ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645
**Indian Journal of Science and Technology,** *Vol 10(28), DOI: 10.17485/ijst/2017/v10i28/97642, July 2017*

# Impedance Matching Techniques for Microstrip Patch Antenna

**Sonia Sharma[1*], C. C. Tripathi[1] and Rahul Rishi[2]**

[1]Department of ECE, University Institute of Engineering and Technology, Kurukshetra University, Kurukshetra – 136119, Haryana, India; sonia 990@gmail.com, tripathiuiet@gmail.com
[2]Department of CSE, University Institute of Engineering and Technology, Maharshi Dayanand University, Rohtak – 124001, Haryana, India; rahulrishi@rediffmail.com

## Abstract

**Objective:** To present a concise and comprehensive summarization of various impedance matching techniques for microstrip patch antennas. **Method:** Designing an impedance matching network is a central issue for optimum performance in every part of RF systems like transceiver, amplifier and antenna to ensure maximum power transfer. Various design formulae to calculate the input impedance of patch antenna and techniques to design a matching network should be known to RF designer. **Finding**: In this paper various impedance matching techniques along with their design equations are presented that utilize quarter wave transformer, taper lines, open or short stubs and lumped elements etc. Methods to calculate the input impedance for various antenna structures like rectangular, circular and triangular patch antenna are described. **Application:** This paper concisely covers some of the existing techniques to design an impedance matching network that can be used to solve the impedance matching problem encountered during antenna design.

**Keywords:** Impedance Matching, Input Impedance, Lumped Circuit, Patch Antenna, Quarter Wave Transformer, Stub, Taper Lines

## 1. Introduction

Impedance matching is an emerging arena of research in almost every aspect of technology[1] viz. communication, electronics, electrical, sound, optical etc. In communication area for the transmission of different types of signal; proper termination is important to reduce reflections and to preserve signal integrity with higher throughput of absolute data[2]. As impedance mismatch in RF network causes power to be reflected back to the source from the impedance mismatch boundary. This reflection creates a standing wave, which leads storage of power instead of transmitting it to the load[3]. Hence, there will be less power delivered from the input to the load or other parts of the system. Along with this, standing waves may damage and overheat the RF device because of increased peak power level. Other advantages of proper termination of load are reduction in amplitude and phase error, reduction in power loss and improvement in the signal to noise ratio.

Impedance matching is a challenging step in the antenna design to achieve optimum performance parameters[4–6] like return loss, efficiency, gain etc. Impedance matching also helps in tuning the antenna frequency with a much easier and faster way than modifying the antenna geometry[7–12]. Proper impedance matching also helps in improving the bandwidth of antenna because impedance matching circuits add some additional resonances. Impedance matching circuits also allow incorporating last minute design change by allowing freedom in choosing the values of discrete components, independently. Mostly,

**Author for correspondence*

impedance of antenna is matched by 50Ω feed line because of the fact that almost all the microwave sources and lines are manufactured with 50Ω characteristic impedance.

Therefore, impedance matching has a great importance in antenna designing application but there is a huge shortage of literature detailing the case specific different methods for calculating the input impedance of microstrip antenna. This paper review the different methods used to calculate the input impedance of microstrip patch antenna along with different impedance matching techniques. Section-2 describes the introduction to microstrip antenna and different impedance matching techniques. In section-3 input impedance of rectangular microstrip patch antenna is calculated by various methods so that after getting the input impedance; any matching technique can be applied. In section-4, complete description of various matching techniques is presented. In the last section as a case study, the design equation for the calculation of input impedance of triangular and circular patch antenna is described which can be extended to design of antenna structure of particular interest. Complete study form the calculation of input impedance on the patch antenna and to match this input impedance with the feed impedance using different matching techniques are tried to cover in this review article.

## 2. Theory

Due to many considerable advantages like lightweight, conformable to planar and non-planar surfaces, simple and inexpensive to manufacture using modern printed-circuit technology, compatible with MMIC designs; microstrip antenna is the best choice for modern wireless and mobile applications[4–6]. The shape of microstrip antenna can be rectangle, square, ellipse, circle, triangle, ring, pentagon, or their variations[4–6]. More complex variations on the basic shapes are frequently used to meet particular design demands and in terms of polarization, bandwidth, gain, etc.

A rectangular microstrip patch antenna of length L, width W printed on a substrate with dielectric constant $\varepsilon_r$ and height h is shown in Figure 1. The CAD formulae[5] for the dimension (L, W) calculation at resonating frequency $f_0$ are listed below:

Effective dielectric constant:

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-\frac{1}{2}} \tag{1}$$

Patch width: $$W = \frac{C}{2f_0\sqrt{\frac{\varepsilon_r + 1}{2}}} \tag{2}$$

Patch length: $$L = \frac{c}{2f_0\sqrt{\varepsilon_{re}}} \tag{3}$$

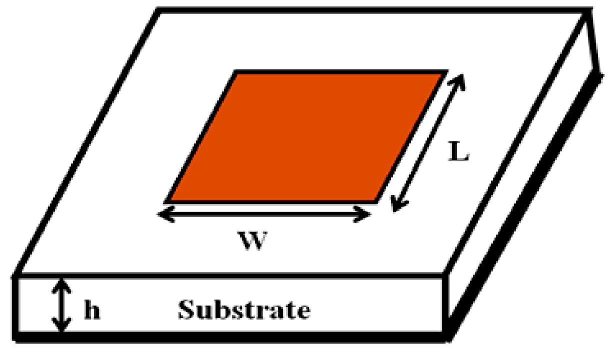


**Figure 1.** Basic Rectangular Patch antenna.

Extended length of patch due to fringing field:

$$\Delta L = 0.412h\frac{(\varepsilon_{re} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{re} - 0.258)\left(\frac{W}{h} + 0.8\right)} \tag{4}$$

Effective patch length: $$L_{eff} = L + 2\Delta L \tag{5}$$

The input impedance[4–6] of an antenna is the impedance presented by an antenna at its terminals and can be written as: $Z_{in} = R_{in} + jX_{in}$ where $Z_{in}$ is the antenna impedance at the terminals, $R_{in}$ is the antenna resistance which consisting of radiation resistance $R_r$ and the loss resistance $R_L$. The imaginary part $X_{in}$ is the antenna reactance and represents the power stored in the near field of the antenna. The power associated with the radiation resistance is the power actually radiated by the antenna, while the power dissipated in the loss resistance in the form of heat is due to dielectric or conducting losses.

To study the impedance distribution over a patch it is necessary to study the electric and current distribution. Patch antenna shown in Figure 2 consists of ground plane, dielectric substrate and radiating patch. The feed probe couples electromagnetic energy in and or out of the patch. The electric field is zero at the center of the patch, maximum on one edge and reverses its direction on opposite edge. This field distribution continuously reverses its direction according to the instantaneous phase of the RF signal. Figure 3 shows the current, voltage and impedance behavior in the radiating patch; the current (magnetic field) is maximum at the center of patch and minimum on the opposite sides of patch, while the voltage (electrical field) is zero in the center and maximum on one edge and reverses its direction (minimum) on opposite edge. Hence the distribution of impedance is minimum at the center and maximum on the both edge of patch. So there is a point lie inside the surface of radiating patch where the impedance is 50Ω; the simplest method for impedance matching is to locate the position of 50 Ω points and connect the feed probe at this point.

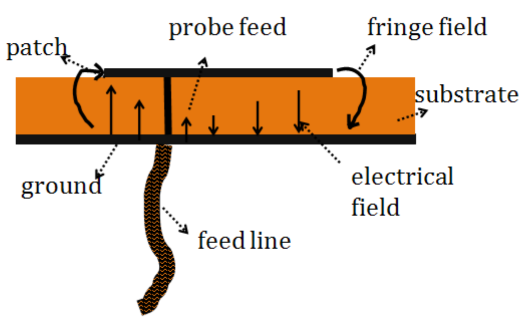


**Figure 2.** Patch antenna showing electric field distribution.

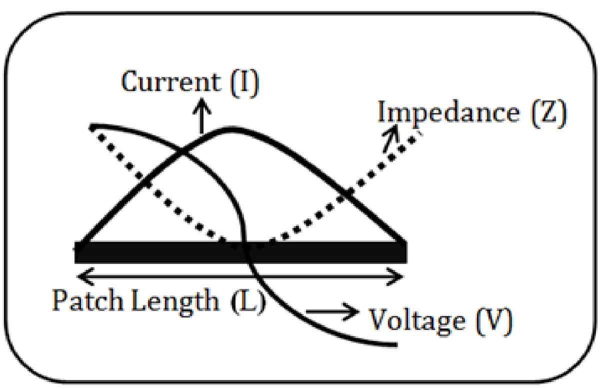


**Figure 3.** Voltage (V), Current (I) and Impedance (Z) distribution along patch resonant length L.

Impedance matching can also be done by calculating the input impedance then applying some impedance matching techniques. Impedance matching techniques can be categorized in two broad categories i.e. Distributed Method and Lumped Element Method as shown in Figure 4.

