and simulation for a given geometry and material. Mathematical theories to describe regenerator operation dates back to 1927 when Nusselt formulated a pair of differential equations that govern the steady state behaviour of an ideal regenerator.<sup>1</sup> Due to the complexity of the conservation equations, analytical solutions are difficult to obtain.<sup>17</sup> Therefore numerical solutions, via a CFD modeling approach, simulate the actual conditions occurring in the regenerator of cryocoolers.<sup>3,18,19</sup> A numerical model provides a powerful tool for estimating parameters, such as enthalpy flow, temperature, velocity, phase between flow and pressure, mass flow which leads to a deeper understanding of heat transfer and fluid-wall interactions in oscillating flow of the working fluid influencing regenerator performance. One such regenerator-design code, developed by NIST is the REGEN 3.2.<sup>20-22</sup>

Rules of thumb useful for designing regenerators have been proposed using the REGEN 3.2 simulation program.<sup>22</sup> For a given mesh size, at constant pressure the length of regenerator is inversely dependent on the cold end temperature and frequency of operation, whereas it is independent of the cooling power. Also, an optimized phase angle exists between the mass flow and pressure at a particular cold end temperature of the regenerator.<sup>22</sup> The cooling power varies linearly with the regenerator cross section area and is also

frequency dependent. Use of higher frequencies to increase cooling power relies on the development of regenerator geometries with smaller hydraulic diameter and operation of compressors at resonant frequencies.<sup>23</sup>

A number of different geometries, focusing on the heat capacity of material, heat transfer and fluid flow friction characteristics and losses in the regenerator have been compared.<sup>12,24-27</sup> Regenerator shapes with a higher ratio between the Colburn factor ' $j$ ' and friction factor ' $f$ ', performed better, independent of the Reynolds number.<sup>7</sup> The factor  $NPH / NTU$  has been used to identify the suitability of a geometrical configuration as a regenerator.  $NPH$  is the Number of Pressure Heads and  $NTU$  is the Number of Transfer Units. Smaller values of the ratio  $NPH/NTU$  leads to better regenerator performance. The parallel plates and lenticular shaped geometric elements give the best possible performance limits, against which other geometries can be compared. The performance shows minor dependence on the Reynolds number.<sup>7</sup>

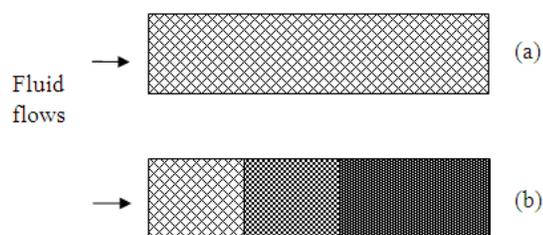
The minimum required volume of a regenerator is calculated by using estimates of the losses due to regenerator ineffectiveness, conduction, and pressure drop within the system.<sup>26</sup> The matrix configuration and size is then selected to the designed regenerator volume

Simultaneously experimental studies of regenerators have been conducted to generate correlations for heat transfer and fluid flow friction.<sup>27-31</sup> These experiments were also used to validate the theoretical analysis prediction. Single mesh regenerator or also called a homogeneous regenerator geometry could not satisfy the overall performance parameters required for a given Cryocooler. The search for a best regenerator that is a configuration having maximum effectiveness and minimum losses continued.

An ideal regenerator is one with zero losses throughout its length. In general, regenerator performance is improved when losses are decreased. A regenerator mesh with a lower hydraulic diameter in the cold region and a larger hydraulic diameter in hot region will lead to lower regenerator losses. The influence of individual regenerator loss mechanism differs at different temperatures. At low temperatures, the heat exchange loss is large, while the pressure drop in this region is negligible. This implies that a homogeneous regenerator will not give an optimum performance.

Varying the volume and the length combinations of mesh sizes is possible, and methodologies have been devised theoretically and experimentally to generate combinations conforming to the physical and geometrical

constraints. A regenerator consisting of varying matrix geometry called the hybrid regenerator could outperform a homogeneous or single mesh regenerator. Cryocoolers with hybrid regenerators have been reported to exhibit improved performance compared to single mesh regenerators. A single mesh size and a multi-layered structure of mesh sizes, or hybrid regenerator is shown in Figure 2. The combination of mesh sizes, lengths of uniform mesh size sections, and overall volume for a given area that would give an optimum performance of the regenerator has to be designed. Theoretical and experimental studies have been carried out to arrive at the optimum combination which is discussed in the next section.



