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Sliding Mode Control with Nonlinear Disturbance Observer based on Genetic Algorithm for Inverted Pendulum with Mismatched Disturbances

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Abstract

Objective: This study proposes the application of optimized Sliding Mode Control (SMC) with Nonlinear Disturbance Observer (NDO) for the Inverted pendulum, which is subjected to mismatched disturbances. **Methods/Statistical Analysis:** Sliding Mode Control (SMC) with Nonlinear Disturbance Observer (NDO) solves the main problem of chattering present in traditional sliding mode controllers while retaining the nominal performance even with mismatched disturbances. Chattering defines the presence of finite frequency and finite amplitude oscillations in the control input. Genetic Algorithm is used for searching the optimal controller parameters, which includes switching function coefficient, switching gain and observer gain. **Findings:** Simulation results shows the advantages of NDO based SMC and proves that NDOSMC can observe the mismatched disturbances very effectively. The conventional SMC method is also used in the control design the Inverted pendulum for comparison studies. The two main issues to be tackled in conventional Sliding Mode Control (SMC) are convergence time and chattering. Chattering and convergence time can be reduced in SMC by getting the optimal values of controller parameters using intelligent optimization techniques such as Genetic Algorithm (GA). In the present study convergence time and chattering of the controlled system are considered as the performance index of objective function. **Application/Improvements:** This study can further be extended for Multi-Input Multi-Output (MIMO) systems subjected to mismatched disturbances and parametric uncertainties.

Keywords: Chattering, Genetic Algorithm, Inverted Pendulum, Mismatched Disturbances, Nonlinear Disturbance Observer (NDO), Sliding Mode Control (SMC)

1. Introduction

The imprecision in nonlinear model of a system may arise from uncertainty about the plant parameters, or from the assumptions made during the mathematical modeling for simplification of the system's dynamics. Inaccuracies occurred during modelling are categorised as: structured uncertainties and unstructured uncertainties. The structured uncertainty refers to those inaccuracies that occur in the terms of the model, while unstructured uncertainty refers to inaccuracies in the order of the system. Continuous research is going on to develop controllers which can work efficiently in spite of the presence of these uncertainties in the system. To cope up with the uncertainties various robust control techniques have been mentioned in the literature such as back-stepping[1], nonlinear adaptive control[2], model predictive control (MPC)[3,4], and sliding mode control (SMC)[5,6]. The sliding mode control (SMC)[7,8] has gain popularity in the past years due to its inherent insensitivity to external disturbances and parametric variations. Two main drawbacks in conventional Sliding Mode Control (SMC) are the convergence time and chattering phenomenon. It could be shown that if the system states arrives the sliding surface in limited convergence time, the stability of the system will be guaranteed. Also, sliding motion of the conven-

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tional Sliding Mode Control (SMC) is not affected by the presence of matched uncertainties[9]. Matched uncertainties means that the uncertainties and the control input exists in the same channel[10].

Hence, NDO based SMC was proposed by[11,12], in order to achieve better control performance by reducing chattering while maintaining the nominal performance in presence of mismatched uncertainties. Chattering and hitting time are further reduced in NDO based SMC by the selection of optimal controller parameters using optimization technique such as Genetic Algorithm (GA). Genetic Algorithm is popularly used in design of optimized complex systems. It is based on genetic and evolutionary mechanism and is used by many researchers for optimal design of controller parameters[13-15].

The paper is divided in following sections. In section 2, the conventional SMC and NDO based SMC methods are explained. In order to prove the efficiency of NDO based SMC method, simulation studies are carried out for Inverted pendulum under mismatched disturbance in section 3 and the optimal controller parameters are found by genetic algorithm. Finally, conclusions are given in section 4.

2. Sliding Mode Control

For robustness against parametric variations and external disturbances, sliding mode control (SMC) that is developed from the theory of variable structure system, has gained much popularity in the literature[16]. In sliding mode control (SMC), the states of the system are moved from their initial states towards a chosen surface in the state space, called the sliding surface. After reaching the sliding surface, the system becomes insensitive to external disturbances and parametric uncertainties. The system is not robust and the system performance can even be affected by matched uncertainties during the reaching phase. Consider the second order system shown by equation (1):

$$\begin{aligned} \dot{x}_1 &= x_2 + w(t) \\ \dot{x}_2 &= p(x) + q(x)u, \\ y &= x_1 \end{aligned} \tag{1}$$

Where x_1 and x_2 represents the states, u is the control input, external disturbances are represented by *w(t)* and output is represented by *y*.