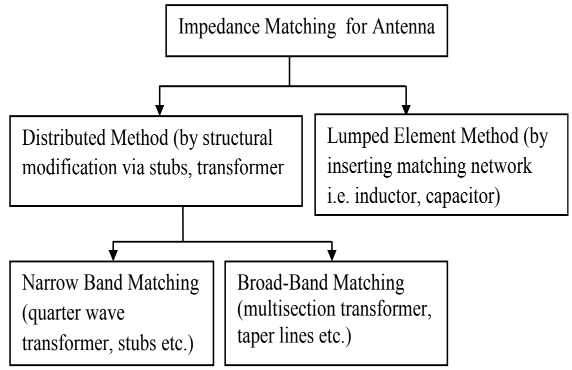


**Figure 4.** Impedance matching techniques.

In distributed[13–17] impedance matching method, antenna can be matched by doing some structural modifications through the use of stubs, single and multi section quarter wave transformer, tapered line, balun and active components as shown by Figure 5. The main advantage of distributed impedance matching method is that there is no requirement to modify the geometry of radiating structure. Therefore, radiation performance of the radiating structure is independent to the matching network and results in easy design optimization. However, this method increases the size of antenna and not recommended for the design of practical array systems. Also system efficiency degrades due to the increase in spurious radiation losses from extra circuitry of matching network. The distributed method can match the impedance in narrow band as well as in broadband. Narrow band impedance matching is achieved by Quarter wave transformer[18–19] and Stubs[20–23]. Whereas for broadband impedance matching is done by multisections quarter wave transformer and taper line[24–25]. These techniques are describes in detail in the section 4.

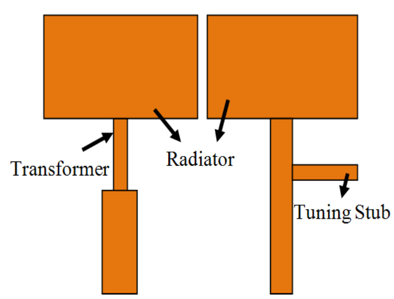


**Figure 5.** Distributed Impedance matching techniques by Quarter wave transformer and Stubs etc.

A lumped networkis introduced to realize impedance matching between antenna and feed structures. Lumped element method can be implemented either by inserting a separate network without changing the antenna structure or by etching slots or notch in the antenna geometry as shown in Figure 6. The main advantage of placing the impedance matching network between antennas and feeding structure is the enhancement in the impedance bandwidth.

In both approaches, extra losses are introduced in the antenna structure. In the distributed approach to impedance matching, loss can be due to spurious loss within the dielectric material. The losses in the lumped element approach are due to the inclusion of finite quality factor components like inductors and capacitors.

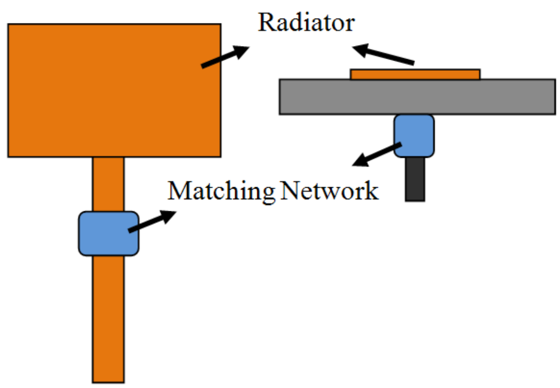


**Figure 6.** Lumped element Impedance matching techniques by matching network.

In order to apply any specific matching technique, input impedance at the edge of antenna, must be known. Therefore, next section presents the different approaches adopted by the researchers to calculate the input impedance at the edge of patch antenna.

## 3. Input Impedance Calculation of Rectangular Microstrip Patch Antenna

In this section input impedance of rectangular microstrip patch antenna is calculated by (A) Transmission line model, (B) Cavity model, (C) Radiation Resistance calculation Method and (D) Quality factor calculation method.

### 3.1 Calculation of Input Impedance by Transmission Line Model

The calculation of input impedance by Transmission line model is case specific depending upon the kind of feed technique used. Therefore, next part is divided in two parts as detailed below.

#### 3.1.1 *For Microstrip Fed Patch Antenna*

The Transmission Line model to represent the microstrip fed rectangular *patch*[4–6] as shown in Figure 7 which consists of a parallel-plate transmission line connected with two *radiating slots* (apertures), each of width W and height h, separated by a transmission line of length L. Each radiating slot of microstrip patch antenna is represented as a parallel equivalent admittance $Y=G+jB$.

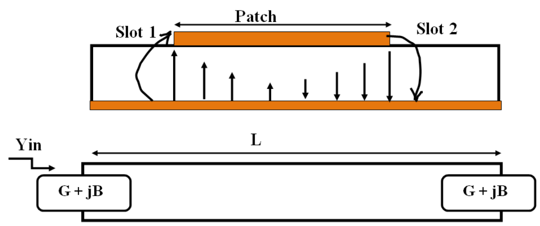


**Figure 7.** Transmission line model for rectangular patch antenna as radiating slot [7].

Since both slots are identical, the total resonant input impedance[26–29] becomes $Z_{in}=1/2G$. Conductance G of single radiating slot1 it is associated with the power radiated and is given by eq (6).

$$G=\begin{cases} 1/90\left(W/\lambda\right)^{2} & W \ll \lambda \\ 1/120\left(W/\lambda\right) & W \gg \lambda \end{cases} \tag{6}$$

Where W = patch width and λ= resonant wave length, B is susceptance due to energy stored in the fringing field near the edge of the patch and given by eq (7)

$$B=\frac{k_0 \Delta l \sqrt{\varepsilon_{eff}}}{Z_0} \tag{7}$$

If $G_{12}$ is the mutual conductance between two slots, $J_o$ is Bessel function of first kind then

$$G_{12}=\int_0^{\pi}\left[\frac{\sin\left\{\frac{k_0 W\cos\theta}{2}\right\}}{120\pi^2\cos\theta}\right]^2 J_0\left(k_0 L\sin\theta\right)\sin^3\theta\, d\theta \tag{8}$$

So, the total input impedance is given by eq (9).

$$Z_{in} = \frac{1}{2(G \pm G_{12})} \tag{9}$$

So using formula given in eq (9) input impedance for microstrip patch antenna can be accurately calculated. This is more reliable method to calculate the input impedance of a rectangular patch antenna.

### *3.1.2 For probe fed patch antenna*

Transmission line equivalent circuit of probe fed patch antenna is shown in Figure 8. The microstrip antenna can be modeled[4–6] as a length of transmission lineof charactersitc impedance $Z_0$ and propagation constant $\gamma = \alpha + j\beta$. Where α is attenuation constant and β phase constant. The input impedance of the patch based on this model can be obtained as:

$$Z_{in} = jX_L + Z_1 \tag{10}$$

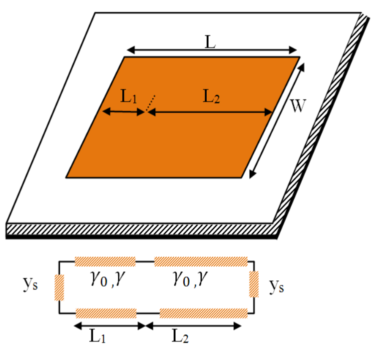


**Figure 8.** Transmission line model for probe feed rectangular patch antenna [7].

Where $X_L$ is the probe reactance and given by eq (11).

$$X_L(\omega L_L) = \eta_0 \mu_r \frac{h}{\lambda_0}\left[-\gamma + \ln\left(\frac{2}{\sqrt{\mu_r \varepsilon_r}\, k_0 a}\right)\right] \tag{11}$$

$$Z_1 = 1/\,Y_1$$

$$Y_1 = Y_0\left[\frac{Y_0 + jY_s \tan(\beta L_1)}{Y_s + jY_0 \tan(\beta L_1)} + \frac{Y_0 + jY_s \tan(\beta L_2)}{Y_s + jY_0 \tan(\beta L_2)}\right] \tag{12}$$

Where $Y_0 = 1/Z_0$, $Y_s$ = self admittance, β= phase constant, Euler constant γ = 0.5772, $\eta_0$ = free space impedance =377.

