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Pareto Optimal Solutions of the Fuzzy Bicriteria Sheet Metal Problem

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Abstract

Objectives: An algorithm has been developed to find Pareto Optimal solutions of the fuzzy bicriteria sheet metal problem with pairwise nesting of designs. **Methods and Statistical Analysis**: The sheet metal problem has been solved by many workers, all of whom have considered the entities of cost and time as crisp numbers. However, in practical situations since cost and time are imprecise, the present work considers them as interval fuzzy numbers. Ordering between overlapping interval numbers is obtained by applying a fuzzy membership approach and a modified Hungarian algorithm is developed to obtain fuzzy Pareto Optimal solutions of the bicriteria problem. The newly developed algorithm is explained by a numerical example. **Findings and Results:** The set of both fuzzy Pareto optimal and other solutions obtained by applying the proposed algorithm, provide the Decision maker a lot of flexibility in making decisions. He can select the solution according to his priority. From amongst the fuzzy Pareto Optimal solutions obtained, he can select the solution which minimizes the cost or the solution which minimizes the time or take the middle path and select the solution which minimizes both cost and time as much as possible. Apart from the three fuzzy Pareto Optimal solutions, other solutions obtained by the proposed method can also be selected by the decision maker as per requirement and conditions. The problem being NP hard, it is very difficult and expensive to find fuzzy Pareto Optimal solutions of the bicriteria problem by analytical methods. The newly developed algorithm is not only easy to understand and implement but also gives good fuzzy Pareto optimal solutions. **Improvements:** The method can also be applied to costs and times being triangular and trapezoidal fuzzy numbers and it can be extended to nesting of up to three designs on a sheet.

Keywords: Interval Number, Nesting, Pareto Optimal, Sheet Metal

1. Introduction

Sheet metal is a very useful form of metal which is formed by mechanically flattening metal. This metal formation has high surface area to volume ratio. The usage of sheet metal spreads over manufacturing various automobile parts and home and office appliances. Sheet metal is sheared or cut into desired shapes with the use of machines and dies. The process of loading the sheet metal on a machine and cutting out pieces from the sheet metal with the help of dies is called blanking. The cut out pieces are used to make objects of daily use and the left out portion of sheet metal which cannot be used for any other purpose is called scrap. In order to fulfill a certain demand, the manufacturers are required to load sheet metal on the machine where the desired shape is punched. This procedure requires time for loading and processing the sheet and results in production of scrap. Profits can maximize only if these two quantities can be controlled. In recent times, manufacturers have adopted the process

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of nesting, which combines various shapes thereby reducing the cost of scrap and total set up and processing time. It can be observed from Figures (1a), (1b) and (1c) that when two designs D_1 and D_2 are nested on a metal sheet, the amount of scrap reduces.

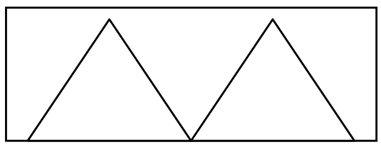

Figure (1a). showing Design D_1.

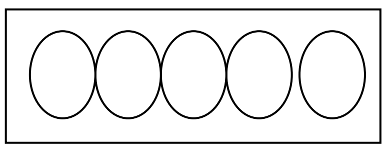

Figure (1b). showing Design D_2.

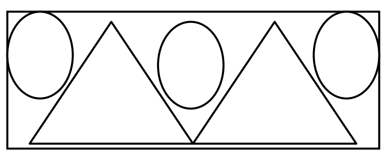

Figure (1c). showing nesting of Designs D_1 and D_2.

Numerous workers have solved the nesting problem by different techniques.In[1,2] have developed a pseudo polynomial dynamic programming algorithm to solve the sheet metal nesting problem.

In[3,4] have developed intelligent algorithms to give optimal nesting. In[5] have applied a compact neighborhood algorithm on large scale nestingand[6–9] have applied genetic algorithms to find optimal solutions of the nesting problem. All the workers have solved the sheet metal nesting problem with a single objective either to minimize the cost or total set up and processing time. In[10] were the first workers to have considered the bicriteria sheet metal nesting problem. They developed a heuristic method to find Pareto optimal solutions of the problem with the two criteria being minimization of scrap and total set up and processing time.