**Figure 2.** Schematic of a single mesh and hybrid regenerator.

a. Single mesh Regenerators. b. Hybrid regenerator.

### 3. Optimization of Regenerators

The commonly used configuration in hybrid regenerators are the wire mesh screens and spheres as their porosity can be quantified. A hybrid regenerator consisting of different sizes arranged sequentially in the regenerator tube is shown in Figure 2. Empirical methods for development of hybrid regenerator outnumber the theoretical efforts. The experimental analyses have mainly depended on the empirical understanding of regenerator functioning for the geometrical design of the combination of regenerator configuration. The experimental and theoretical methods of optimization are discussed.

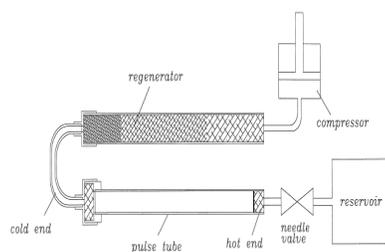
#### 3.1 Experimental Methods

A hybrid regenerator was first used in a Stirling machine by Andeen, consisting of two zones where the matrix geometry differed.<sup>2</sup> The modification produced an increase of 5% in the system efficiency. It was concluded that decreasing the hydraulic diameter as the gas temperature decreases can reduce the sum of the losses due to pressure drop,

ineffectiveness and conduction loss. The regenerator performance was improved by maintaining a low pressure drop across the regenerator while allowing the required surface area for heat transfer. A regenerator consisting of layered mesh screens with high mesh numbers at the cold-end and smaller mesh numbers at the warm end would improve the performance of the Stirling cryocooler, and a Stirling type pulse tube cryocooler.<sup>32,33</sup> Experimental results on hybrid regenerators were obtained in a single stage Gifford-McMahon refrigerator.<sup>34</sup> The hybrid regenerator utilized a nickel microsphere bed in the warm end and lead microsphere bed at the cold end of the regenerator. A minimum temperature of 35 K and a net refrigeration power of 6 W at 77 K were attained.

The effect of a combination of materials on the effectiveness of a regenerator with an oscillating flow in the temperature range 50°C to 0°C was experimentally investigated.<sup>10</sup> It was found that a combination of mesh sizes, sequenced in size as 60-120-200 from the hot side of the regenerator, produced a higher effectiveness and lower pressure drop than when using a single 120 mesh size that maintained a constant porosity of 0.7. A sequence of mesh sizes arranged as 200-120-60 resulted in the lowest effectiveness and highest pressure drop compared to the above combination or the single mesh configuration. This shows that a proper sequence of regenerator materials is important in order to enhance the regenerator effectiveness.

Based on an empirical understanding of regenerator performance, the hydraulic radius of the mesh along the stack length was varied experimentally and an improved performance was reported.<sup>35</sup> A hybrid regenerator with 400, 200 and 150 mesh screens was used in the single stage orifice pulse tube refrigerator (Figure 3). The dimensional design or the comparative performance of the hybrid regenerator with a single mesh screen regenerator is not provided in the paper. The paper also does not provide the design method to find the volume of individual mesh sizes.



**Figure 3.** Schematic of the single stage Orifice Pulse Tube Refrigerator showing the use of a hybrid regenerator.<sup>35</sup>

### 3.2 Theoretical Methods

Some of the theoretical methods in optimization have been based on maximizing efficiency of refrigerator, losses minimization, entropy generation minimization, thermodynamic analysis, harmonic approximation, optimal cooling rate and Carnot efficiency and unconstrained optimization.<sup>36-38</sup>

In general, regenerator performance can be improved by decreasing its losses. The summation of energy loss terms in a regenerator normalized by the PV work provides a convenient form for an optimization study.<sup>1</sup>

$$\frac{W_{net}}{W_{pv}} + \frac{Q_{reg}}{W_{pv}} + \frac{W_{\Delta P}}{W_{pv}} + \frac{Q_z}{W_{pv}} + \frac{\sum Q_l}{W_{pv}} = 1 \quad (1)$$