## 3.2 Calculation of Input Impedance by Cavity Model

To calculate the input impedance at the edge of patch using cavity model, the interior region of the patch antenna is modeled as a cavity bounded by electric walls on the top and bottom, and a magnetic wall along the periphery[4]. Input impedance in this model is calculated as:

$$Z_{in} = \frac{V_{in}}{I_0} \tag{13}$$

where $V_{in}$ is the RF voltage at the feed point

$$V_{in} = -E_z(x_0, y_0)\,h \tag{14}$$

Now to calculate the electric field at center of probe $(x_0, y_0)$the following computations should be done[9–11]. The electric field in the patch cavity can be expressed in terms of various modes of the cavity as:

$$E_z(x,y) = \sum_m \sum_n A_{mn}\varphi_{mn}(x,y) \tag{15}$$

Where $\varphi_{mn}$ is the an electric field mode vector or ortho-normalized eigenfunctions which must satisfy the homogenous wave equation boundary conditions and is given by:

$$\varphi_{mn}(x,y) = \sqrt{\frac{\varepsilon_m \varepsilon_n}{LW}} \cos(k_m x)\cos(k_n y) \tag{16}$$

Where $k_m = \frac{m\pi}{L}, k_n = \frac{n\pi}{W}$

$$k_{mn}^2 = k_m^2 + k_n^2 \text{ where } m,n = 0,1,2,p,\ldots$$

$\varepsilon_p$ = 1 for $p = 0$ and $\varepsilon_p$ = 0 for $p \neq 0$. $A_{mn}$ are the amplitude coefficients corresponding to the electric field mode vectors and eigenfunctions $\varphi_{mn}$

$$A_{mn} = \frac{j\omega\mu_0 J_0}{k^2 k_{mn}^2}\sqrt{\frac{\varepsilon_m \varepsilon_n}{LW}} \cos(k_m x_0)\cos(k_n y_0)\, G_{mn} \tag{17}$$

$$\text{where } G_{mn} = sinc\left(\frac{n\pi D_x}{2L}\right) sinc\left(\frac{m\pi D_y}{2W}\right) \tag{18}$$

$D_x$, $D_y$ equal to the cross-sectional area of the probe centered at $(x_0, y_0)$. For a microstrip line feed connected along the width of the patch, we should set $D_x=0$ and $D_y$

equal to the effective width of the feed line. On solving eq. (15) by substituting the value of $A_{mn}$, gives:

$$E_z(x,y) = j\omega\mu_0 J_0 \sum_m^\infty \sum_{n=0}^\infty \frac{\varphi_{mn}(x,y)\varphi_{mn}(x_0,y_0)}{k^2-k_{mn}^2} G_{mn} \tag{19}$$

By calculating the value $E_z(x_0, y_0)$(from eq.19) by putting $x_0$, $y_0$ and solving eq (14)

$$V_{in} = -j\omega\mu_0 h J_0 \sum_m^\infty \sum_{n=0}^\infty \frac{\varphi^2{}_{mn}(x_0,y_0)}{k^2-k_{mn}^2} G_{mn} \tag{20}$$

Therefore, the input impedance becomes (eq.13):

$$Z_{in} = -j\omega\mu_0 h \sum_m^\infty \sum_{n=0}^\infty \frac{\varphi^2{}_{mn}(x_0,y_0)}{k^2-k_{mn}^2} G_{mn} \tag{21}$$

Where value of $\varphi_{mn}(x_0,y_0)$ ,$k^2$ is given in eq (22) and (23):

$$\varphi_{mn}(x_0,y_0) = \sqrt{\frac{\varepsilon_m\varepsilon_n}{LW}}\cos(k_m x_0)\cos(k_n y_0) \tag{22}$$

$$k^2 = k_0^2\varepsilon_r\left(1-j\delta_{eff}\right) \tag{23}$$

$$\delta_{eff} = tan\delta + \frac{\Delta}{h} + \frac{P_r}{\omega W_T} \tag{24}$$

$$Z_{in} = -j\omega\mu_0 h \sum_m^\infty \sum_{n=0}^\infty \frac{\varphi^2{}_{mn}(x_0,y_0)}{k_0^2\varepsilon_r\left(1-j\delta_{eff}\right)-k_{mn}^2} G_{mn} \tag{25}$$

On solution of the eq (25) gives the input impedance of rectangular antenna. This is investigative approach and it is quite complicated to implement as a lot of complex calculation are needed to be performed.

## 3.3 Radiation Resistance Calculation Method

If a patch is fed at a distance $x_f$ from one of the radiating edges, then the input impedance can be calculated[15–17] by eq (26):

$$R_{in} = R_r\cos^2\left(\frac{\pi x_f}{L}\right) \tag{26}$$

Radiation resistance $R_r$ decreases with the increase in substrate thickness and patch width because of the increase in radiated power. Approximate formula for radiation resistance is given by eq. (27).

$$R_r = \frac{V_0^2}{2P_r} = \varepsilon_{re}\frac{Z_0^2}{120I_2} \tag{27}$$

Where $Z_0$ = characteristic impedance, $V_0$= applied voltage, $\varepsilon_{re}$ = effective dielectric constant, $P_r$ is power radiated by the antenna and can be calculated by integrating the real part of pointing vector over the hemisphere above the patch. The power radiated by antenna can be approximated by eq (28).

$$P_r = \frac{(E_0 h)^2 A\pi^4}{23040}\left[(1-B)\left(1-\frac{A}{15}+\frac{A^2}{420}\right)+\frac{B^2}{5}\left(2-\frac{A}{7}+\frac{A^2}{189}\right)\right] \tag{28}$$

Radiation resistance $R_r$ also calculated by substituting the value of characteristic impedance and approximate value of $I_2$ value in eq. (27). For $\varepsilon_r \le 5$ $I_2$ is given by eq (29)

$$I_2 = (k_0 h)^2\left[0.53-0.03795\left(\frac{k_0 W}{2}\right)^2\right]-\frac{0.03553}{\varepsilon_{re}} \tag{29}$$

If $5<\varepsilon_r\le 10$ than $I_2 = I_L / I_1$

$$I_1 = (\varepsilon_r-1)/9\left(1.29-3.57h\sqrt{\frac{\varepsilon_r}{\lambda_0}}\right) \tag{30}$$

$$I_L = (k_0 h)^2\left[1.3-\frac{4}{3\varepsilon_r}+\frac{0.53}{\varepsilon_r^2}\right]-\left(\frac{k_0 W}{2}\right)^2\left[0.08856-\frac{0.08856}{\varepsilon_r}+\frac{0.03795}{\varepsilon_r^2}\right] - \left[\frac{0.248714}{\varepsilon_r}-\frac{0.373071}{\varepsilon_r^2}+\frac{0.159887}{\varepsilon_r^3}\right] \tag{31}$$

After selecting the patch dimension $L$, $W$ for a given substrate the next point is to calculate the 50 Ω feed locations ($x_0$, $y_0$). It is observed that with the change in feed location the input impedance of the patch changes hence it provides a simple method for impedance matching. So by calculating the Radiation resistance and radiated power; input impedance can be calculated.

## 3.4 Quality Factor Calculation Method

The input impedance of the patch at resonance frequency $f_0$ depends upon edge resistance which is further a function of quality factor, length, width and operating frequency of the patch[4–6].An approximate expression for $R$ is given in eq. (32) which is simply computed by calculating the value of total quality factor.

$$R = R_{edge}\cos^2\left(\frac{\pi x_0^e}{L_e}\right) \tag{32}$$

Where the input resistance $R_{edge}$ when fed at the edge ($x_0$=0) is:

$$R_{edge} = \eta_0 \mu_r \left[ \frac{4L_e hQ}{\pi W_e \lambda_0} \right] \quad (33)$$

The effective feed locations are $x_0^e = x_0 + \Delta L$, $y_0^e = y_0 + \Delta W$ accounting for fringing field. Where $\eta_0$ = free space impedance =377, $\mu_r$ = permeability constant, Q= total quality factor, $L_e$ and $W_e$ are effective length and width of the patch.

