

Two Methods for Simultaneous Calibration of Four Phase Plates

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Abstract

Objectives: Phase plates are optical polarization elements widely used in applications and measurements of polarized light. This work is concerned with the calibration of four phase plates simultaneously. **Method /Analysis:** Two methods are employed for calibrating the plates based on a previously presented model for simultaneous calibration of two plates without the need for a standard plate. Analysis of the results and the used mathematical expressions are quite simple. **Findings:** The four mica plates were found to have retardances and birefringence in agreement with previously published data of muscovite mica. **Application / Improvements:** The work provides simple and time saving methods for the calibration of phase plates. It also allows for calibrating samples of small retardances which are difficult to calibrate accurately using other methods.

Keywords: Birefringence, Calibration, Phase Plates, Retardance

1. Introduction

In a previous work¹, a method was presented for simultaneous calibration of two phase plates at any wavelength. The method has the advantage that no reference plate is used and each of the two plates acts as a reference to the other plate while both of them are of unknown retardance. The optical system is shown in Figure 1, where P, A are polarizing prisms (polarizer and analyzer), C₁, C₂ are the calibrated plates, D is the detecting system and S is monochromatic light source of wavelength λ . The two plates are oriented with their fast axes at c₁ = 45°, c₂ = 0° and P, A are simultaneously rotated for extinction (small letters p, a, c₁ and c₂ are used for the settings of the optical elements and angles are measured in a CCW sense with respect to the transmission axes of the polarizers and the fast axes of the plates by an observer receiving the radiation). At extinction, the retardances δ₁ and δ₂ of the two plates are given by the simple expressions

$$\cos \delta_1 = -\cos 2a / \cos 2p, \quad (1)$$

$$\cos \delta_2 = -\sin 2p / \sin 2a. \quad (2)$$

Note that there are two extinction pairs (p₁, a₁) and (p₂, a₂) such that

$$p_2 = p_1 \pm 90^\circ, \quad (3)$$

$$a_2 = a_1 \pm 90^\circ. \quad (4)$$

Equations (1) and (2) apply for both of the two pairs.

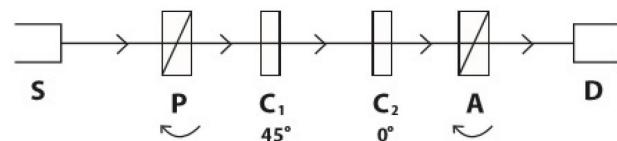


Figure 1. Optical arrangement for simultaneous calibration of two phase plates. S - light source, P, A - polarizing prisms, C₁ and C₂ - the calibrated plates and D - the detecting system.

As the trigonometric functions can have two values, it is necessary to know whether each of the two plates has retardance less than or greater than π . In another work², it was shown that if the retardance of one of the two plates

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is known to be less than or greater than π and the other plate is of completely unknown value, it is still possible to determine their retardances by considering two auxiliary equations of the form

$$\tan \delta_1 = \tan 2a \sin \delta_2, \quad (5)$$

$$\tan \delta_2 = \cot 2p \sin \delta_1. \quad (6)$$

Finally, the restrictions imposed on the retardance values were removed by considering two measurements at two adjacent wavelengths³.

In this work, a 2-step procedure using this calibration model is introduced to calibrate four plates. Another 3-step procedure is presented for the calibration process with the advantage that each plate is calibrated two times. Experimental results are presented.

2. Optical Arrangement

Four muscovite mica plates C_1 , C_2 , C_3 and C_4 of thicknesses 47.4, 22.8, 33.5 and 27.8 μm respectively cleaved from the same crystal are used in the calibration process. The accuracy of thickness measurement is $\pm 0.2 \mu\text{m}$. Laser source of $\lambda = 543.5 \text{ nm}$ is used in the work. The optical arrangement for calibrating the four plates is shown in Figure 2, where the two plates C_1 and C_2 are set with their fast axes at 45° while the plates C_3 and C_4 are oriented at 0° . The system is similar to that of Figure 1 except that four plates are investigated. Now, rotating P and A simultaneously for extinction, we get

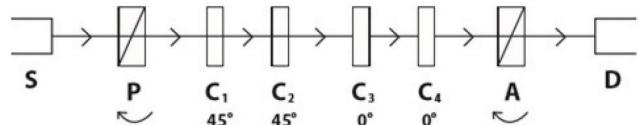
$$\cos(\delta_1 + \delta_2) = -\cos 2a_1 / \cos 2p_1, \quad (7)$$

$$\cos(\delta_3 + \delta_4) = -\sin 2p_1 / \sin 2a_1. \quad (8)$$

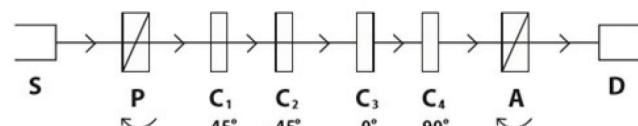
Now, we rotate the holder of the plate C_1 about a vertical axis through 180° while keeping it following C_1 , it will be oriented now at -45° and also we rotate the scale of the plate C_4 so that its fast axis is set at 90° , Figure 2. Repeating the above step, then at extinction we get

$$\cos(\delta_1 - \delta_2) = -\cos 2a_2 / \cos 2p_2, \quad (9)$$

$$\cos(\delta_3 - \delta_4) = -\sin 2p_2 / \sin 2a_2. \quad (10)$$



(a)



(b)

Figure 2. The optical system for simultaneous calibration of four phase plates.

Solving equations (7) and (9), we get values for δ_1 and δ_2 . Similarly, solving equations (8) and (10), we get δ_3 and δ_4 .

