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Establishment of an EOQ with Non-Increasing Demand for Two Credit Periods under Deterioration and Time Discounting

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Abstract

Objective: In this paper, we consider time dependent demand under constant deterioration. The main objective of this paper is to obtain optimal cycle time which minimizes the total relevant cost. **Methods/Statistical Analysis:** We know that every business refers to the input- out relation. If the firm expands output by employing more and more the variable input, it alters the proportion between fixed and variable inputs. In this study, the truncated Taylor's series approximations are used for exponential terms to find closed form numerical solution. The optimal value of cycle time is obtained by differentiating cost function. **Findings**: We compared all results with the optimal solution we also show that the total relevant cost function in minimum. Mathematical models are established to validate the proposed model considering four different situations i.e. case (1) T ≥ M1 Case (2) T < M1. Case (3) T ≥ M2 and case (4) T < M2. **Application/Improvements:** Capital is the most essential element of production process. At present inflation plays a crucial role in each type of business transaction. Numerical examples are given for four different situations, those show that the validity of the model. Sensitive analysis is included for different parameters. Cost is a factor which is directly related to production cost analysis helps in achieving high quality production at low cost.

Keywords: Credit Period, Deterioration, EOQ Model, Optimal, Time Discounting, Time Sensitive Demand

1. Introduction

The one of the most important areas in business is management of inventories. EOQ (Economic Order Quantity) model considers that the demand of commodities is constant. However, the consideration is not true in general and more realistic to consider that demand depends on time, stock etc.

Several EOQ models have been published for spoiling items.[1] It was Proposed an EOQ model for damageable commodities. Established a model for perishable product for Weibull deterioration rate.[2,3] Presented inventory models with time dependent demand. Designed the impact of trade credit in inventory systems.[4] Described an EOQ model of deteriorating items under the condition of trade credits.[5] In the presented article pointed out

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EOQ models for the retailer -buyer supply chain.[6] Shah established a stock inventory model for damazable items & trade credits.[7] In the article presented an EOQ model for stock sensitive demand.[8] In the article analyzed EOQ models for a seller by incorporating with the different situation.[9] In the article presented a model for deteriorating commodities.[10] In the article developed a review of deteriorating inventory on trends in modeling.[11] In the presented article published a model for stock –sensitive demand.[12] Several works can be seen In the article this area and their citations.[13-20]

All the above inventory models have not taken in to consideration of the effect of the cash discount. In this direction developed inventory models by considering the cash discount to fit today's business.[21-24]

2. Notation and Assumptions

2.1 Notations

p : Unit purchasing cost

λ : Rate of deterioration, $0 < \lambda < 1$

I_d : Interest earned /$/ year

I_c : Interest charged/$ in stock/year, $I_c > I_d$

s : ordering cost/ order

Q : Initial demand

$D(t) = a - b.t$: rate of demand,

r : Rate of cash discount,

h : Unit holding cost / year

$M_1 \& M_2$: period of cash discount & trade credits

T : Cycle time

$I(t)$: Inventory level at any instant t, $0 \le t \le T$

$T_i, i = 1,2,3,4$: optimal cycle time for cases (1), (2), (3) and (4) respectively

$Z_i(T)$: Total relevant costs/year for all four cases

$Q^*(T_i)$: Optimal order quantity for all four cases.

2.2 Assumptions

i. Demand rate is non- increasing function of time.

ii. Lead time is negligible

iii. Shortages are not considered.

iv. The account is settled at the end of time M_1 or, $M_2 \& M_1 > M_2$.

v. Cash discount rate is considered to be constant and lies between 0 &1.