Total quality factor is given by eq (34):

$$\frac{1}{Q} = \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}} + \frac{1}{Q_d} + \frac{1}{Q_c} \quad (34)$$

$Q_{sp}$, $Q_{sw}$, $Q_d$, and $Q_c$ denote the space-wave, surface-wave, dielectric, conductor quality factors. A microstrip antenna has dielectric and conductor and surface-wave loss. The surface-wave loss depends on the environment surrounding the patch. If there is a substrate that surrounds the patch and the surface-wave power launched by the antenna is gradually dissipated by an absorber, then the power launched into the surface wave by the patch is a loss. Now mathematical expression to calculate total quality factor is calculated as below. $Q_{sp}$ accounts for the desired radiation into the space given by eq (35):

$$Q_{sp} = \frac{3\varepsilon_r L_e \lambda_0}{16 p_r c_1 W_e h} \quad (35)$$

Where effective length of patch antenna $L_e = L + 2\Delta L$, effective width of antenna $W_e = W + 2\Delta W$, fringing width $\Delta W = h\left(\frac{\ln 4}{\pi}\right)$ and the terms $p_r$ and $c_1$ are geometry terms constant. Substrate absorb the surface wave so surface wave power $Q_{sp}$ is a loss from the antenna radiation point of view and given in eq. (36).

$$Q_{sw} = Q_{sp}\left(\frac{e_r^{sw}}{1 - e_r^{sw}}\right) \quad (36)$$

$e_r^{sw}$ is the radiation efficiency of the patch when accounting only surface loss and given in eq (37).

$$e_r^{sw} = \left(\frac{P_{sp}}{P_{sp} + P_{sw}}\right) \quad (37)$$

$P_{sp}$ is the power radiated into the space and $P_{sw}$ power launched into the surface wave. Dielectric quality factor is simply given by $Q_d = 1/tan\delta$ where $tan\delta$ $\varepsilon''/\varepsilon'$ loss tangent of the substrate:

Conductor quality factor is given by eq. (38):

$$Q_c = \mu_r \frac{\eta_0 h k_0}{2R_s^{ave}} \quad (38)$$

Where $R_s^{ave}$ denotes the average of ground plane and patch metal surface resistances $R_{sg}$ and $R_{sp}$. The surface resistance is related to the conductivity of the metal and the skin depth $\delta$ by the eq (40):

$$R_s = \frac{1}{\sigma\delta}, \delta = \sqrt{\frac{2}{\omega\sigma\mu_0}} \quad (39)$$

This section covers almost all the method by which input impedance of rectangular patch antenna can be calculated. From the above calculation it is clearly observed that impedance is not 50Ω at the edge of antenna. So, it is essential to implement some impedance matching techniques so that antenna can be properly matched with feed line impedance. The next section gives the complete design sketch of different matching techniques.

## 4. Impedance Matching Techniques

As explained earlier in Section-2 the impedance matching techniques can be broadly classified in to two categories: distributed method and lumped element method. In this section the detail of each method is extended for broader understating.

### 4.1 Distributed Impedance Matching Method

In this method antenna impedance is matched by doing some structural modifications through the use of stubs, double stubs, open or short circuit stubs, quarter wave transformer, tapered lines etc. Single section quarter wave transformer and single stub is used to match the antenna impedance with feed line at a single frequency (narrow band matching), but in many application there is a immense need to match the antenna over a large bandwidth (broadband matching)[18–19] for which multi section transformer and taper lines are used.

In this section complete design rules and design equation for following matching techniques are explained in detail.

1. Impedance matching through Quarter-Wavelength transformer consists of
   a) Narrow band matching through single transformer
   b) Broadband matching through multisection transformer consisting of chebyshev type and binomial type depending upon the response in the pass band.
2. Broad band Impedance matching through tapered line uses the design equation of
   a) Exponential Taper
   b) Triangular Taper
   c) Klopfenstein Taper
3. Impedance matching through Stub
   a) Shunt Stub matching through open and short circuit stub
   b) Series Stub matching through open and short circuit stub
   c) Double Stub matching

### 4.1.1 Impedance Matching through Quarter-Wavelength Transformer

Impedance transformer allows perfect matching of two different in a system. If the load in the system is not match with the source, then due to reflection from load; standing wave pattern are generated and complete power is not transfer to the load instead it get stored. This stored power can damage and overheat the system when delivered back to the input source[18–20]. Simple impedance transformer is the quarter wavelength transformer which is suitable for matching two real impedances at a single frequency[18]. The quarter-wave transformer provides narrow-band impedance matching by giving zero reflection at the operating frequency as shown in Figure 9(a). However, broadband matching is strongly desired in many applications. This problem can be solved by multi-section matching transformer and Tapered lines. Multi-section matching transformer increases the impedance bandwidth with the increase number of sections shown in Figure 9(b).

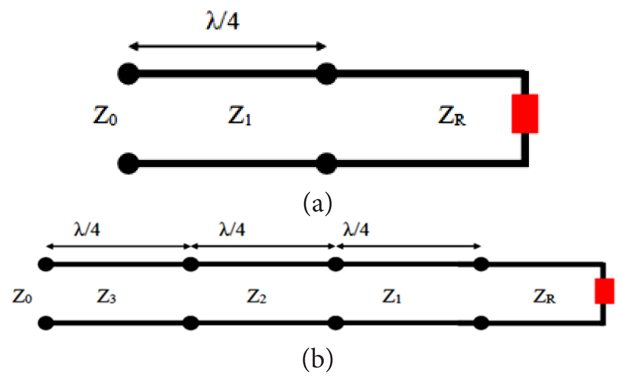


**Figure 9.** (a) Single section quarter wave transformer (b) Multisection quarter wave transformer

#### 4.1.1.1 Single Quarter Wave Transformer

The microstrip patch antenna can be matched to feed line ($Z_0$ Ω) by using a quarter-wavelength transmission line[18-19] ($Z_q$ Ω) as shown in Figure 10.

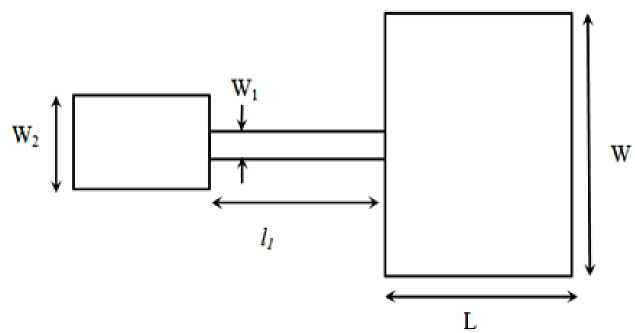


**Figure 10.** Quarter wave transformer.

The aim of adding Quarter wave transformer is to match the input impedance of antenna $Z_a$ exactly with impedance of the feed line ($Z_0$). The input impedance at the beginning of the quarter-wavelength lineis given by eq (40)

$$Z_{in} = Z_0 = \frac{Z_q^2}{Z_a} \tag{40}$$

By calculating impedance[30] of quarter wave transformer $Z_q$ such that $Z_{in}$=$Z_0$; input impedance $Z_{in}$ can be matched at a particular operating frequency. The impedance of quarter wave transformer $Z_q$ inversely proportional to $W_1$ width of strip. Input impedance of antenna is approximated by eq.(41 and 42)

$$Z_a = \frac{45\lambda_0^2}{W^2} \tag{41}$$

$$Z_a = \frac{90\varepsilon_r^2}{\varepsilon_r - 1}\left(\frac{L}{W}\right)^2 \tag{42}$$

$$Z_q = \sqrt{50 * Z_a} \tag{43}$$

Width of quarter wave transformer can be calculated by putting the value $Z_q$ in eq (44) and solving it for $W_1$

$$Z_q = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{8d}{w_1} + \frac{w_1}{4d}\right) \tag{44}$$

Length $L_1$ quarter wave transformer $L_1 = \frac{\lambda_g}{4\sqrt{\varepsilon_{re}}}$.

Width of 50Ωmicrostrip feed[4–6] can be found eq (45):

$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{eff}}\left(1.393 + \frac{W}{h} + \frac{2}{3}\ln\left(\frac{W}{h} + 1.444\right)\right)} \tag{45}$$

#### 4.1.1.2 Multisection Transformer

Multisection transformer method is used for designing Broadband matching networks. Multisection transformer uses more than one quarter wave transformer and depending upon the response in the operating region it can be divided into two types i.e. equiripple (chebyshev type) and maximally flat (binomial type).

*(i) Chebyshev Type Multisection Transformer*

A Chebyshev multi-section transformer offer larger bandwidths compared to binomial multi-section transformer for the same number of sections[31–32]. But the increment in bandwidth of the Chebyshev transformer is at the cost of larger ripple in the operating band.