None of the above workers have considered cost and time as fuzzy numbers. However in real life situations the two entities of cost and time are not crisp but imprecise. To overcome this difficulty the present work considers the bicriteria problem with the two entities of cost and time as fuzzy interval numbers. In solving problems with fuzzy numbers one of the most difficult parts is to define a suitable ordering or ranking approach. Fuzzy numbers were first introduced by[11] and one of the frontrunners in defining ranking approach of interval fuzzy numbers[12], followed by[13–19] to name a few.

In the present work, ordering between overlapping interval numbers is obtained by applying a fuzzy membership approach and a modified Hungarian algorithm is developed to obtain Pareto Optimal solutions of the bicriteria problem. A constraint of the problem is that orders are nested at most in pairs. The problem being NP hard, it is very difficult and expensive to find Pareto Optimal solutions by analytical methods. The newly developed heuristic technique is not only easy to understand and implement but also gives good Pareto optimal solutions.

The rest of the paper is organized as follows: Section 2 of the paper gives some definitions, in Section 3 mathematical formulation of the problem is discussed, in Section 4 the proposed algorithm is discussed, in Section 5 a numerical example is given in detail to explain the proposed algorithm and Section 6 is the conclusion followed by Acknowledgements and References.

2. Definitions

Definition 1: Interval numbers arithmetic

If A= $[a^L, a^R]$ and B = $[b^L, b^R]$ are two interval numbers then

(i) Center of A= [aL , aR] is Ac = (aR + aL)/2
(ii) Width of A= $[a^L, a^R]$ is $A^w = (a^R - a^L)/2$.
(iii) $A + B = [a^L + b^L, a^R + b^R]$
(iv) $A - B = [a^L - b^R, a^R - b^L]$

Definition 2: Interval ordering

According to[13], $A \leq_{LR} B$ (Interval A is less than or equal to interval B) iff $a^L \leq b^L$ and $a^R \leq b^R$ as shown in Figures (2a) and (2b).

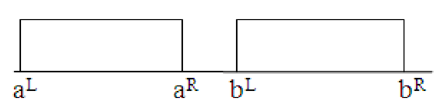

Figure (2a). showing $A \leq_{LR} B$.

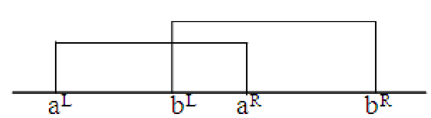

Figure (2b). showing $A \leq_{LR} B$.

However for intervals A and B satisfying $A^w > B^w$, $a^L \leq b^L$ and $a^R \geq b^R$ the above mentioned approach fails to define the ordering. In such a case the approach discussed by[19] is considered. In this approach a fuzzy membership function f(A, B)is defined as

$$f(A,B) = \begin{cases} 0, & b^L = a^L, b^R < a^R \\ \dfrac{(b^L - a^L)}{2(a^w - b^w)}, & a^L < b^L < b^R < a^R \\ 1, & a^L < b^L, b^R = a^R \end{cases}$$

f(A, B) = 1 implies A $\preccurlyeq$ B [Figure (3a)] and f(A, B) = 0 implies B $\preccurlyeq$ A [Figure (3b)].

For $a^L < b^L < b^R < a^R$, f(A, B) = $\dfrac{(b^L - a^L)}{2(a^w - b^w)}$ gives the degree of acceptability of A $\preccurlyeq$ B, if 0.5 < $\dfrac{(b^L - a^L)}{2(a^w - b^w)}$ < 1 then A $\preccurlyeq$ B and if 0 < $\dfrac{(b^L - a^L)}{2(a^w - b^w)}$ < 0.5 then B $\preccurlyeq$ A. If $\dfrac{(b^L - a^L)}{2(a^w - b^w)}$ = 0.5 then either A $\preccurlyeq$ B or B $\preccurlyeq$ A can be considered [Figure (3c)].