The NTU, matrix capacity ratio and void volume ratio are determined assuming values for the loss ratios. The optimization process is continued till the losses are a minimum. The optimum NTU of a regenerator mesh is one that minimizes the combined losses associated with pumping and heat transfer. For specified operating conditions and regenerator stack length, it is achieved by varying the hydraulic radius.<sup>39</sup> To optimize the regenerator operation, the pumping losses and heat transfer losses are added. The resulting sum is differentiated with respect to the hydraulic radius, set equal to zero, and then written in terms of the NTU.<sup>39</sup>

$$NTU_{opt} = \frac{6(N_T - 1)^{3/2} \sqrt{(\gamma/(\gamma-1))}}{(1 + f_{NTCR^{-1}}) N_{FL} \frac{N_{MA}}{\alpha_{ff}} f_{NT} \delta_r} \frac{1}{\sqrt{\left( N_T (N_T^3 - 1) N_{pr}^{2/3} \frac{C_f}{j} \right)}} \quad (2)$$

Another theoretical method is to calculate the minimum volume of a regenerator by estimating the losses due to regenerator ineffectiveness, conduction, and the pressure drop of the system.<sup>8</sup>

Using the efficiency of the engine as the objective function to be maximised, two optimisations of the regenerator matrix were performed.<sup>5</sup> In the first optimisation, the wire diameter and fill factor were varied uniformly throughout the matrix to maximise efficiency. The regenerator matrix in the second optimisation was divided into three sections with the two end sections at 5 mm length and a central section of 51 mm. The 5 mm length end sections were chosen such that the sections were used to refine the discretisation locally. Now the individual wire diameters and fill factors were adjusted to optimise efficiency. MusSim software was used to perform the optimisations applying the conjugate

gradients method. In the second optimisation, the electrical efficiency of the engine improved from 32.9% to 33.2%, while the power output and the regenerator loss decreased by 3% and 5% respectively.

Finding a suitable balance between the losses within a regenerator is another method for the optimization of regenerator performance.<sup>6</sup> An ideal regenerator has no entropy production and zero enthalpy flow and heat flow. Hence the entropy production rate within the regenerator is minimized, thereby increasing the coefficient of performance for the entire system. A finer matrix structure can reduce the entropy production associated with heat transfer. However, a larger hydraulic diameter, with a coarser matrix, reduces the pressure drop. For a constant regenerator volume, a low aspect ratio (short and wide regenerator) reduces the pressure drop. A high aspect ratio (tall and narrow) regenerator improves the heat exchange. Minimum entropy production is at the cold end for smaller hydraulic diameter meshes than in the warm end of the regenerator. Entropy production is due to heat exchange over a finite temperature difference, heat conduction through gas and matrix, and flow resistance. Optimizing a regenerator is to find the combination between the above contributions, so that the total entropy produced is a minimum. Equation 3 containing these terms are:<sup>6</sup>

$$\sigma_r = \beta \frac{(T_r - T_g)^2}{T_g T_r} + \frac{\kappa_{eff,r}}{T_r^2} \left( \frac{\partial T_r}{\partial l} \right)^2 + \frac{\kappa_{eff,r}}{T_g^2} \left( \frac{\partial T_g}{\partial l} \right)^2 + \frac{\mu \epsilon_r (j_m)^2}{T_g} \quad (3)$$

From Equation 3, the total rate of entropy production is then given by:<sup>6</sup>

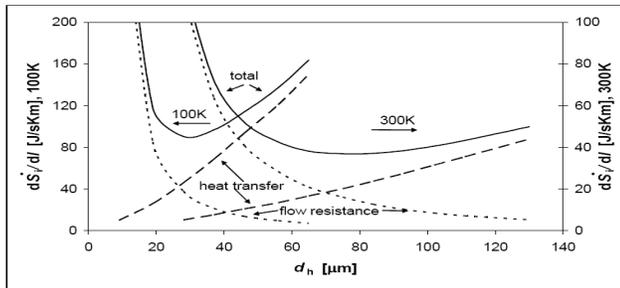
$$\dot{S}_{ir} = A_r \int_0^{L_r} \sigma_{ir} dl \quad (4)$$

The entropy produced per cycle is

$$S_{ir} = \int_0^c \dot{S}_{ir} dt \quad (5)$$

Along the two ends of the regenerator at 300 K and 100 K, the entropy production rate per unit length as a function of hydraulic diameter is shown in Figure 4. Near the 100 K temperature region, the heat exchange contribution is very large, while the contribution of pressure drop to entropy production in this region is negligible. This implies that a regenerator with a single mesh size will have a high entropy production. If at the cold end of the regenerator, a much lower hydraulic diameter than at the warm end is

positioned, a lower entropy production rate can be achieved. Hence a regenerator consisting of different matrix geometry would lead to better performances than a single mesh regenerator. The expected increase in performance is predicted with the first 2 cm of regenerator near the cold heat exchanger replaced with gauzes of 25 μm gauze.<sup>6</sup> The regenerator losses decrease by 16.9%, whereas the cooling power of the refrigerator increases by 58.3%. The lower pressure drop leads to an increase in COP by 38.8%.<sup>6</sup>