Note that the bandwidth defined by $\Gamma_m$ increases as the number of sections N increases. The function $T_N(\cos\theta \sec\theta_m)$ is a Chebyshev polynomial of order N. We can determine higher order Chebyshev polynomials using the recursive formula:

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \tag{46}$$

Steps to design a Chebyshev multisection transformer:

1. Determine the value N required to meet the bandwidth and ripple $\Gamma_m$ requirements.
2. Determine the Chebyshev function.

$$\Gamma(\theta) = Ae^{-jN\theta} T_N(\cos\theta \sec\theta_m) \tag{47}$$

For maximally flat $\Gamma_m = A$

$$\Gamma_m = A = \frac{Z_L - Z_0}{Z_L + Z_0}\frac{1}{T_N \sec\theta_m} \tag{48}$$

$$\sec\theta_m = \cos h\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{2\Gamma_m}\left|\ln\frac{Z_L}{z_0}\right|\right)\right] \tag{49}$$

Also Chebyshev transformers are symmetric, i.e.

$$\Gamma_0 = \Gamma_n and \Gamma_1 = \Gamma_{n-1}$$

3. Determine all $\Gamma_n$ by equating terms with the symmetric multisection transformer expression given in eq (50)

$$\Gamma(\theta) = 2e^{-jN\theta}\left[\begin{array}{l}\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \\ \Gamma_n \cos(N-2n)\theta + G(\theta)\end{array}\right] \tag{50}$$

$$G(\theta) = \left\{\begin{array}{l}\frac{1}{2}\Gamma_{N/2} \text{for N even} \\ \Gamma_{(N-1)/2} \cos\theta \text{for N odd}\end{array}\right\} \tag{51}$$

4. Calculate all $Z_n$ using the approximation

$$\Gamma(\theta) = \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n} \tag{52}$$

5. Determine section length $l = \frac{\lambda_0}{4}$

*(ii) Binomial Type Multisection Transformer*

Reflection coefficient approximation for the $N$ section Binomial typematching transformer[31–32] is written according to binomial series as given in eq (53)

$$\Gamma(\beta l) = A\left(1 + e^{-2j\beta l}\right)^N = A\sum_{n=0}^{N} C_n^N e^{-j2n\beta l} \tag{53}$$

$A$ = amplitude coefficient and $C_n$ is the binomial coefficient given by $C_n^N = \frac{N!}{(N-n)!n!}$

The impedance of cascaded multiple section can be calculated using eq (54):

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \approx \frac{1}{2}\ln\frac{Z_{n+1}}{Z_n} \tag{54}$$

In practice, there is no need to design N section; two or three section transformer is sufficient. To save timein solving complex design equations, simple design equations are used and illustrated below.

*(a) Two Section Quarter-Wave Transformer*

If $Z_1$ source impedance is to be matched with $Z_2$ load impedance by two section quarter-wave transformer as shown by Figure 11 then characteristic impedance of two quarter-wave transformer $Z_A$,$Z_B$ are given by eq (55a and 55b).

$$Z_A = Z_1\left(\frac{Z_2}{Z_1}\right)^{1/4} \quad (55a)$$

$$Z_B = Z_1\left(\frac{Z_2}{Z_1}\right)^{3/4} \quad (55b)$$

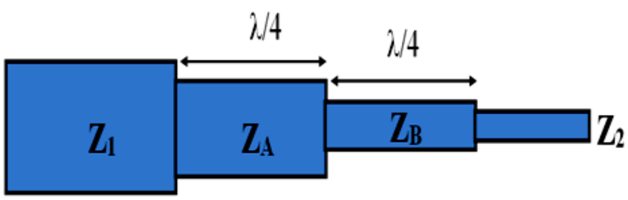


**Figure 11.** Two section quarter-wave transformer.

*(b) Three Section-Quarter-Wave Transformer*

If $Z_1$ source impedance is to be matched with $Z_2$ load impedance by three section quarter-wave transformer as shown in Figure 12 then characteristic impedance of three quarter-wave transformer $Z_A$, $Z_B$, $Z_C$ are given by eq (56a-56c):

$$Z_C = Z_2 e^{-2\Gamma_1} \quad (56a)$$

$$Z_B = Z_1\sqrt{\frac{Z_2}{Z_1}} \quad (56b)$$

$$Z_A = Z_1 e^{2\Gamma_1} \quad (56c)$$

$$\Gamma_1 = 0.125\left\{0.5\ln\left(\frac{Z_2}{Z_1}\right)\right\} \quad (56d)$$

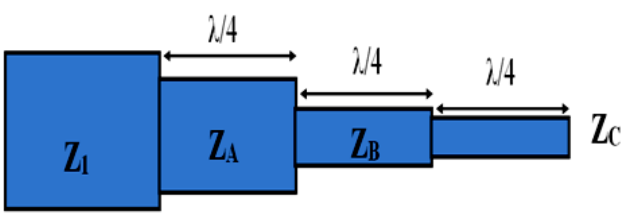


**Figure 12.** Three section quarter-wave transformer.

### 4.1.2 Broad Band Impedance Matching through Tapered Line

Instead of having an impedance matching network which have step change in characteristic impedance (i.e., a multi-section transformer), another matching structure can be implemented which has continuous varying impedance along its length (function of distance z). A tapered impedance broadband matching network depends upon length L of taper line and taper function $Z_1(z)$. Depending on the behavior of taper function $Z_1(z)$; taper line can be classified in three category: exponential taper, triangular taper and Klopfenstein taper[24–25] as shown in Figure 13.

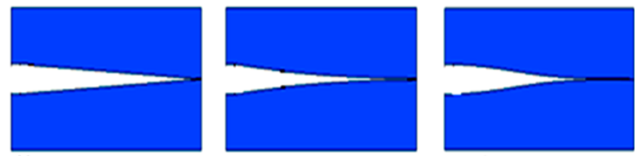

**Figure 13.** Exponential, Triangular and Klopfenstein taper line.

#### 4.1.2.1 Exponential Taper Transformer

In the exponential taper line, the natural logarithm of taper line's characteristic impedance varies linearly from $Z_L$ to $Z_0$. The exponential taper has the form given in eq (57):

$$Z_1(z) = Z_0 e^{az}\ 0 < z < l \quad (57)$$

where $a = \frac{1}{L}\ln\frac{Z_L}{Z_0}$

Reflection coefficients are given by eq (58):

$$\Gamma = \frac{\ln Z_L / Z_0}{2} e^{-i\beta l}\frac{\sin\beta L}{\beta L} \quad (58)$$

The bandwidth of a tapered line will typically increase as the length L is increased.

#### 4.1.2.2 Triangular Taper Transformer

Characteristic impedance of Triangular Taper lines varies from $Z_L$ to $Z_0$ according to the taper function as given by eq (59).

$$Z_{(z)} = \begin{cases} Z_0 e^{2\left(\frac{z}{L}\right)^2}\ln\left(\frac{Z_L}{Z_0}\right) & \text{for} 0 \le z \le L/2 \\ Z_0 e^{\left(4z/L - 2z^2/L^2 - 1\right)^2}\ln\left(\frac{Z_L}{Z_0}\right) & \text{for} 0 \le z \le L/2 \end{cases} \quad (59)$$

Reflection coefficient for the triangular taper are given by eq (60):

$$\Gamma = 0.5e^{-\beta L}\ln\left(\frac{Z_L}{Z_0}\right)\left[\frac{\sin\left(\beta L/2\right)}{\beta L/2}\right]^2 \quad (60)$$

### 4.1.2.3 Klopfenstein Taper Transformer

R.W. Klopfenstein presented equations which can be used to design transmission line taper which represents an improved alternative to the exponential taper. This structure can either achieve better match on the same length, or comparable match on the shorter length than the exponential taper[24–25]. Compared to the exponential taper, Klopfenstein design has one more degree of freedom in the taper definition, represented by the variable A in the relation $I_1$ is a modified Bessel function and $\Gamma_0$ is the maximum reflection coefficient at the zero frequency.

$$lnZ(z) = 0.5\ln Z_0 Z_L + \frac{\Gamma_0}{\cosh A}A^2\phi\left(2z/L - 1, A\right) \quad (61)$$

$$\phi(x, A) = \int_0^x \frac{I_1\left(A\sqrt{1-y^2}\right)}{\left(A\sqrt{1-y^2}\right)}dy \; for x < 1 \quad (62)$$

$$\Gamma = \Gamma_0 e^{-j\beta z}\frac{\cos\sqrt{(\beta L)^2 - A^2}}{\cosh A} \; for \; \beta L > A \quad (63)$$

Drawback of the Klopfenstein taper is that an impedance discontinuity or step occurs at the ends of the taper.

## 4.1.3 Impedance Matching Through Stubs

Impedance matching using stub is one of the most widely used method. In this technique, the stub is positioned at a specific distance (*'d'* from the load) where the real part of the normalized load impedance/ admittance becomes unity[20–23]. Then the stub of length *l* is connected at the point (*d* from load) such that it offers capacitive or inductive reactance/ susceptance which are same in magnitude but opposite in sign to that of load at same point. Thus, the reactive part of stub impedance and load impedance cancels to provide impedance matching. Figure 14 shows a microstrip patch antenna matched by single stub and double stub on the sides of patch.