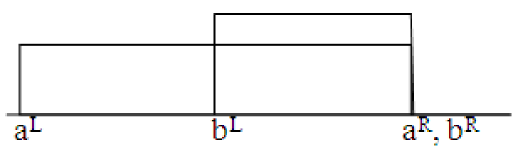

Figure (3a). showing A $\preccurlyeq$ B.

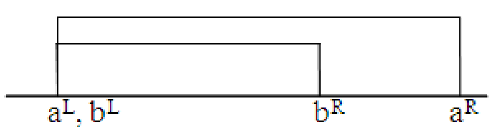

Figure (3b) showing B $\preccurlyeq$ A.

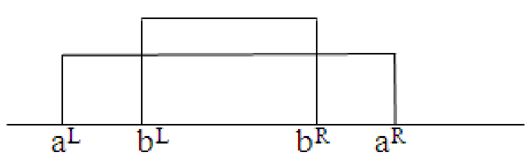

Figure (3c). showing A $\preccurlyeq$ B with a degree of acceptability f(A, B) = $\dfrac{(b^L - a^L)}{2(a^w - b^w)}$.

Definition 3: Pareto optimal solutions

Bicriteria solutions (C1, T1) and (C2, T2) to minimize cost C and time T are said to be Pareto Optimal (Ignizio[20]; Steuer[21]) if C1 ≤ C2 and T1 ≥ T2 with strict inequality holding in at least one of the two cases.

3. Mathematical Formulation of Problem

A sheet metal problem with *n* different designs of dies to be made is considered. The constraint is that no more than two orders can be nested on a sheet. The two objectives are to minimize the total cost of scrap and to minimize total set up and processing time of nesting and blanking. Two tables are formed, the first one showing the total cost of scrap and the second one showing the total set up and processing time in case of no nesting and nesting. Cost and time are taken as interval numbers.

Let $C_{ij} = [C_{ij}^L, C_{ij}^R]$ be the cost of scrap and $T_{ij} = [T_{ij}^L, T_{ij}^R]$ denote total set up and processing time in the blanking operation when designs *i* (*i*= 1,2,…*n*) and *j* (*j* =1,2,…*n*) are nested; let C denote the total cost of scrap and T denote the total set up and processing time after nesting of designs, let x_{ij} be the integer variable taking the values 1 or 0 according as allocation is made or not made to the cell (*i, j*) in the cost table.

The objective of the problem is to minimize

$$C = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} C_{ij} \quad (1)$$

$$T = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} T_{ij} \quad (2)$$

Subject to the constraints

$$x_{ij} = 1 \text{ or } 0 \quad (3)$$

$$C_{ij} = C_{ji}, T_{ij} = T_{ji} \quad (4)$$

$$\sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} \leq n \quad (5)$$

Constraint (5) ensures pairwise nesting of some or all designs.

4. Solution Procedure

The sheet metal problem with *n* different designs of dies is considered. Two symmetric tables are formed – the first one denoting total cost of scrap left after the blanking operation and the second one denoting the total set up and processing times when either designs *i*(*i* =1,2,…*n*) and *j*(1,2,…*n*) are pairwise nested or there is no nesting. The costs and durations are taken as interval fuzzy numbers. A modified Hungarian algorithm is developed and the ordering between intervals numbers is obtained as defined in Section2.

Step 1: In the cost table with costs as interval numbers, select the smallest number in each row and subtract it from every other number. The selection of smallest interval number and subsequent subtraction are done by applying the ordering and subtraction formula as explained in Section 2.

Step 2: In the row reduced Cost table, select the smallest interval number in each column and subtract it from every other number in that column using the ordering used in Step 1.

Step 3: In the row reduced and column reduced cost table reduce each interval number to its center as defined in Section 2.

Step 4: Select the row having a single cell whose interval cost has center at 0 and make assignment in the corresponding cell(*i*, *j*).

Step 5: After making assignment in the cell (*i*, *j*) delete the *i*th row and column and *j*th row and column. This is because allocation in cell (*i*, *j*) implies that designs *i* and *j* have been nested. In case allocation is in cell (*i*, *i*) it implies no nesting. Delete the *i*th row and *i*th column.

Step 6: Repeat Steps 4 and 5with all the rows and columns to obtain all the assignments.