**Figure 4.** Entropy production rates per unit of length in the regenerator as functions of hydraulic diameter, at the cold end of the regenerator (100 K) and warm end (300 K).<sup>6</sup>

In a similar work done, the optimum grain size  $d_h$  was determined for a minimum entropy production rate.<sup>40</sup> An expression for a quantity of regenerator material, giving the best heat capacity is obtained by equating  $\partial S_{ir} / \partial \delta = 0$ .

$$\delta_o \approx \left[ g_\beta g_z \frac{R^2}{C_p^2} \eta \kappa_g \frac{L^2 T_L^2}{T_H P^2} \right]^{1/4} \quad (6)$$

The unconstrained optimisation technique, wherein the search for the optimum mesh combination with a maximum effectiveness and minimum pressure drop is by a direct search method has been reported by the author.<sup>41</sup> The optimisation method is the univariate method of search using the unconstrained optimisation technique. For a three-mesh hybrid regenerator, the mesh sizes have to be suitably arranged within the regenerator tube for optimal performance. Combinations of 250, 300 and 400 mesh sizes are possible and a methodology is devised and incorporated in the simulation program of the regenerator to generate combinations confirming to the physical constraints of the Orifice Pulse Tube Refrigerator system.

The design methodology for the three zone matrix is as follows:-

a) Total Length of the regenerator

$$L_1 + L_2 + L_3 = tl \quad (7)$$

where  $L_1$ ,  $L_2$ , and  $L_3$  are the individual lengths of the three meshes.

b) Volume of the Regenerator meshes

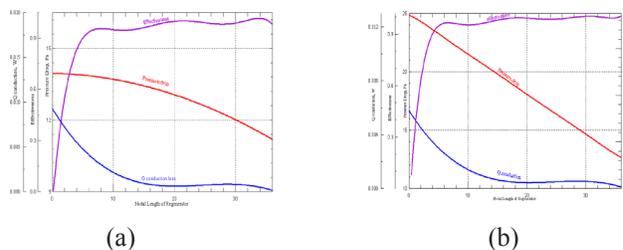
The regenerator void volume of the meshes should be equal to the total regenerator void volume.

$$V_{r1} + V_{r2} + V_{r3} = V_r \text{ and } \text{por}_1 L_1 + \text{por}_2 L_2 + \text{por}_3 L_3 = V_r / A_r \quad (8)$$

The void volume of the regenerator  $V_r$ , total length of the regenerator  $tl$ , regenerator frontal area  $A_r$  are design parameters obtained from the system optimisation. As there are 3 unknowns and 2 Equations., 1 unknown  $L_1$  is assumed and the other 2  $L_2$  and  $L_3$  are calculated.

c) Equations are written in matrix form  $AX = C$ , where  $A$  is the coefficient matrix,  $X$  is the unknown matrix and  $C$  is the constant matrix. The solution of  $X$  is obtained by Cramer's rule  $X = A^{-1}C$ .  $A^{-1}$  is the inverse of matrix  $A$  and is obtained by Gauss Jordan rule.

The specifications of Orifice Pulse Tube Refrigerator (OPTR) developed by Kral are input into the computer program developed and performance parameters are predicted.<sup>42</sup> The regenerator model is integrated with the complete OPTR simulation program. Performance of the 400 mesh regenerator as shown in Figure 5a is calculated. Next the performance of hybrid meshes in Kral's OPTR configuration with the length of 250 mesh size varying from 10 mm to 80 mm inside the regenerator tube are calculated and shown in Figure 5b. For a hybrid regenerator of 250-300-400 mesh combination with lengths 20-78.086-4.913 mm respectively, the effectiveness is 0.9923 for a pressure drop of 0.0897 bar. The regenerator effectiveness has increased by 0.16% and the pressure drop during compression has decreased by 29.2% when compared to the 400 mesh regenerator.