3. Mathematical Model

From the above consideration the state of inventory at any instant of time are:

$$\frac{dI(t)}{dt} + \lambda I(t) = -D(t) \tag{1}$$

The solution of (1) using $I(T) = 0$ is

$$I(t) = \left(\frac{b + a\lambda}{\lambda^2}\right)\left(e^{\lambda(T-t)} - 1\right) - \frac{b}{\lambda}\left\{Te^{\lambda(T-t)} - t\right\} \tag{2}$$

Order quantity

$$Q = \left(\frac{b + a\lambda}{\lambda^2}\right)\left(e^{\lambda T} - 1\right) - \frac{\mathrm{bT}e^{\lambda T}}{\lambda} \tag{3}$$

The total cost is obtained by the following components:

(a) Cost of placing order (A) $= \dfrac{s}{T}$ (4)

(c) Holding cost (HC) $= \dfrac{h}{T}\left\{\left(\dfrac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda T} - 1\right) - \left(\dfrac{b + a\lambda}{\lambda^2}\right)T + \dfrac{bT^2}{2\lambda}\right\}$ (5)

Case (1). $T \geq M_1$: In this case, the customer saves $rpQ/$ cycle due to price discount. Now

Discount / year $(DC) = = \dfrac{rp}{T}\left\{\left(\dfrac{b + a\lambda}{\lambda^2}\right)\left(e^{\lambda T} - 1\right) - \dfrac{bTe^{\lambda T}}{\lambda}\right\}$ (6)

The commodities in stock have to finance after M_1. Therefore, Interest payable/year

$$(IE_1) = \frac{pI_c}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_1)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_1)+\frac{b}{2\lambda}\left(T^2-M_1^{\ 2}\right)\right\} \tag{7}$$

Also during $[0, M_1]$ the retailer sells the product & deposits revenue that earns interest I_d /\$/year. Thus,

Interest earned/ year (IE_1)

$$= \frac{pI_d}{T}\int_0^{M_1}(a-bt)\,tdt = \frac{pI_d M_1^{\ 2}}{T}\left(\frac{a}{2}-\frac{bM_1}{3}\right) \tag{8}$$

Total relevant cost is

$$Z_1(T) = A + HC + IP_1 - IE_1 - CD$$

$$= \frac{s}{T}+\frac{h}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{bT^2}{2\lambda}\right\}+\frac{pI_c}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_1)}-1\right)\right.$$

$$\left.-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_1)+\frac{b}{2\lambda}\left(T^2-M_1^{\ 2}\right)\right\}-\frac{pI_d M_1^{\ 2}}{T}\left(\frac{a}{2}-\frac{bM_1}{3}\right)-$$

$$\frac{rp}{T}\left\{\left(\frac{b+a\lambda}{\lambda^2}\right)\left(e^{\lambda T}-1\right)-\frac{bTe^{\lambda T}}{\lambda}\right\} \tag{9}$$

Case (2). $T < M_1$**:** Interest earned/ year (IE_2) is

$$\frac{pI_d}{T}\left\{\int_0^T(a-bt)\,tdt+(M_1-T)\int_0^T(a-bt)\,dt\right\} = pI_d\left\{aM_1-\left(\frac{a+bM_1}{2}\right)T+\frac{b}{6}T^2\right\} \tag{10}$$

Now Total relevant cost/ year is

$$Z_2(T)=\frac{s}{T}+\frac{h}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{b}{2\lambda}T^2\right\}-pI_d\left\{aM_1-\left(\frac{a+bM_1}{2}\right)T+\frac{b}{6}T^2\right\}$$

$$-\frac{rp}{T}\left\{\left(\frac{b+a\lambda}{\lambda^2}\right)\left(e^{\lambda T}-1\right)-\frac{b}{\lambda}\left(Te^{\lambda T}\right)\right\} \tag{11}$$

Case (3). $T\geq M_2$: Since the payment is made in M_2 , therefore, 'r' is zero. Interest payable /year is

$$\frac{pI_c}{T}\int_{M_2}^{T}I(t)dt=\frac{pI_c}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_2)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_2)+\frac{b}{2\lambda}\left(T^2-M_2{}^2\right)\right\} \tag{12}$$

Interest earned/ year

$$(IE_3)=\frac{pI_d}{T}\int_0^{M_2}\left(a-bt\right)tdt=\frac{pI_dM_2{}^2}{T}\left(\frac{a}{2}-\frac{bM_2}{3}\right) \tag{13}$$