Step 7: Find the total interval cost from the cost table and the corresponding interval time from the time table to get the 1stPareto Optimal solution (C_1, T_1).

Step 8: To obtain the 2ndPareto Optimal solution, the assignment table obtained in Step 4 is considered and the cell with next minimum (non-zero) cost is identified. The first assignment is made in this cell and thereafter assignments are made in the zero cost cells by applying Steps 5-7 to get the second Pareto optimal solution denoted by (C_2, T_2) with $C_1 \leq C_2$ and $T_1 \geq T_2$. In case the solution obtained is not Pareto Optimal then the cell with second next minimum (non-zero) cost is identified. Assignment is first made to this cell and then to the zero cost cells by applying Steps 5-7. In case there is a tie in the next minimum (non-zero) cost cells corresponding to different nestings of designs both the cells are considered one at a time with the remaining zero cost cells and of all the solutions obtained the Pareto optimal solutions are considered. In case there is a tie in the next minimum cost cells corresponding to the same nestings of designs then any one cell is chosen arbitrarily and the remaining 0 cost cells are considered for assignment. To obtain 3rdPareto Optimal solution the third next minimum (non-zero) cost cell is selected in the assignment Table obtained in Step 4 and allocations are first made to that cell and then to the zero cost cells. The fourth and subsequent efficient solutions are obtained by proceeding exactly as in case of the third efficient solution by selecting the next higher minimum (non-zero) cost cell in the assignment Table obtained in Step 4.The process terminates when all cases are exhausted. The third and subsequent Pareto Optimal solutions obtained are denoted by (C_3, T_3), (C_4, T_4),… satisfying $C_1 \leq C_2 \leq C_3 \leq C_4 \leq \dots$ and $T_1 \geq T_2 \geq T_3 \geq T_4 \geq T_5 \geq \dots$

5. Numerical Example

Let there be 5 designs of dies. Table 1 shows the cost of scrap and Table 2 shows the total set up and processing times in case of no nesting and pairwise nesting of the designs of dies on the metal sheet. Both cost and time are interval fuzzy numbers.

Table 1. Denoting cost of scrap in case of no nesting and nesting

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	[4 5]	[1 2]	[2 3]	[3 6]	[1 3]
D_2	[1 2]	[3 4]	[1 4]	[2 3]	[2 5]
D_3	[2 3]	[1 4]	[3 5]	[1 4]	[2 4]
D_4	[3 6]	[2 3]	[1 4]	[2 3]	[2 7]
D_5	[1 3]	[2 5]	[2 4]	[2 7]	[1 2]

Table 2. Denoting total set up and processing time in case of no nesting and nesting

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	[1 2]	[3 4]	[3 5]	[4 7]	[5 8]
D_2	[3 4]	[2 3]	[3 4]	[2 5]	[3 6]
D_3	[3 5]	[3 4]	[2 4]	[3 4]	[1 2]
D_4	[4 7]	[2 5]	[3 4]	[1 2]	[2 4]
D_5	[5 8]	[3 6]	[1 2]	[2 4]	[2 3]

On applying Step 1 to Table 1 the row reduced Table 3 is obtained.

Table 3. Row reduced cost table

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	[2 4]	[-1 1]	[0 2]	[1 5]	[-1 2]
D_2	[-1 1]	[1 3]	[-1 3]	[0 2]	[0 4]
D_3	[-2 2]	[-3 3]	[-1 4]	[-3 3]	[-2 3]
D_4	[-1 5]	[-2 2]	[-3 3]	[-2 2]	[-2 6]
D_5	[-1 2]	[0 4]	[0 3]	[0 6]	[-1 1]

On applying Step 2 on Table 3 the column reduced Table 4 is obtained.

Table 4. Column reduced cost table

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	[1 5]	[-2 2]	[-3 5]	[-2 8]	[-2 3]
D_2	[-2 2]	[0 4]	[-4 6]	[-3 5]	[-1 5]
D_3	[-3 3]	[-4 4]	[-4 7]	[-6 6]	[-3 4]
D_4	[-2 6]	[-3 3]	[-6 6]	[-5 5]	[-3 7]
D_5	[-2 3]	[-1 5]	[-3 6]	[-3 9]	[-2 2]

On applying Step 3 to Table 3 all the interval costs reduced to their centers are shown in Table 5.