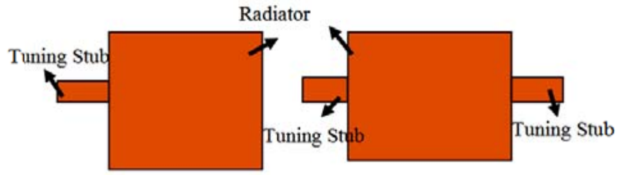


**Figure 14.** Single stub and double stub matching of microstrip patch antenna.

A single stub will only achieve a perfect match at one specific frequency because as the frequency changes, the wavelength changes which corresponds to change in reactance at the point of attachment of the stub[26–28]. For wideband matching, several stubs may be used, spaced along the main transmission line. The resulting structure is filter-like and filter design techniques are applied. Impedance matching technique may be simplified by using the SMITH chart for calculations and design.

A brief overview to design of short or open stub in series and shunt connection is given as below:

### 4.1.3.1 Shunt Stub

In this method, a open or short circuit stub[20–23] is attached at a distance d from the load as shown in Figure 15 and given in eq (41) so that total stub input admittance $j\omega C$ and $j\omega L$ cancel the imaginary part of load admittance. Shunt stubs are primarily preferred for microstrip and strip line types of transmission lines. As the stub here is connected in shunt to main line, therefore, the calculations are preferably done in admittance.

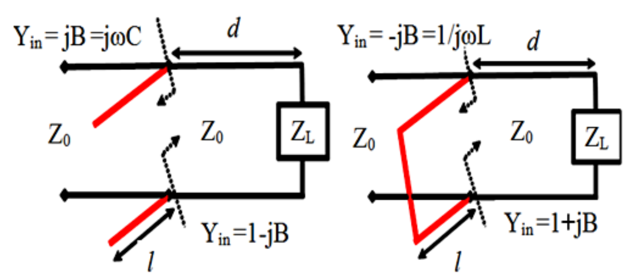


**Figure 15.** Shunt stub matching using open and short circuit stubs.

When the load impedance $Z_L = R_L + jX_L$ is connected by a shut stub then the admittance at this point is $Y=G+jB$. Length for open and short circuited shut stubs is given by eq (64) where B is stub susceptance.

$$\frac{1}{\lambda} = \begin{cases} -\frac{1}{2\pi}\tan^{-1}\left(\frac{B}{Y_0}\right) & \textit{for open stub} \\ \frac{1}{2\pi}\tan^{-1}\left(\frac{Y_0}{B}\right) & \textit{for short stub} \end{cases} \quad (64)$$

Distance of stub from the load is given by eq (65):

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi}\tan^{-1} t & \text{for} t \geq 0 \\ \frac{1}{2\pi}\left(\pi + \tan^{-1} t\right) & \text{for} t \leq 0 \end{cases} \quad (65)$$

Where t is given by eq (66):

$$t = \begin{cases} \dfrac{X_L \pm \sqrt{\dfrac{R_L\left[\left(Z_0 - R_L\right)^2 + X_L^2\right]}{Z_0}}}{\left(R_L - Z_0\right)} & \text{for } R_L \neq Z_0 \\ \dfrac{-X_L}{2Z_0} & \text{for } R_L = Z_0 \end{cases} \quad (66)$$

#### 4.1.3.2 Series Stub

In this method, a open or short circuit stub is attached at a distance d from the load as shown in Figure 16. So that total stub input impedance $jX=1/j\omega C$ or $j\omega L$ cancel the imaginary part of load impedance. Series stubs are primarily preferred for slotline and coplanar waveguide types of transmission lines[21]. As the stub is connected in series to main line, therefore, the calculations are preferably done using impedance. When the load impedance $Y_L= G_L+jB_L$ is connected by a series stub of length $l$ down a distance $d$ then the at this point is impedance is $Z=R+ jX$. Length for open and short circuited series stubs is given by eq (67) where X is stub reactance.

$$\frac{l}{\lambda} = \begin{cases} -\frac{1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right) & \text{for short stub} \\ \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X}\right) & \text{for open stub} \end{cases} \quad (67)$$

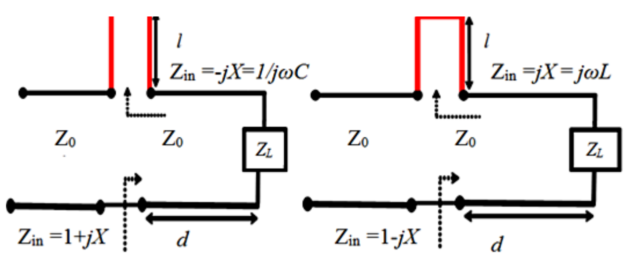


**Figure 16.** Series stub matching using open and short circuit stubs.

Distance of stub from the load is given by eq (68):

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi}\tan^{-1} t & \text{for } t \geq 0 \\ \frac{1}{2\pi}\left(\pi + \tan^{-1} t\right) & \text{for } t \leq 0 \end{cases} \quad (68)$$

Where t is given by eq (69):

$$t = \begin{cases} \dfrac{B_L \pm \sqrt{\dfrac{G_L\left[\left(Y_0 - G_L\right)^2 + B_L^2\right]}{Y_0}}}{\left(G_L - Y_0\right)} & for G_L \neq Y_0 \\ \dfrac{-B_L}{2Y_0} \; for G_L = Y_0 \end{cases} \quad (69)$$

#### 4.1.3.3 Double Stub Matching

The single stub tuner is very flexible at matching any load impedance to a given transmission line. However, if the load impedance varies, an adjustable tuner is necessary. For the single stub tuner, the position of the stub must be varies, to match the variable load impedance. However, double stub tuner allows an adjustable matching at a fixed position by varying the length of stubs. Thus, matching over a wide range of load impedance and frequencies can be achieved at the cost of increased circuit size.

### 4.2 Impedance Matching by Lumped Element Method

In this approach instead of modifying the antenna geometry a passive network attempts to equalize the impedance mismatch between the source and the antenna[33–34]. Lumped elements like Capacitor: chip capacitor, MIM capacitor, inter digital gap capacitor; Inductor: chip inductor, loop inductor, spiral inductor; Resistor: chip resistor, planar resistor are used to match the antenna impedance with the feed. For frequencies near to 1 GHz, matching networks through lumped elements can be done easily because the size of lumped element is small enough relative to wavelength of operation. Smith chart is the best tool to analyze the L-networks.

Here two cases to determine the value of lumped element are given. Circuit arrangement of lumped element for both case $R_L > Z_o$ and $R_L < Z_o$ are shown in Figure 17. Also the necessary design equations to calculate value of susceptance B and reactance X are given[35–37].

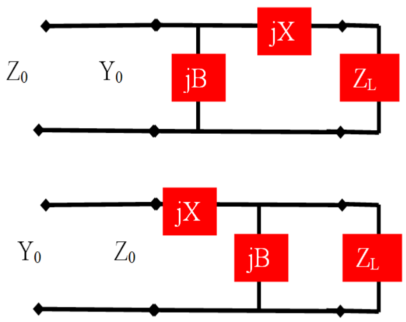


**Figure 17.** Lumped Element Matching Network for (a) $R_L > Z_o$ (b)$R_L < Z_o$.

### 4.2.1 *Lumped Element Matching Network for $R_L > Z_o$*

Any combination of capacitors ($X < 0$, $B > 0$) or inductors ($X > 0$, $B < 0$) is used to realize the reactance $jX$ and susceptance $jB$. For a matched network, the input impedance $Z_{in}$ must be equal to $Z_o$ which gives

$$Z_{in} = Z_0 = jX + \left(jB + \frac{1}{Z_L}\right)^{-1} \tag{70}$$

Now by equating the real and imaginary terms on both sides of the eq (70), unknowns $X$ and $B$ can be evaluated. Hence lumped circuit can be designed by inserting the calculated value of X and B.

### 4.2.2 *Lumped Element Matching Network for $R_L < Z_o$*

For a matched network, the input admittance $Y_{in}$ must be equal to $1/Z_o$ which gives:

$$Y_{in} = 1/Z_0 = jB + \frac{1}{R_L + j(X + jX_L)} \tag{71}$$

Now by equating the real and imaginary terms on both sides of the eq (71), unknowns $X$ and $B$ can be evaluated. Hence lumped circuit can be designed by inserting the calculated value of X and B.

The above techniques can also be applied on triangular and circular shaped patch antenna, by calculating the input impedance of these shapes. In the next section; different design equation for triangular and circular microstrip patch antenna is given and various formulae to calculate the input impedance is also discussed.

## 5. Input Impedance Calculation of Triangular Microstrip Patch Antenna

The triangular geometry of the microstrip patch antenna appears to be a better option than its rectangular counterpart as it is physically smaller and consequently the weight and volume of antenna structure are reduced. Interestingly, triangular patch is typically a narrow impedance bandwidth structure which may limit its operations, yet it may be used profitably in many applications such as designing microstrip band pass filters, for use in compact arrays with reduced coupling between adjacent elements and for being used on curved surfaces because of its conformability.