Total relevant cost/ year is

$$Z_3(T)=\frac{s}{T}++\frac{h}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{bT^2}{2\lambda}\right\}+$$

$$\frac{PI_c}{T}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_2)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_2)+\frac{b}{2\lambda}\left(T^2-M_2{}^2\right)\right\}-\frac{PI_dM_2{}^2}{T}\left(\frac{a}{2}-\frac{bM_2}{3}\right) \tag{14}$$

Case (4). $T<M_2$: In this case, only interest earned is applicable by supplier. Thus, interest earned/ year (IE_4) is

$$\frac{pI_d}{T}\left\{\int_0^T\left(a-bt\right)tdt+(M_2-T)\int_0^T\left(a-bt\right)dt\right\}=pI_d\left\{aM_2-\left(\frac{a+bM_2}{2}\right)T+\frac{b}{6}T^2\right\} \tag{15}$$

Now, Total relevant cost /year is

$$Z_4(T) = \frac{s}{T} + \frac{h}{T}\left\{\left(\frac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda T} - 1\right) - \left(\frac{b + a\lambda}{\lambda^2}\right)T + \frac{b}{2\lambda}T^2\right\} - pI_d\left\{aM_2 - \left(\frac{a + bM_2}{2}\right)T + \frac{b}{6}T^2\right\} \quad (16)$$

4. Determination of Optimal Values

From Equation (9), (11), (14) & (16), we obtain

$$\frac{dZ_1(T)}{dT} = -\frac{s}{T^2} + \frac{h}{T}\left\{\left(\frac{a - bT}{\lambda}\right)\left(e^{\lambda T} - 1\right)\right\} - \frac{h}{T^2}\left\{\left(\frac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda T} - 1\right) - \left(\frac{b + a\lambda}{\lambda^2}\right)T + \frac{bT^2}{2\lambda}\right\} +$$

$$\frac{PI_c}{T}\left\{\left(\frac{a - bT}{\lambda}\right)\left(e^{\lambda(T - M_1)} - 1\right)\right\} - \frac{PI_c}{T^2}\left\{\left(\frac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T - M_1)} - 1\right) - \left(\frac{b + a\lambda}{\lambda^2}\right)\left(T - M_1\right) + \frac{b}{2\lambda}\left(T^2 - M_1^{\,2}\right)\right\}$$

$$+\frac{PI_d M_1^{\,2}}{T^2}\left(\frac{a}{2} - \frac{bM_1}{3}\right) - \frac{rp}{T}\left\{\left(a - bT\right)\, e^{\lambda T}\right\} + \frac{rp}{T^2}\left\{\left(\frac{b + a\lambda}{\lambda^2}\right)\left(e^{\lambda T} - 1\right) - \frac{bTe^{\lambda T}}{\lambda}\right\} \quad (17)$$

$$\frac{dZ_2(T)}{dT} = -\frac{s}{T^2} + \frac{h}{T}\left\{\left(\frac{a - bT}{\lambda}\right)\left(e^{\lambda T} - 1\right)\right\} - \frac{h}{T^2}\left\{\left(\frac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda T} - 1\right) - \left(\frac{b + a\lambda}{\lambda^2}\right)T + \frac{b}{2\lambda}T^2\right\}$$

$$+pI_d\left\{\left(\frac{a + bM_1}{2}\right) - \frac{b}{3}T\right\} - \frac{rp}{T}\left\{\left(a - bT\right)e^{\lambda T}\right\} + \frac{rp}{T^2}\left\{\left(\frac{b + a\lambda}{\lambda^2}\right)\left(e^{\lambda T} - 1\right) - \frac{bTe^{\lambda T}}{\lambda}\right\} \quad (18)$$

$$\frac{dZ_3(T)}{dT} = -\frac{s}{T^2} + \frac{h}{T}\left\{\left(\frac{a - bT}{\lambda}\right)\left(e^{\lambda T} - 1\right)\right\} - \frac{h}{T^2}\left\{\left(\frac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda T} - 1\right) - \left(\frac{b + a\lambda}{\lambda^2}\right)T + \frac{bT^2}{2\lambda}\right\}$$