**Figure 5.** Effectiveness and losses along the Nodal length of (a) 400 Mesh Regenerator and (b) 250-300-400 Mesh in Kral's OPTR system.<sup>41</sup>

Thus a combination of mesh sizes with continuously varying hydraulic diameter gives a higher effectiveness with a lower pressure drop compared to a single mesh regenerator.

### 3.3 Regenerators at Low Temperatures

Rare earth magnetic materials exhibit higher heat capacities at certain narrow temperature range called the inversion temperature. In the colder parts of the regenerator, if these materials are placed at appropriate volume proportions, an increase in the heat capacity of the material is possible. Thus heat capacities of regenerator material higher than the working fluid along the entire length of the regenerator are possible with combinations of conventional and rare-earth materials. This hybrid regenerator should have appropriate volumes to ensure a maximum effectiveness and minimum pressure drop.

A combination of three regenerator materials - lead,  $\text{Er}_3\text{Ni}$  and  $\text{ErNi}_{0.9}\text{Co}_{0.1}$ , in the ratio 30%, 30% and 40% in sequence was used and predicted a 34% increase in the 4.2 K cooling power of the refrigerator compared to that of a regenerator using only  $\text{Er}_3\text{Ni}$  grains.<sup>43</sup> An experimental optimization of the structural parameters and geometrical configuration of the hybrid refrigerator using two zone and three zone mesh types with varying volumes was presented.

An experimental study utilizing the regenerator within a 4K-Gifford McMohan cryocooler was presented in the reported article.<sup>44</sup> It was found that a large heat-transfer area and high heat transfer coefficients produced by decreasing the size of the Lead spheres resulted in a higher NTU, and improved efficiency of the regenerator. Smaller spheres created a higher pressure drop in the regenerator, thereby reducing the expansion work at cold end. The study established that an optimal diameter exists when the porosity of the lead spheres within the packed bed is kept constant.

A layered structural regenerator using magnetic regenerator materials in a two-stage GM refrigerator was investigated.<sup>45</sup>  $\text{Er}_{0.75}\text{Gd}_{0.25}\text{Ni}$  was placed in the high temperature part of the second stage regenerator. The NIST numerical model REGEN3.3 was used to find the optimum set of parameters that yield high second-law efficiency.<sup>46</sup> This software incorporates the properties of  $^4\text{He}$  and  $^3\text{He}$  and includes 30 regenerator materials. An optimized 4 K He-3 regenerator using GOS (Gadolinium Oxysulfate) and ErPr with the warm end at 30 K and average pressure of

1.0 MPa reduced the regenerator loss to 0.36 and a second law efficiency of 25 % was achieved. A regenerator with GOS spheres and  $\text{Er}_{0.5}\text{Pr}_{0.5}$  spheres was found to be the best material combination.  $\text{HoCu}_2$  could be substituted for GOS and a slightly higher efficiency could be achieved when the hot end temperature was 30 K.

## 4. Conclusion

This paper presents the state of the art in the design of cryogenic regenerators. An assessment of the methods adopted in the optimization of regenerators for 300 K to 30 K temperature range is discussed. A hybrid regenerator consisting of different mesh sizes would lead to higher performances than homogeneous regenerators. The regenerator mesh with a lower hydraulic diameter in the cold region and a larger hydraulic diameter in hot region will lead to lower regenerator losses. Regenerators with components in a parallel orientation provide a lower pressure drop than screen mesh regenerators. Theoretical analyses of hybrid regenerators below 30 K have been difficult due to high heat capacities at inversion temperatures. This has led to empirical understanding of the regenerator behavior with increasing use of hybrid regenerators. Theoretical prediction of the transition or inversion temperature and the heat capacity of rare earth materials at temperatures below 30 K has to be taken up for its economical and optimum usage.

Regenerative cryocoolers in the future will have to be designed for higher capacities with smaller geometries. They will need to operate at frequencies above 100 Hz with reductions in size for a given cooling power. The regenerator has been able to keep pace with the developments in cryocooler technology.

## 5. References

1. Ackerman RA. Cryogenic Regenerative Heat Exchangers; New York, Plenum Press; 1997. Crossref
2. Andeen BR. Heat Capacity and Geometry Impacts on Regenerator Performances. Proceedings of the Cryogenic Engineering Conference - Advances in Cryogenic Engineering. 1982 Aug; 27:611-9.
3. Gary J, Denny DE, Radebough R. A computational model for a regenerator. National Bureau of Standards Boulders. Special Publication; Boulder, Colorado. 1985 May; 698:199-11.