Table 5. Interval costs in terms of their centers

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	3	0	1	3	0.5
D_2	0	2	1	1	2
D_3	0	0	1.5	0	0.5
D_4	2	0	0	0	2
D_5	0.5	2	1.5	3	0

5.1 First Pareto Optimal Solution

In Table 5 by applying Step 4 it is observed that Row 1 has single 0 cost cell. The cell in Row 1- Column 2 with 0 corresponding to nesting of designs D_1 and D_2 is selected and assignment is made to that cell. Thereafter the rows and columns showing scrap costs for designs D_1 and D_2 are deleted to obtain Table 6.

Table 6. Rows and columns corresponding to D_1 and D_2 deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2					
D_3			1.5	0	0.5
D_4			0	0	2
D_5			1.5	3	0

In Table 6 it is observed that Row 3 has a single 0 cost cell corresponding to the nesting of designs D_3 and $D_{4.}$ Assignment is made to the Row 3- Column 4 cell and the rows and columns corresponding to designs D_3 and D_4 are deleted to obtain Table 7.

Table 7. Rows and columns corresponding to D_3 and D_4 deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2					
D_3					
D_4					
D_5					0

From Table 7 it is observed that assignment is made to Row 5- Column 5 corresponding to Design 5 which cannot be nested. The assignments in first Pareto Optimal solution are nesting of D_1 and $D_{2,}$ nesting of designs D_3 and D_4 and no nesting of design $D_{5.}$ From the cost Table 1 the total cost of scrap $C_{1} =$ [1 2] + [1 4] + [1 2] = [3 8] and from Table2 the total set up and processing time T_1= [3 4] + [3 4] + [2 3] = [8 11]. The first Pareto Optimal solution obtained is (C_1 , T_1) = ([3 8] , [8 11]).

5.2 Second Pareto Optimal Solution

To obtain the second Pareto Optimal solution, Table 5 is considered. The next minimum (non-zero) cost is 0.5 at Row 1- Column 5, Row 5- Column 1 and Row 3- Column5. Since Row 1- Column 5, and Row 5- Column 1 correspond to the same nesting of designs D_1 and D_5, so the cell in Row 1- Column 5 is considered among the two and the cells in Row 1- Column 5 and Row 3- Column5 are considered one by one in sections 5.2.1 and 5.2.2.

5.2.1

In table 5 the cell in Row 1- Column 5 with 0.5 corresponding to nesting of designs D_1 and D_5 is selected and assignment is made to that cell. Thereafter the rows and columns showing scrap costs for designs D_1 and D_5 are deleted to obtain Table 8.

Table 8. Rows and columns corresponding to D_1 and D_5 deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2		2	1	1	
D_3		0	1.5	0	
D_4		0	0	0	
D_5					

In table 8 it is observed that Column 3 has a single 0 cost cell corresponding to the nesting of designs D_4 and D_3. Assignment is made to the Row 4- Column 3 cell and the rows and columns corresponding to designs D_4 and D_3 are deleted to obtain Table 9.

Table 9. Rows and columns corresponding to D_4 and D_3 deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2		2			
D_3					
D_4					
D_5					

In Table 9, since no zero cost cell remains so assignment is not possible.

5.2.2

In Table 5 the cell in Row 3 – Column5 with 0.5 corresponding to nesting of designs D_3 and D_5 is selected and assignment is made to that cell. Thereafter the rows and columns showing scrap costs for designs D_3 and D_5 are deleted to obtain Table 10.

Table 10. Rows and columns corresponding to D_3 and D_5 deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	3	0		3	
D_2	0	2		1	
D_3					
D_4	2	0		0	
D_5					

In Table 10, the 0 in cell Row 1-column 2 is considered and allocation is made to the cell. The allocation corresponds to nesting of designs D_1 and D_2. Row and column corresponding to designs D_1 and D_2 are deleted to obtain Table 11.