$$+\frac{PI_c}{T}\left\{\left(\frac{a - bT}{\lambda}\right)\left(e^{\lambda(T - M_2)} - 1\right)\right\} - \frac{PI_c}{T^2}\left\{\left(\frac{b + a\lambda - b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T - M_2)} - 1\right) - \left(\frac{b + a\lambda}{\lambda^2}\right)\left(T - M_2\right)\right.$$

$$+\frac{b}{2\lambda}\left(T^2-M_2{}^2\right)\Big\}+\frac{PI_dM_2{}^2}{T^2}\left(\frac{a}{2}-\frac{bM_2}{3}\right) \tag{19}$$

$$\frac{dZ_4(T)}{dT}=-\frac{s}{T^2}+\frac{h}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}-\frac{h}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{b}{2\lambda}T^2\right\}$$

$$+pI_d\left\{\left(\frac{a+bM_2}{2}\right)-\frac{b}{3}T\right\} \tag{20}$$

$$\frac{d^2Z_1}{dT^2}=\frac{2s}{T^3}+\frac{h}{T}\left\{(a-bT)\ e^{\lambda T}-\frac{b}{\lambda}\left(e^{\lambda T}-1\right)\right\}-\frac{2h}{T^2}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}+$$

$$\frac{2h}{T^3}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{bT^2}{2\lambda}\right\}+\frac{pI_c}{T}\left\{(a-bT)\ e^{\lambda(T-M_1)}-\frac{b}{\lambda}\left(e^{\lambda(T-M_1)}-1\right)\right\}$$

$$-\frac{2pI_c}{T^2}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda(T-M_1)}-1\right)\right\}+\frac{2pI_c}{T^3}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_1)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_1)+\right.$$

$$\left.\frac{b}{2\lambda}\left(T^2-M_1{}^2\right)\right\}-\frac{2pI_dM_1{}^2}{T^3}\left(\frac{a}{2}-\frac{bM_1}{3}\right)-\frac{rp}{T}\left[\lambda\{(a-bT)-b\}\ e^{\lambda T}\right]+\frac{2rp}{T^2}\left\{(a-bT)\ e^{\lambda T}\right\}-$$

$$\frac{2rp}{T^3}\left\{\left(\frac{b+a\lambda}{\lambda^2}\right)\left(e^{\lambda T}-1\right)-\frac{bTe^{\lambda T}}{\lambda}\right\} \tag{21}$$

$$\frac{d^2Z_2(T)}{dT^2}=\frac{2s}{T^3}+\frac{h}{T}\left\{(a-bT)e^{\lambda T}-\frac{b}{\lambda}\left(e^{\lambda T}-1\right)\right\}-\frac{2h}{T^2}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}+$$

$$\frac{2h}{T^3}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{b}{2\lambda}T^2\right\}-\frac{bpI_d}{3}-\frac{rp}{T}\left[\lambda\left\{(a-bT)-b\right\}e^{\lambda T}\right]$$

$$+\frac{2rp}{T^2}\left\{(a-bT)e^{\lambda T}\right\}-\frac{2rp}{T^3}\left\{\left(\frac{b+a\lambda}{\lambda^2}\right)\left(e^{\lambda T}-1\right)-\frac{bTe^{\lambda T}}{\lambda}\right\} \qquad (22)$$

$$\frac{d^2Z_3}{dT^2}=\frac{2s}{T^3}+\frac{h}{T}\left\{(a-bT)\ e^{\lambda T}-\frac{b}{\lambda}\left(e^{\lambda T}-1\right)\right\}-\frac{2h}{T^2}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}+$$

$$\frac{2h}{T^3}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{bT^2}{2\lambda}\right\}+\frac{pI_c}{T}\left\{(a-bT)\ e^{\lambda(T-M_2)}-\frac{b}{\lambda}\left(e^{\lambda(T-M_2)}-1\right)\right\}-$$

$$\frac{2pI_c}{T^2}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda(T-M_2)}-1\right)\right\}+\frac{2pI_c}{T^3}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_2)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_2)+\right.$$