The second method for simultaneous calibration of four phase plates which requires 3 steps is represented in Figure 3. Thus, if instead of C_1 in Figure 1. We set successively (C_1, C_2) , (C_1, C_3) and (C_1, C_4) with their fast axes at 45° and instead of C_2 we set (C_3, C_4) , (C_2, C_4) and (C_2, C_3) at 0° , then by solving the resulting equations at extinction, we can get the retardances of the four plates. The advantage of this procedure is that the consistency of the results indicates the accuracy of measurements and each plate is calibrated two times.

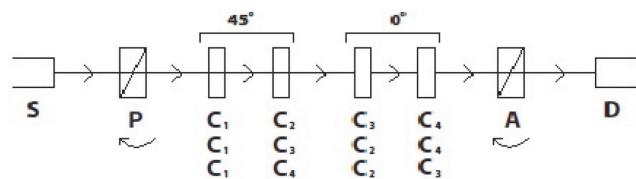


Figure 3. Second arrangement for calibrating four plates simultaneously.

3. Results and Discussion

In the first method, for each of the two calibration steps, two extinction pairs were recorded and the mean values were considered. The results are shown in Table 1. Birefringence of mica is represented by the relation⁴

$$\delta = \pm 2\pi.d.\Delta n / \lambda \quad (11)$$

where d is the plate thickness, Δn is the birefringence and λ is the working wavelength. The \pm sign is used since mica is of negative birefringence and retardances are measured only between 0° and 360° .

Table 1. Results for the retardances and birefringence values for the plates at 543.5 nm

Plate	Thickness (μm)	Retardance ($^\circ$)	Birefringence	δ/d ($^\circ/\mu\text{m}$)
1	47.4	132.30	-0.00421	2.791
2	22.8	63.75	-0.00422	2.786
3	33.5	92.94	-0.00419	2.791
4	27.8	77.62	-0.00422	2.792

Results showed that birefringence at 543.5 nm is

$$\Delta n = -0.00421 \pm 0.00002 \quad (12)$$

This result highly agrees with previously published data for muscovite mica⁴. Note that birefringence of mica may vary from one crystal to another even if they were from the same source. Small differences between the results of birefringence arise probably from errors in thickness measurements or experimental errors. The last column in Table 1 shows the retardance per thickness of one micron of mica for our samples at 543.5 nm which is given as $\sim 2.79^\circ/\mu\text{m}$ which means that a plate of thickness $32.3\ \mu\text{m}$ assumes quarterwave retardance at this wavelength.

Some notes are to be mentioned now. First, if the plate C_2 is of higher retardance value than C_1 , then if C_2 is oriented at -45° , the resultant effect of the two plates will be a plate oriented at -45° . This has no effect on the results⁵ as the same formulas (1) and (2) apply for the case of $c_1 = -45^\circ$ and $c_2 = 0^\circ$. This also applies if δ_4 is greater than δ_3 . This could also be easily understood since $\cos(a - b) = \cos(b - a)$. Also, from published data of mica⁴, it was concluded from the thicknesses of the four plates that the expected value for $(\delta_1 + \delta_2)$ is greater than π and that for $(\delta_3 + \delta_4)$ is less than π .

In the second method, we get the following results

$$\cos(\delta_1 + \delta_2) = -\cos 2a_1 / \cos 2p_1, \quad (13a)$$

$$\cos(\delta_3 + \delta_4) = -\sin 2p_1 / \sin 2a_1, \quad (13b)$$

$$\cos(\delta_1 + \delta_3) = -\cos 2a_2 / \cos 2p_2, \quad (14a)$$

$$\cos(\delta_2 + \delta_4) = -\sin 2p_2 / \sin 2a_2, \quad (14b)$$

$$\cos(\delta_1 + \delta_4) = -\cos 2a_3 / \cos 2p_3, \quad (15a)$$

$$\cos(\delta_2 + \delta_3) = -\sin 2p_3 / \sin 2a_3, \quad (15b)$$

Again, two readings are recorded according to Eqs. (3, 4) and the mean values are considered. Method for calculating values of δ_1 is now presented and results for other plates are calculated in similar ways. It follows from measurements of (13a) and (14a) that

$$2\delta_1 + \delta_2 + \delta_3 = 421.74^\circ \quad (16)$$

Also, from (15b), we get

$$\delta_2 + \delta_3 = 157.20^\circ \quad (17)$$

From (16) and (17), we get

$$\delta_1 = 132.27^\circ \quad (18)$$

Similarly, from (13a), (15a) and (14b) we obtained

$$2\delta_1 + \delta_2 + \delta_4 = 407.02^\circ$$

And

$$\delta_2 + \delta_4 = 141.82^\circ$$

which gives

$$\delta_1 = 132.60^\circ \quad (19)$$

Retardance values for other plates are calculated in similar way and are presented in Table 2 with the mean value of the two results considered. The uncertainty in retardance measurements using these models is $\pm 0.25^\circ$.

Table 2. Retardance values of the four calibrated plates using the 3-step method at 543.5 nm

Plate	Retardance
C1	132.44
C2	63.33
C3	93.15
C4	77.92

Finally, an important advantage of these calibration methods is that if one of the plates is of small retardance,

it could be inserted in the system in place of any plate. As was shown in a previous work⁶, it is difficult to measure the retardance of a sample of exceedingly small retardance using different methods as a small error in the settings of elements results in large errors in the retardance value. Attaching such sample to a plate of moderate retardance in our methods will increase the sensitivity of measurement and we can get accurate results.

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