Table 11. Rows and columns corresponding to D_1 and D_2 deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2					
D_3					
D_4				0	
D_5					

From Table 11 it is observed that assignment is made to Row 4- Column 4 corresponding to Design 4 which cannot be nested. The assignments in the next solution are nesting of D_1 and D_2, nesting of D_3 and D_5 and no nesting of D_4. From the Cost Table 1 the total cost of scrap $C_2 =$ [1 2] + [2 4] + [2 3] = [5 9] and from Table2 the total set up and processing time T_2 = [3 4] + [1 2] + [1 2] = [5 8]. On comparing with the Ist Pareto optimal solution (C_1, T_1) = ([3 8] , [8 11]) it is observed that [3 8] $\prec$ [5 9] and [5 8] $\prec$ [7 13]. Hence (C_2 , T_2) = ([5 9] , [5 8]) is a Pareto Optimal solution.

5.3 Third Pareto Optimal Solution

The cells with next higher minimum (non-zero) cost, which is 1, are considered. These cells are in Row 1- Column 3, Row 2 - Column 3 and Row 2 - Column 4. Since the three cells correspond to different nestings of designs so they are considered one by one in Sections 5.3.1, 5.3.2 and 5.3.3.

5.3.1

In Table 5 the cell in Row 1- Column3 with 1 corresponding to nesting of designs D_1 and D_3 is selected and assignment is made to that cell. Thereafter the rows and columns showing scrap costs for designs D_1 and D_3 are deleted to obtain Table 12.

Table 12. Rows and columns corresponding to designs D_1 and D_3 are deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2		2		1	2
D_3					
D_4		0		0	2
D_5		2		3	0

From Table 12 it is observed that assignment is made to Row 5- Column 5 corresponding to Design 5 which cannot be nested. Row and column corresponding to D_5 are deleted to obtain Table 13.

Table 13. Rows and columns corresponding to design D_5 are deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2		2		1	
D_3					
D_4		0		0	
D_5					

In Table 13 there are two possible assignments, one to the 0 in Row 4- Column 2 corresponding to nesting of designs D_2 and D_4 and the other to the 0 in Row 4- Column 4 corresponding to no nesting of design D_4. Since nesting always reduces the scrap cost so assignment is made to the 0 in Row 4- Column 2 corresponding to nesting of designs D_2 and D_4.

The assignments obtained give nesting of designs D_1 and D_3, nesting of D_2 and D_4 and no nesting of D_5.From the Cost Table 1 the total cost of scrap $C_3 =$ [2 3] + [2 3] + [1 2] = [5 8] and from Table2 the total set up and processing time T_3 = [3 5] + [2 5] + [2 3] = [7 13]. The third solution obtained is (C_3 , T_3) = ([5 8] , [7 13]). Since [3 8] $\prec$ [5 8] and [8 11] $\prec$ [7 13], hence (C_3, T_3) is not a Pareto Optimal solution.

5.3.2

In Table 5 the cell in Row 2- Column 3 with 1 corresponding to nesting of designs D_2 and D_3 is selected and assignment is made to that cell. Thereafter the rows and columns showing scrap costs for designs D_2 and D_3 are deleted to obtain Table 14.

Table 14. Rows and columns corresponding to designs D_2 and D_3 are deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	3			3	0.5
D_2					
D_3					
D_4	2			0	2
D_5	0.5			3	0

From Table 14 it is observed that assignment is made to Row 4- Column 4 corresponding to Design 4 which cannot be nested. Row and column corresponding to D_4 are deleted to obtain Table 15.

Table 15. Rows and columns corresponding to design D_4

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	3				0.5
D_2					
D_3					
D_4					
D_5	0.5				0

From Table 15 it is observed that assignment is made to Row 5- Column 5 corresponding to Design 5 which cannot be nested. The assignments obtained are nesting of D_2 and D_3, no nesting of D_4 and no nesting of D_5. From the Cost Table 1 the total cost of scrap $C_4 =$ [1 4] + [2 3] + [1 2] = [4 9] and from Table2 the total set up and processing time T_4 = [3 4] + [1 2] + [2 3] = [6 9]. On comparing (C_4 , T_4) = ([4 9] , [6 9]) with the Ist Pareto optimal solution (C_1 , T_1) = ([3 8] , [8 11]) it is observed that [3 8] $\prec$ [4 9] and [6 9] $\prec$ [8 11] and on comparing with the second Pareto Optimal solution (C_2 , T_2) = ([5 9] , [5 8]) it is observed that [4 9] $\preccurlyeq$ [5 9] and [5 8] $\preccurlyeq$ [7 13]. Since $C_1 \preccurlyeq C_4 \preccurlyeq C_2$ and $T_2 \preccurlyeq T_4 \preccurlyeq T_1$, the solution (C_4, T_4) = ([4 9] , [6 9]) is a Pareto Optimal solution.