$$\left.\frac{b}{2\lambda}\left(T^2-M_2{}^2\right)\right\}-\frac{2pI_dM_2{}^2}{T^3}\left(\frac{a}{2}-\frac{bM_2}{3}\right) \qquad (23)$$

$$\frac{d^2Z_4(T)}{dT^2}=\frac{2s}{T^3}+\frac{h}{T}\left\{(a-bT)e^{\lambda T}-\frac{b}{\lambda}\left(e^{\lambda T}-1\right)\right\}-\frac{2h}{T^2}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}$$

$$+\frac{2h}{T^3}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{b}{2\lambda}T^2\right\}-\frac{bpI_d}{3} \qquad (24)$$

For finding minimum value of $Z_i(T)$, we get $\frac{dZ_i(T)}{dT}=0$, for which $\frac{d^2Z_i(T)}{dT^2}>0$, i = 1, 2, 3, 4, putting $\frac{dZ_i(T)}{dT}=0$, i = 1, 2, 3, 4, we get

$$-\frac{s}{T^2}+\frac{h}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}-\frac{h}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{bT^2}{2\lambda}\right\}+$$

$$\frac{PI_c}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda(T-M_1)}-1\right)\right\}-\frac{PI_c}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_1)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_1)+\frac{b}{2\lambda}\left(T^2-M_1{}^2\right)\right\}$$

$$+\frac{PI_dM_1{}^2}{T^2}\left(\frac{a}{2}-\frac{bM_1}{3}\right)-\frac{rp}{T}\left[(a-bT)\ e^{\lambda T}\right]+\frac{rp}{T^2}\left\{\left(\frac{b+a\lambda}{\lambda^2}\right)\ \left(e^{\lambda T}-1\right)-\frac{bTe^{\lambda T}}{\lambda}\right\}=0\,. \tag{25}$$

$$-\frac{s}{T^2}+\frac{h}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}-\frac{h}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{b}{2\lambda}T^2\right\}+$$

$$pI_d\left\{\left(\frac{a+bM_1}{2}\right)-\frac{b}{3}T\right\}-\frac{rp}{T}\left\{(a-bT)e^{\lambda T}\right\}+\frac{rp}{T^2}\left\{\left(\frac{b+a\lambda}{\lambda^2}\right)\ \left(e^{\lambda T}-1\right)-\frac{bTe^{\lambda T}}{\lambda}\right\}=0\,. \tag{26}$$

$$-\frac{s}{T^2}+\frac{h}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}-\frac{h}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{bT^2}{2\lambda}\right\}$$

$$-\frac{PI_c}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda(T-M_2)}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)(T-M_2)+\frac{b}{2\lambda}\left(T^2-M_2{}^2\right)\right\}$$

$$+\frac{PI_c}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda(T-M_2)}-1\right)\right\}+\frac{PI_dM_2{}^2}{T^2}\left(\frac{a}{2}-\frac{bM_2}{3}\right)=0\,. \tag{27}$$

$$-\frac{s}{T^2}+\frac{h}{T}\left\{\left(\frac{a-bT}{\lambda}\right)\left(e^{\lambda T}-1\right)\right\}-\frac{h}{T^2}\left\{\left(\frac{b+a\lambda-b\lambda T}{\lambda^3}\right)\left(e^{\lambda T}-1\right)-\left(\frac{b+a\lambda}{\lambda^2}\right)T+\frac{b}{2\lambda}T^2\right\}$$

$$+pI_d\left\{\left(\frac{a+bM_2}{2}\right)-\frac{b}{3}T\right\}=0. \tag{28}$$

5. Numerical Examples and Sensitivity Analysis

Example 1 (case 1).

Let $s=500;\ h=10;\ a=100;\ b=20;\ \lambda=0.4;$ p = 200; $I_c=0.15;\ I_d=0.13;\ r=0.01;\ M_1=0.010245$ in appropriate units. Using these values in Equation (25) & solving; we get $T=T_1=0.441705$ years; the corresponding $Q^*(T_1)=46.1161$ units; & $Z^*(T_1)=\$1510.86$, which verified case (1) i.e. $T\geq M_1$ and $\frac{d^2Z_1}{dT^2}=8873.62>0$.