Assuming that the disturbances in the above system are bounded by $w^* = \sup_{t>0} |w(t)|$, the following equations represent the sliding mode surface and sliding mode control law:

$$\sigma = x_2 + \lambda x_1, \quad u = -q^{-1}(x)\left[p(x) + \lambda x_2 + \eta\,\mathrm{sgn}(\sigma)\right] \tag{2}$$

The control law in (2) which satisfies the condition $\frac{1}{2}\frac{d}{dt}\sigma^2 \le -\beta|\sigma|$, where $\beta > 0$ a constant can also be defined as:

$$u = u_{eq.} - \eta\,\mathrm{sgn}(\sigma)$$

Where $u_{eq.}$ is the equivalent control law.

Combining (1) - (2) yields

$$\dot{\sigma} = -\eta\,\mathrm{sgn}(\sigma) + \lambda w(t) \tag{3}$$

It is shown by equation (3) that if the switching gain η is designed such that $\eta > \lambda w^*$ then in a finite time the states of the system (1) will arrive at the sliding surface $\sigma = 0$.

It is a difficult task to know the upper bound of the uncertainty in real time application and very often this bound is overestimated resulting in excessive gain. To overcome these difficulties, NDO based SMC was proposed by (Yang et al., 2013)[11,12].

Consider a system represented by

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + lw \\ y &= x_1 \end{aligned} \tag{4}$$

Where

$f(x) = [x_2, \quad p(x)]^T$, $g(x) = [0, \quad q(x)]^T$ *and* $l = [1, \quad 0]^T$.

To estimate the disturbance in (4), a nonlinear disturbance observer (NDO) is given by

$$\begin{aligned} \dot{z} &= -m(x)[l(x)(n(x) + z) + f(x) + g(x)u] \\ \hat{w} &= z + n(x) \end{aligned} \tag{5}$$

Where z represents the internal state vector of nonlinear observer and $\hat{w}$ is the estimation of disturbance *w*. Nonlinear function that is to be designed is represented by *n(x)* . Observer gain *m(x)* is given as

$$m(x) = \frac{\partial n(x)}{\partial x} \tag{6}$$

The sliding mode surface and control law of NDO-SMC for the system (4) under mismatched disturbance are given as follows:

$$\sigma = x_2 + \lambda x_1 + \hat{w}$$
$$u = -q^{-1}(x)\left[p(x) + \lambda(x_2 + \hat{w}) + \eta \, \mathrm{sgn}(\sigma) \right] \tag{7}$$

Where $\hat{w}$ is the disturbance estimation that is given by NDO.

Theorem 1: Assuming that the disturbances in the above system are bounded by $w* = \sup_{t>0} |w(t)|$, the derivative of the disturbances satisfies $\lim_{t \to \infty} \dot{d}(t) = 0$ and the estimation of error in disturbance is bounded by $e_d* = \sup_{t>0} |e_d(t)|$ where $e_d = w - \hat{w}$, the closed loop system will be asymp totically stable if the switching gain in (7) is designed such that $\eta > (\lambda + m(x)l) e_d*$ and the gain of the observer $m(x)$ is chosen such that $m(x)l > 0$ holds.

3. Simulation Studies

In order to verify the advantages of the proposed NDO based SMC, it is applied to a single inverted pendulum[17]. A Linear Single Inverted Pendulum, shown in Figure 1, consists of a vertical pendulum pivoted in the end, over a cart moving linearly. To retain the pendulum around its upper unstable equilibrium position is the objective. The reason for considering inverted pendulum for implementing the control strategy is its characteristics such as nonlinearity and inherent instability. For doing the mathematical modelling of Linear Single Inverted Pendulum, certain assumptions are made, e.g., all gearbox backlash and frictions are neglected. The derived equations are as follows:

$$\dot{x}_1 = x_2 + d(t)$$
$$\dot{x}_2 = \frac{g \sin x_1 - amLx_2^2 \sin x_1 \cos x_1 + au \cos x_1}{L(4/3 - am\cos^2 x_1)} \tag{8}$$