5.3.3

In Table 5 the cell in Row 2- Column4 with 1 corresponding to nesting of designs D_2 and D_4 is selected and assignment is made to that cell. Thereafter the rows and columns showing scrap costs for designs D_2 and D_3 are deleted to obtain Table 16.

Table 16. Rows and columns corresponding to designs D_2 and D_4 are deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1	3		1		0.5
D_2					
D_3	0		1.5		0.5
D_4					
D_5	0.5		1.5		0

From Table 16 it is observed that assignment can be made to Row 3- Column 1 corresponding to the nesting of designs D_3 and D_1. Rows and columns corresponding to designs D_3 and D_1 are deleted to obtain Table 17.

Table 17. Rows and columns corresponding to designs D_3 and D_1 are deleted

Design → ↓	D_1	D_2	D_3	D_4	D_5
D_1					
D_2					
D_3					
D_4					
D_5					0

From Table 17 it is observed that assignment can be made to Row 5- Column 5 corresponding to Design 5 which cannot be nested. The assignments obtained are nesting of D_2 and D_4, nesting of D_1 and D_3 and no nesting of D_5. From the cost Table 1 the total cost of scrap C_5 = [2 3] + [2 3] + [1 2] = [5 8] and from Table2 the total set up and processing time T_5 = [2 5] + [3 5] + [2 3] = [7 13]. On comparing (C_5 , T_5) = ([5 8] , [713]) with the Ist Pareto optimal solution (C_1 , T_1) = ([3 8] , [8 11]) it is observed that [3 8] $\preccurlyeq$ [5 8] and [8 11] $\preccurlyeq$ [7 13] . So (C_5 , T_5) is not Pareto Optimal solution.

On proceeding similarly by selecting the cell with the next minimum (non-zero) number together with 0 cost cells it is observed that no more new Pareto optimal solutions can be obtained.

The results obtained are summarized in Table 18.

Table 18. Solutions obtained by applying the newly developed algorithm

Sl. No	Cells selected for assignment	Nesting of designs	Solution	Pareto Optimal solution
1	All the 0 cost cells of table 5	Nesting of D_1 and D_2, nesting of D3 and D_4 and no nesting of D_5.	(C_1 , T_1) = ([3 8], [8 11]).	1st Pareto optimal solution
2	Cell at Row 1 –Column 5 with cost 0.5 and all the 0 cost cells of table 5	Solution not possible		
3	Cell at Row 3 –Column 5 with cost 0.5 and all the 0 cost cells of table 5	Nesting of D_1 and D_2 , nesting of D_3 and D_5 and no nesting of D_4	(C_2 , T_2) = ([5 9] , [5 8])	3rd Pareto Optimal solution
4	Cell at Row 1 –Column 3 with cost 1 and all the 0 cost cells of table 5	Nesting of D_1 and D_3, nesting of D_2 and D_4 and no nesting of D_5	(C_3 , T_3) = ([5 8] , [7 13])	Not Pareto optimal
5	Cell at Row 2 –Column 3 with cost 1 and all the 0 cost cells of table 5	nesting of D_2 and D_3 ,no nesting of D_4 and no nesting of D_5	(C_4 , T_4) = ([4 9] , [6 9])	2nd Pareto Optimal solution
6	Cell at Row 2 –Column 4 with cost 1 and all the 0 cost cells of table 5	nesting of D_2 and D_4, nesting of D_1 and D_3 and no nesting of D_5	(C_5 , T_5) = ([5 8] , [7 13])	Not Pareto Optimal