Example 2 (case 2). Let $s=\$700/order;\ h=\$10/unit/year;\ a=1000;\ b=20;\ \lambda=0.4;$ p = \$ 30 / unit; $I_d=0.15/year;\ r=0.01;\ M_1=0.50445 year$. Using these data in Equation (26) and solving; we get optimal $T=T_2=0.303833$ years; the corresponding $Q^*(T_2)=322.066$ units; and $Z^*(T_2)=\$1512.27$, that proves case (2) i.e. $T<M_1$ and $\frac{d^2Z_2}{dT^2}=51176.6>0$.

Example 3 (case 3). Let $s=700;\ h=10;\ a=100;\ b=20;\ \lambda=0.4;$ p = 200; $I_c=0.12;\ I_d=0.11;$ $M_2=0.010543$. Putting these data in Equation (27) & solving; we get $T=T_3=0.645262$ years; the corresponding $Q^*(T_3)=68.6642$ units; and $Z^*(T_3)=\$2151.54$ which proves case (3) & $\frac{d^2Z_3}{dT^2}=5157.24>0$.

Example 4 (case 4). Let $s = \$700/order$; $h = \$10/unit/year$; $a = 1000$; $b = 20$; $\lambda = 0.4$; p = \$ 30 /unit; $I_d = 0.15/year$; $M_2 = 0.6215 year$. Putting these values in Equation (28) and solving for T; we obtain $T = T_4 = 0.302535$ years; the corresponding $Q^*(T_4) = 320.609$ units; & $Z^*(T_4) = \$1774.02$, that proves case 4 and $\frac{d^2 Z_4}{dT^2} = 51837 > 0$.

6. Sensitivity Analysis

From the Table the following observations can be made:

Case (1).

From Table 1(a), we see that, s, Q^* and $Z_1^*(T)$ moves in the same direction.

Table 1. A. Variation of s taking remaining elements same as in Ex.1

s	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_1}{dT^2}$
300	0.330359	34.1256	1073.52	11912.7
400	0.389457	40.4592	1305.86	10084.7
700	0.532728	56.0991	1866.89	7325.83
800	0.573453	60.618	2025.65	6790.23
900	0.61178	64.9001	2174.72	6350.12

Table 1. B. Change of discount rate r

r	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_1}{dT^2}$
0.0112	0.435101	45.3981	1458.9	8999.97
0.0113	0.434557	45.339	1454.59	9010.5
0.012	0.430768	44.9275	1424.56	9084.81
0.013	0.425429	44.3481	1381.97	9191.53
0.0131	0.4249	44.2907	1377.73	9202.22

Table 1. C. Variation of I_d

I_d	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_1}{dT^2}$
0.131	0.441704	46.116	1510.86	8873.66
0.141	0.441699	46.1154	1510.84	8873.76
0.143	0.441698	46.1153	1510.832	8873.77
0.146	0.441696	46.1151	1510.831	8873.84
0.149	0.441695	46.1150	1510.82	8873.83

Table 1. D. Variation of I_C

I_c	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_1}{dT^2}$
0.155	0.436983	45.6026	1534.82	9197.70
0.156	0.436057	45.502	1539.57	9262.98
0.157	0.435138	45.4021	1544.31	9328.35
0.158	0.434224	45.3028	1549.04	9393.93
0.159	0.433316	45.2042	1553.76	9459.66

From Table 1(b), we see that, if cash discount rate r increases Q^* and $Z_1^*(T)$ both will decrease.

From Table 1(c), we see that, increase of interest earned /dollar I_d causes, no significant change in Q^* and $Z_1^*(T)$.

From Table 1(d), we see that increase of interest charged per dollar I_c causes decrease in Q^* and increase in $Z_1^*(T)$.

Case (2).

From Table 2(a) & 3(a), we see that increase of ordering cost *s* causes increase in Q^* & $Z_2^*(T)$.