For equilateral triangular patch antenna shown in Figure 18 resonant frequency is given by[38–39] eq (72).

$$f_{r,nm} = \frac{2c}{3a_{eff}\sqrt{\varepsilon_{reff}}}\sqrt{n^2 + m^2 + nm} \tag{72}$$

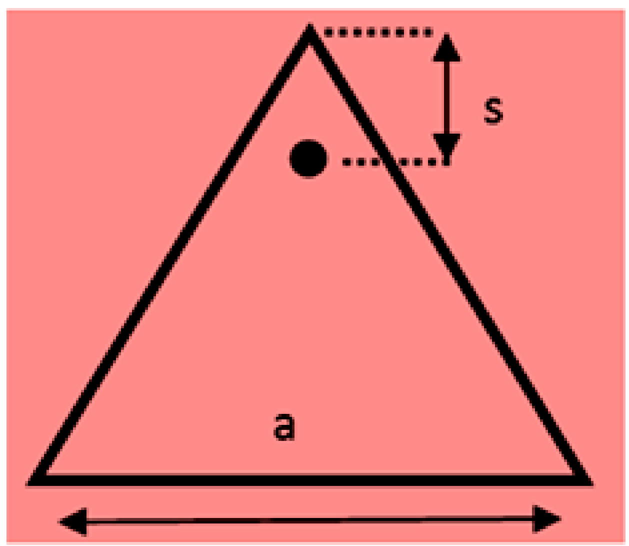


**Figure 18.** Triangular microstrip patch antenna.

c = velocity of light in free space; $a_{eff}$ = effective length of triangular side; $\varepsilon_{reff}$ = effective relative permittivity; m, n, and l are integers which should fulfill this condition *m+n+l=0*

$$a_{eff} = a(1 + p) \tag{73}$$

Due tofringing fields at the edge of the patch $p$ is equal to:

$$p=\left[\frac{h}{a}\left\{\begin{array}{c}0.3849\frac{a}{h}+0.5879+0.1093\left(\frac{\varepsilon_{r,eff}-1}{\varepsilon_{reff}^{2}}\right)+\\ \left(\left(\frac{\varepsilon_{r,eff}+1}{2\pi\varepsilon_{r,eff}}\right)\left(1.9346+1.33\ln\left(0.2887\frac{a}{h}+0.94\right)\right)\right)\end{array}\right\}-0.3849\right]\left(\frac{\varepsilon_{reff}+0.3}{\varepsilon_{reff}-0.258}\right) \quad (74)$$

Accurate calculation of input impedance of the patch antenna is necessary for achieving the optimum performance. Input impedance from cavity model of a coaxial fed triangular patch antenna with side length *a is* given by eq (75).

$$Z_{in}=R+jX=\frac{R_r}{1+Q_T^2\left(\frac{f_{r,nm}}{f}-\frac{f}{f_{r,nm}}\right)^2}+j\left[\frac{R_rQ_T\left(\frac{f_{r,nm}}{f}-\frac{f}{f_{r,nm}}\right)}{1+Q_T^2\left(\frac{f_{r,m}}{f}-\frac{f}{f_{r,nm}}\right)^2}\right] \quad (75)$$

Where radiation resistance $R_r$ when the feed is located at a distance $\rho$ from the edge of the triangle given in eq (76).

$$R_r=\frac{2\eta a\varepsilon_{reff}P_{lnm}}{(1+\pi)a_{eff}\varepsilon_{re}} \quad (76)$$

Where the term $f_{rnm}$ $a_e, \varepsilon_{reff}$ are the resonant frequency effective radius and effective dielectric constant. The field factor $P_{lmn}$ written by eq (77).

$$P_{lmn}=\left[\cos\left(\frac{2\pi l\rho}{\sqrt{3}a}\right)j_0\left(\frac{2\pi lg}{\sqrt{3}a}\right)+\cos\left(\frac{2\pi n\rho}{\sqrt{3}a}\right)j_0\left(\frac{2\pi ng}{\sqrt{3}a}\right)+\cos\left(\frac{2\pi m\rho}{\sqrt{3}a}\right)j_0\left(\frac{2\pi mg}{\sqrt{3}a}\right)\right]^2 \quad (77)$$

$Q_T$ is the total quality factor depends upon quality factor due to radiation loss ($Q_r$), quality factor due to dielectric loss ($Q_d$) and quality factor due to conductor loss ($Q_c$) and is given in eq (78).

$$Q_T=\left[\frac{1}{Q_r}+\frac{1}{Q_d}+\frac{1}{Q_c}\right]^{-1} \quad (78)$$

The quantity $Q_r$ can be computed as $Q_r=\frac{\pi}{4G_rZ_r}$ where radiation conductance ($G_r$) and characteristics impedance ($Z_r$) is given by in eq (79) and (80) respectively.

$$G_r=\frac{a}{\sqrt{3}}\frac{7.75+2.2hk_0+4.8(hk_0)^2}{3000\lambda_0}\left[1+\frac{(\varepsilon_r-2.45)(hk_0)^3}{1.3}\right] \quad (79)$$

$$Z_r=\frac{2\eta_0}{\sqrt{\varepsilon_{reff}}\left[a/h+2.494+0.556\ln\left(a/h+1.333\right)\right]} \quad (80)$$

The quality factor due to dielectric loss $Q_d$ is given eq (81).

$$Q_d=\frac{1}{tan\delta} \quad (81)$$

The quantity $Q_c$ is given by eq (82).

$$Q_c=h\sqrt{\pi\sigma f_{r,nm}\mu_0} \quad (82)$$

## 6. Input Impedance Calculation of Circular Microstrip Patch Antenna

The resonant frequency ($f_{nm}$) of a circular patch antenna[40–42] shown in Figure 19 having radius $a$ and printed on a substrate with relative permittivity $\varepsilon_r$ substrate thickness $h$ for each TM mode is given by eq (83)[43].

$$f_{nm}=\frac{A_{mn}.c}{2\pi a_{eff}\sqrt{\varepsilon_r}} \quad (83)$$

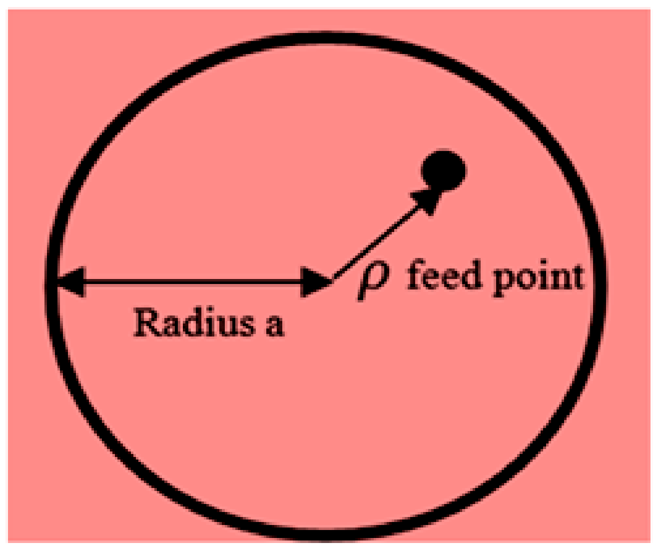


**Figure 19.** Geometry of Circular patch antenna.

where $A_{nm}$ = $m^{th}$ derivative of the n order Bessel function. $a_{eff}$ = effective radius of the patch and given by eq (84).

$$a_{eff}=a\left[1+\frac{2h}{\pi a\in_r}\left\{\text{In}\left(\frac{a}{2h}\right)+(1.41\varepsilon_r+1.77)+\frac{h}{a}(0.268\varepsilon_r+1.65)\right\}\right]^{1/2} \quad (84)$$

The input resistance at resonance [9] is given by eq (85).

$$R_{in}=R_r\frac{J_1^2(k_{11}\rho_0)}{J_1^2(k_{11}a)}\text{where}\,k_{11}a=1.84118 \quad (85)$$

The radiation resistance is given in eq (86).

$$R_r = {}^{1}\!/_{G_r} \tag{86}$$

Resonant radiation conductance $G_r$ is calculated by putting the value of radiated power from eq (88) in eq (87).

$$P_r = \frac{1}{2} G_r V_0^2 \tag{87}$$

$$P_r = \frac{(E_0 h)^2 \pi^3 a^2}{2\lambda_0^2 \eta_0}\left[\frac{4}{3} - \frac{8}{15}(k_0 a)^2 + \frac{11}{105}(k_0 a)^4 - \dots\right] \tag{88}$$

$$a = \frac{1.841}{k_0\sqrt{\varepsilon_r}} \tag{89}$$

To calculate the feed location at a 50 ohm point put the value of $R_r$ in eq (90).

$$\mathrm{J}_1(\mathrm{k}_{11}\rho_0) = \sqrt{\frac{\mathrm{R_{in}}}{\mathrm{R_r}}}\mathrm{J}_1(\mathrm{k}_{11}\mathrm{a}) = \sqrt{\frac{\mathrm{R_{in}}}{\mathrm{R_r}}}\mathrm{J}_1(1.84118) = 0.5819\sqrt{\frac{\mathrm{R_{in}}}{\mathrm{R_r}}} \tag{90}$$

After getting the value of input impedance of circular patch antenna from eq (90) and for triangular patch from eq (75), it is very simple job to match the input impedance of circular and triangular antenna with any matching technique described in section 4.