7	Cell at Row 3 –Column 3 with cost 1.5 and all the 0 cost cells of table 5	nesting of D_1 and D_2 ,no nesting of $D_{3,}$ no nesting of D_4 and no nesting of D_5	$(C_6 , T_6) = ([7\ 12] , [8\ 13])$	Not Pareto Optimal
8	Cell at Row 5 –Column 3 with cost 1.5 and all the 0 cost cells of table 5	nesting of D_1 and D_2, nesting of D_3 and D_5 and no nesting of D_4.	$(C_7 , T_7) = ([5\ 9] , [5\ 8])$	3rd Pareto Optimal solution, same as (C_2 , T_2)
9	Cell at Row 2 –Column 5 with cost 2 and all the 0 cost cells of table 5	nesting of D_5 and D_2, nesting of D_3 and D_1 and no nesting of D_4.	$(C_8 , T_8) = ([6\ 11] , [7\ 13])$	Not Pareto Optimal
10	Cell at Row 2 –Column 2 with cost 2 and all the 0 cost cells of table 5	nesting of D_3 and D_1, no nesting of D_2, no nesting of D_4, no nesting of D_5	$(C_9 , T_9) = ([8\ 12] , [8\ 13])$	Not Pareto Optimal
11	Cell at Row 4 –Column 1 with cost 2 and all the 0 cost cells of table 5	nesting of D_3 and D_2, nesting of D_4 and D_1 and no nesting of D_5	$(C_{10} , T_{10}) = ([5\ 12] , [9\ 14])$	Not Pareto Optimal
12	Cell at Row 4 –Column 5 with cost 2 and all the 0 cost cells of table 5	Solution not possible		
13	Cell at Row 4 –Column 1 with cost 2 and all the 0 cost cells of table 5	nesting of D_3 and D_1, nesting of D_5 and D_2 and no nesting of D_4	$(C_{11} , T_{11}) = ([6\ 11] , [7\ 13])$	Not Pareto Optimal
14	Cell at Row 1 –Column 1 with cost 3 and all the 0 cost cells of table 5	Solution not possible		
15	Cell at Row 1 –Column 4 with cost 3 and all the 0 cost cells of table 5	nesting of D_3 and D_2, nesting of D_4 and D_1 and no nesting of D_5	$(C_{12} , T_{12}) = ([5\ 12] , [9\ 14])$	Not Pareto Optimal
16	Cell at Row 5 –Column 4 with cost 3 and all the 0 cost cells of table 5	Solution not possible		

It can be seen that all solutions obtained are not Pareto Optimal and in some cases solution is not obtained. The set of solutions obtained, both Pareto optimal and other, provide the Decision maker a lot of flexibility in making decisions. He can select the solution according to his requirement. For example if his primary objective is to minimize the cost he will select the 1st Pareto Optimal solution and do nesting of designs D_1 and D_2 , nesting of designs D_3 and D_4 and no nesting of design $D_{5.}$; if his primary objective is to minimize the total set up and processing time he will select the 3rd Pareto Optimal solution and do nesting of designs D_1 and D_2 , nesting of designs D_3 and D_5 and no nesting of design D_4 ; if his objective is to minimize both as much as possible he will take the middle path and consider the 2nd Pareto Optimal solution and do nesting of designs D_2 and D_3 ,no nesting of design D_4 and no nesting of design D_5. Apart from the three Pareto Optimal solutions the other solutions obtained by the proposed method can also be selected by the decision maker as per requirement and conditions.

6. Conclusion

The algorithm developed in this paper provides a heuristic technique to find Pareto Optimal solution of the fuzzy bicriteria sheet metal problem. The set of Pareto Optimal solutions obtained provides flexibility to the DM and he can select the solution according to his priority. The method can also be applied to costs and times being triangular and trapezoidal fuzzy numbers. By considering a proper ranking approach the triangular and trapezoidal fuzzy numbers can be converted to crisp numbers and thereafter the newly developed algorithm can be applied to get Pareto Optimal solutions. The heuristic technique developed is very easy to understand and implement and can thus be applied extensively in the fuzzy nesting problem. In the present work the nesting is considered to be at most in pairs. However this can be also extended to nesting of up to three designs on a sheet.

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