Table 2. A. Change of s adjusting rest parameters same as in Ex.2

s	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_2}{dT^2}$
800	0.324258	345.079	1828.73	48199
900	0.343379	366.786	2126.44	45735.3

Table 2 A Continued

1000	0.361407	387.397	2408.46	43652.1
1100	0.378503	407.073	2677.09	41861.2
1200	0.394792	425.939	2934.13	40300.2

Table 2. B. Variation of I_d

I_d	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_2}{dT^2}$
0.153	0.302952	321.077	1480.72	51612.4
0.154	0.30266	320.749	1470.19	51757.9
0.155	0.302369	320.423	1459.66	51903.6
0.156	0.302078	320.097	1449.12	52049.8
0.157	0.301789	319.772	1438.58	52195.5

From Table 2(b), we see that increase of interest earned/dollar I_d results decrease of Q^* & $Z_2^*(T)$.

Case (3).

From Table 3(b), we see that increase of interest earned/ dollar I_d, results, no significant change in Q^* &

Table 3. A. Variation of s

s	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_3}{dT^2}$
800	0.690333	73.765	2301.29	4805.08
900	0.73277	78.6032	2441.83	4511.95
1000	0.773011	83.2226	2574.66	4262.70
1100	0.811385	87.656	2700.89	4047.15
1200	0.848149	91.9293	2821.41	3858.13

Table 3. B. Variation of I_d

I_d	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_3}{dT^2}$
0.112	0.645261	68.6641	2151.54	5157.25
0.113	0.64526	68.6639	2151.54	5157.26
0.115	0.645259	68.6638	2151.53	5157.27
0.117	0.645258	68.6637	2151.53	5157.28
0.119	0.645257	68.6636	2151.53	5157.29

Table 3. C. Variation of Ic

I_c	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_3}{dT^2}$
0.16	0.580049	61.3529	2386.38	7115.6
0.17	0.566618	59.8573	2441.36	7636.97
0.18	0.55408	58.4643	2495.06	8170.41
0.19	0.542339	57.1626	2547.56	8715.73
0.20	0.531315	55.9429	2598.93	9272.61

$Z_3^*(T)$.

From Table 3(c), we see that increase of I_c, results decrease in Q^* & increase in $Z_3^*(T)$.

Case (4).

From Table 4(a), it is clear that s, Q^* & $Z_4^*(T)$ move in the same direction.

Table 4. A. Variation of s

s	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2Z_4}{dT^2}$
800	0.322874	343.514	2093.8	48821.1
900	0.341913	365.116	2394.64	46324.9

Table 4 A Continued

1000	0.359865	385.628	2679.63	44214.4
1100	0.376889	405.21	2951.09	42400.1
1200	0.393109	423.984	3210.82	40818.7

Table 4. B. Variation of I_d

I_d	T_1	$Q^*(T_1)$	$Z^*(T_1)$	$\frac{d^2 Z_4}{dT^2}$
0.10	0.318236	338.275	2471.22	44732.6
0.11	0.314906	334.52	2332.74	46122.0
0.12	0.311676	330.882	2193.77	47527.1
0.13	0.30854	327.353	2054.32	48948.3
0.14	0.305495	323.932	1914.4	50384.7

From Table 4(b), we see that increase of interest earned/dollar I_d results decrease of Q^* & $Z_4^*(T)$.

7. Conclusion

This paper employs a deterministic inventory model for deteriorating commodities under non- increasing time induced demand. The cash discount and trade credits to settle the account have been also considered. We examine the cost minimization behavior in term of optimal cycle time. Sensitivity analysis is discussed with respect to major parameters. From managerial point of view the observations can be made as follows:

Increase of ordering cost causes increase in Q^*& Z^*.

Increase of cash discount rate results decrease of Q^* & Z^*.

Increase of interest earned per dollar causes, no significant change in Q^* & Z^*.

Increase of interest charged per dollar causes decrease in Q^* & increase in Z^*.

Note: Mathematica 9 is used to find the numerical results.

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