## 7. Conclusion

This paper covers the impedance matching methods including distributed as well as lumped for microstrip patch antenna along with their complete design equations. Narrow and broad band matching techniques through quarter wave transformer, taper lines, stubs and lumped elements etc. have discussed in detail. Different techniques opted by the researcher to compute the input impedance of rectangular, triangular and circular patch antenna have also been discussed.

## 8. References


1. Hector J, De Los, Santos. RF circuit design for wireless applications. Artech house: Boston London, 2002. PMCid:PMC125947.
2. Haykin S. Communication systems. 3rd Edn. John Wiley & Sons, 2005.
3. Pozar D.M. Microwave engineering. 3rd Edn. John Wiley & Sons, 2005. PMid:15998154.
4. Garg R, Bhartia P, Bahl I, Ittipiboon A. Microstrip antenna design handbook. Boston, USA: Artech House, 2001.
5. Kumar G, Ray K.P. Broadband microstrip antennas. Artech House, MA, 2003.
6. Pozar D.M, Schaubert D.H. Microstrip antenna. John Wiley & Sons, 1995. Crossref.
7. Sharma S, Tripathi C.C. A wide spectrum sensing and frequency reconfigurable antenna for cognitive radio, Progress in Electromagnetics Research C. 2016; 67:11–20. Crossref.
8. Sharma S, Tripathi C.C. An integrated frequency reconfigurable antenna for cognitive radio application, Radioengineering. 2017.
9. Sharma S, Tripathi C.C. A versatile reconfigurable antenna for cognitive radio, Asia Pacific Microwave Conference. 2016 Dec; 1–4. Crossref.
10. Sharma S, Tripathi C.C. Frequency reconfigurable U-slot antenna for SDR application, Progress in Electromagnetics Research Letters. 2015; 55:129–36. Crossref.
11. Sharma S, Tripathi C.C. Wideband to concurrent tri- band frequency reconfigurable microstrip patch antenna for wireless communication, International Journal of Microwave and Wireless Technologies. 2017 May; 9(4):915–22. Crossref.
12. Sharma S, Tripathi C.C. A novel reconfigurable antenna with separate sensing mechanism for CR system, Progress in Electromagnetics Research C. 2017; 72:187–96. Crossref.
13. William H. Engineering electromagnetics. 5th Edn. McGraw-Hill, 1989.
14. Edwards T.C, Steer M.B. Foundation of microstrip circuit design. 3rd Edn. John Wiley & Sons, 2000. Crossref.
15. Chen W.K. Theory and design of broadband matching networks. NY: Pergamon Press, Gilbert. 1976.
16. Matthaei G.L, Young L, Jones E.M.T. Microwave filters, impedance-matching networks and coupling structures. MA: Artech House, 1980.
17. Ying H, Jackson D.R, Williams J.T, Long S.A. A design approach for inset-fed rectangular microstrip antennas, IEEE Antennas and Propagation Society International Symposium. 2006 Jul; 1491–94. Crossref.
18. Khodier M, Dib N, Ababneh J. Design of multi-band multisection transmission line transformer using particle swarm optimization, Electrical Engineering. 2008 Apr; 90(4):293–300. Crossref.
19. Stiles J. The Chebyshev matching transformer. The University of 20. Kansas, Department of EECS. 2010, p. 1–16.
21. Plessis M.D, Cloete J. Tuning stubs for Microstrip-Patch antennas, IEEE Antennas and Propagation Magazine. 1994 Dec; 36(6):52–56. Crossref.
22. Araii H, Durnan G.J, Saitol S. A dual element patch array antenna structure with Microstrip Triple Stub matching,

IEEE Antennas and Propagation Society International Symposium. 2003; 4:528–31. Crossref.

23. Yanagi T, Nishioka Y, Ohtsuka M, Makino S. Basic study on wideband Microstrip patch antenna incorporating matching-stubs onto patch conductor, IEEE Antennas and Propagation Society International Symposium. 2007 Jun, p. 885–88.
24. Deshmukh A.A, Ray K.P, Chine P.N. Multi-band stub loaded Square ring Microstrip antennas, Applied Electromagnetics Conference (AEMC). 2009 Dec, p. 1–4. Crossref.
25. Esa M, Malik N.N.N.A, Latif N.A, Marimuthu J. performance investigation of Microstrip exponential tapered line impedance transformer using math CAD, Progress in Electromagnetics Research Symposium Proceedings. 2009 Aug; p. 1209–13. PMid:19786392.
26. Panda J.R, Kshetrimayum R.S. Notched antenna with triangular tapered feed lines for tri-band operation, International Journal of Recent Trends in Engineering. 2009 May; 1(3):277–79.
27. Pozar D.M. Input impedance and mutual coupling of rectangular Microstrip antennas, IEEE Transactions on Antennas and Propagation. 1982 Nov; 30(6):1191–96. Crossref.
28. Samaras T, Kouloglou A, Sahalos J.N. A note on the impedance variation with feed position of a rectangular Microstrip-patch antenna, IEEE Antennas and Propagation Magazine. 2004 Apr; 46(2):90–92. Crossref.
29. Hu Y, Lundgren E, Jackson D.R, Williams J.T, Long S.A. A study of the input impedance of the inset-fed rectangular Microstrip antenna as a function of notch depth and width, IEEE Antennas and Propagation Society International Symposium. 2005 July; 4A:330–33. PMid:16248426.
30. Chatterjee D, Chettiar E. Analytical calculation of input impedance of rectangular Microstrip patch antennas on finite ground planes, IEEE International Conference on Wireless Communications and Applied Computational Electromagnetics. 2005; p. 960–63. Crossref.
31. Bahl I. Lumped elements for RF and Microwave circuits. Artech House, 2003.
32. Khare R, Nema R. Reflection coefficient analysis of Chebyshev impedance matching network using different algorithms, International Journal of Innovative Research in Science, Engineering and Technology. 2012 Dec; 1(2):214–18.
33. Pues H.F, Capelle V.D. An impedance matching technique for increasing the bandwidth of Microstrip antennas, IEEE Transactions on Antennas and Propagation. 1989 Nov; 37(11):1345–54. Crossref.
34. Iobst K.W, Zaki K.A. An optimization technique for lumped-distributed two ports, IEEE Transactions on Microwave Theory and Techniques. 1982 Dec; 30(12):2167–71. Crossref.
35. Glibert E. Impedance matching with lossy components, IEEE Transactions on Circuits and Systems. 1975 Feb; 22(2):96–100.
36. Abboud F, Damiano J.P, Papiernik A. A new model for calculating the input impedance of coax-fed circular Microstrip antennas with and without air gaps, IEEE Transactions on Antennas and Propagation. 1990 Nov; 38(11):1882–85. Crossref
37. Antoszkiewicz K, Shafai L. Impedance characteristics of circular Microstrip patches, IEEE Transaction on Antennas and Propagation. 1990 Jun; 38(6):942–46. Crossref.
38. Sussman-Fort S.E. Microwave matching network synthesis software and user's manual, Automated Synthesis of Low Pass, High-Pass, and Bandpass Lumped and Distributed Matching Networks. MA: Artech House, 1991.
39. Biswas M, Mandal A. CAD model to compute the input impedance of an equilateral triangular Microstrip patch antenna with Radome, Progress in Electromagnetics Research M. 2010; 12:247–57. Crossref.
40. Biswas M, Guha D. Input impedance and resonance characteristic of superstrate loaded triangular Microstrip patch, IET Microwaves, Antennas and Propagation. 2009 Feb; 3(1):92–98. Crossref.
41. Banik S, Biswas M. Improved CAD model to compute the input impedance of tunable circular patch antenna, Antenna Week (IAW). 2011 Dec; p. 1–3.
42. Guha D, Antar Y.M.M, Siddiqui J.Y, Biswas M. Resonant resistance of probe and Microstrip line-fed circular Microstrip patches, IEEE Proceedings- Microwaves, Antennas and Propagation. 2005 Dec;152 (6):481–84. Crossref.
43. Bhattacharyya A.K. Characteristics of circular patch on thick substrate and superstrate, IEEE Transaction on Antennas and Propagation. 1991 Jul; 39(7):1038–41. Crossref.