4. De Waele A. Optimization of pulse tubes. *Cryogenics*. 1999; 39:13–5. Crossref
5. Andersen SK, Carlsen H, Thomsen PG. Numerical study on optimal Stirling engine regenerator matrix designs taking into account the effects of matrix temperature oscillations. *Energy Conversion and Management*. 2006 May; 47(7-8):894–908. Crossref
6. Willems DWJH. High Power Cryocooling [Ph.D. Thesis]. Technical University: Eindhoven; 2007. p. 1–183.
7. Rühlich I, Quack H. Investigation on Regenerative Heat Exchangers. Proceedings of the International Cryocoolers Conference. *Cryocoolers 10*: Kluwer Academic/Plenum Publishers; 1999. p. 265–74.
8. Radebaugh R, Louie B. A simple first step to the optimization of regenerator geometry. *NBS Special Publication Cryocooler*. 1985; 698:177–98.
9. Superczynski WF, Green GF. Photoetched Regenerator for use in a High Frequency Pulse Tube. Proceedings of the International Cryocoolers Conference. *Cryocoolers 14*, Arnold, MD: Kluwer Academic/Plenum Publishers; 2007. p. 389–96.
10. Lee GT, Lee JH, Kang BH. Effectiveness Enhancement of a Thermal Regenerator in an Oscillating Flow. *Applied Thermal Engineering*. 1998 Apr; 18(8):653–60. Crossref
11. Mitchell MP. Assembly methods for etched foil regenerators. *Advances in Cryogenic Engineering*. New York, Anchorage, Alaska (USA): Plenum Press. 2004; 710:1592–7.
12. Nam K, Jeong S. Development of parallel wire regenerator for cryocoolers. *Cryogenics*. 2006 Apr; 46(4):278–87. Crossref
13. Tao Y, Liu Y, Gao F, Chen X, He Y. Numerical analysis on pressure drop and heat transfer performance of mesh regenerators used in cryocoolers. *Cryogenics*. 2009; 49:497–503. Crossref
14. Buschow KHJ, Olijhoek JF, Miedema AR. Extremely large Heat capacities between 4 and 10 K. *Cryogenics*. 1975; 15(5):261–64. Crossref
15. Mérida WR, Barclay JA. Monolithic regenerator technology for low temperature (4K) Gifford McMahon cryocoolers. *Advances in Cryogenic Engineering*. Plenum Press: New York. 1998 Jan; 43:1597–04.
16. Tanaeva IA, Ikeda H, van Bokhoven LJA, Matsubara Y, de Waele ATAM. Heat capacities and magnetic moments of potential regenerator materials at low temperatures. *Cryogenics*. 2003 May; 43(7):441–8. Crossref
17. Willmott AJ, Schmidt FW. *Thermal Energy Storage and Regeneration*. Hemisphere Publishing Corporation; 1981.
18. Atrey MD, Bapat SL, Narayankhedkar KG. Theoretical analysis and performance investigation of Stirling cycle regenerators. *Cryogenics*. 1991 Apr; 31(12):1044–52. Crossref
19. Hegggs PJ. Design and Operational Flexibility of Thermal Regenerators Heat Exchange Engineering by Foumeny and Hegggs. Ellis Horwood Publication: UK. 1991; 1:1088–96.
20. Zhu S, Matsubara Y. A numerical method of regenerator. *Cryogenics*. 2004 Feb; 44(2):131–40. Crossref
21. Shi J. Optimization of single and multi-stage pulse tube refrigerators [Ph.D. Thesis]. University of Wisconsin–Madison; 2008. p. 1–161.
22. Pfothenauer JM, Shi JL, Nellis GF. A Parametric Optimization of a Single Stage Regenerator Using REGEN 3.2. *Cryocoolers 13*. Springer Science+Business Media Inc: New York; 2004. p. 463–70.
23. Radebaugh R. Cryocoolers: The state of the art and recent developments. *Journal of Physics: Condensed matter*. 2009; 21(16):1–9. Crossref PMID:21825399
24. Atrey MD, Bapat SL, Narayankhedkar KG. Influence of regenerator matrix and working fluid on optimisation of design parameters of Stirling cryocoolers. *Cryogenics*. 1994 Jun; 34(1):211–4. Crossref
25. Waele DATAM, Steijaert PP, Koning JJ. Thermodynamical aspects of pulse tubes. *Cryogenics*. 1997; 37(6):313–24. Crossref
26. Miyabe H, Takahashi S, Hamaguchi K. An approach to the design of Stirling engine regenerator matrix using packs of wire gauzes. Proceedings of 17th Intersociety Engineering Conference on Energy Conversion; US; 1982 Aug. p. 1839–44.
27. Thomas B, Pittman D. Update on the evaluation of different correlations for the flow friction factor and heat transfer of Stirling engine regenerators. 35th Intersociety Energy Conversion Engineering Conference; Las Vegas; 2000. p. 76–84. Crossref
28. Gedeon D, Wood JG. Oscillating-flow regenerator Test Rig: Hardware and theory with derived correlations for screens and felts. NASA Contractor Report 198442: US; 1996 Feb. p. 1–74.
29. Isshiki S, Takasaki Y, Ushiyama I. An experimental study on flow resistance of regenerator wire meshes in oscillating flow. 32nd Intersociety energy conversion engineering conference; Honolulu; 1997. p. 1027–32.
30. Ju Y, Jiang Y, Zhou Y. Experimental study of the oscillating flow characteristics for a regenerator in a pulse tube cryocooler. *Cryogenics*. 1998; 38(6):649–56. Crossref
31. Qiu LM, He YL, Gan ZH, Zhang XB, Chen GB. Regenerator performance improvement of a single-stage pulse tube cooler reached 11.1 K. *Cryogenics*. 2007; 47(6):49–55. Crossref
32. Orłowska AH, Davey G. Measurement of losses in a Stirling cycle. *Cryogenics*. 1987 Nov; 27(11):645–51. Crossref
33. Imura J, Iwata N, Yamamoto H, Ohashi Y, Nomachi H, Okumura N, et al. Optimization of regenerator in high capacity stirling type pulse tube cryocooler. *Physica C*. 2008; 468:2178–80. Crossref

34. Borchi E, Zoli M, Lombardini L. On the efficiency of hybrid regenerators with different matrix configurations. *Cryogenics*. 1987; 27(6):314–6. Crossref
35. Huang BJ, Yu GJ. Experimental study on the design of orifice pulse tube refrigerator. *International Journal of Refrigeration*. 2001 Aug; 24(5):400–8. Crossref
36. Das SK, Sahoo RH. Thermodynamic optimization of regenerators. *Cryogenics*. 1991; 31:862–8. Crossref
37. De Waele ATAM. Regenerator dynamics in the harmonic approximation II. *Cryogenics*. 2001; 41:195–200. Crossref
38. deBoer PCT. Optimal performance of regenerative cryocoolers. *Cryogenics*. 2011; 51:105–13 Crossref
39. Finkelstein T, Organ AJ. *Air Engines*. New York: ASME Press. 2001; 1:1–351.
40. Will ME, De Waele ATAM. Heat exchanger versus regenerator: A fundamental comparison. *Cryogenics*. 2005 Jul; 45(7):473–80. Crossref
41. Jayaraman B, Kumar S P. Design Optimisation and performance analysis of orifice pulse tube cryogenic refrigerators. *Indian Journal of Science and Technology*. 2010 Apr; 3(4):425–36.
42. Kral SF, Dallas H, Jon R, Joseph J. Test Results of an Orifice Pulse Tube Refrigerator. *Advances in Cryogenic Engineering*. 1992 Feb; 37:931–37. Crossref
43. Ju YL, Wang L. On the numerical design of a new type of 4 K GM/PT hybrid refrigerators. *Cryogenics*. 2002 Sept; 42(9):533–42. Crossref
44. Kuriyama T, Sagawa K, Tachibana H, Okamura T. Effect of Regenerator Material Configuration on 4K-GM Cryocooler Performance. *Proceedings of the International Cryocoolers Conference*. Cryocoolers 15, California: Kluwer Academic/Plenum Publishers; 2009. p. 317–24.
45. Tsukagoshi T, Matsumoto K, Hashimoto T, Kuriyama T, Nakagome H. Optimum structure of multilayer regenerator with magnetic materials. *Cryogenics*. 1997 Jan; 37(1):11–4 Crossref
46. Radebaugh R, Yonghua H, Gallagher AO, Gary J. Optimization Calculations for a 30 HZ., 4 K Regenerator with Helium-3 Working Fluid. *Proceeding IEEE/CSC & ESAS AIP conference proceedings*; Colorado. 2010; 1218:1581–92.