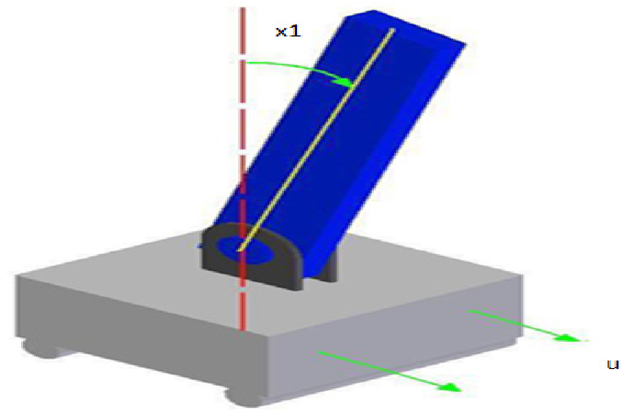

Figure 1. Linear Inverted Pendulum System.

where x_1 denotes the angular position of inverted pendulum (rad), x_1 denotes its angular speed (rad/s), mass of the cart is represented by M, mass of inverted pendulum by m and half length of inverted pendulum by L. The control input is denoted by u, external disturbance is represented by $d(t)$ and the coefficient $a = m/(M + m)$. The values taken for the simulation parameters are: $M = 1$ kg, $m = 0.2$ kg, $L = 0.5$m, $g = 9.8\text{m/s}^2$, and the initial values of the state vector is $[x(1) \quad x(2)] = [-\pi/18 \quad 0]$.

At time t = 4 sec, the system is subjected to by an external disturbance d = 0.2. The conventional SMC method is also used in the control design for system (8) for comparison studies. In this study GA is used to obtain the optimal value of the controller parameters switching function coefficient λ, switching gain η and observer gain m. The two main issues to be tackled in conventional Sliding Mode Control (SMC) are convergence time and chattering. Chattering and convergence time can be reduced in SMC by getting the optimal values of controller parameters using intelligent optimization techniques such as Genetic Algorithm (GA). **Selection of parameters in GA is given in Table 1**. In the present study convergence time and chattering of the controlled system are considered as the performance index of objective function. The objective function taken here is:

$$f = \int \sigma \, dt + \left(u - u_{eq}\right)^2 \tag{9}$$

The controller parameters are chosen by the proposed fitness function such that the convergence time and chattering phenomenon are reduced. The first part of the proposed fitness function (9) results in shorter convergence time by reducing the area under the curve of the sliding surface. The second term of the fitness function results in reduction of chattering amplitude.

Table 1. Parameter value in GA

Parameter	Value
Population size	50
Number of generations	100
Crossover rate	0.8
Mutation rate	0.04

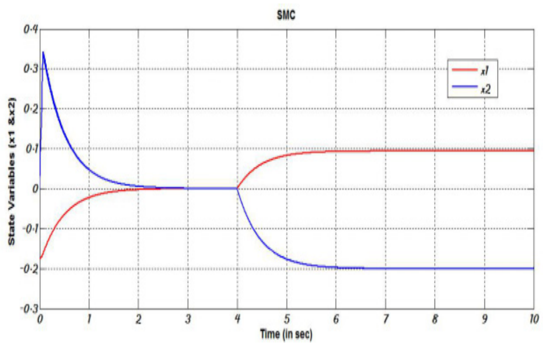

Figure 2. State variables of Inverted Pendulum for GA-SMC.

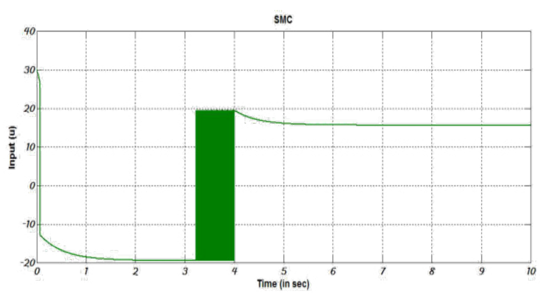

Figure 2. Control input of Inverted Pendulum for GA-SMC.

It can be observed from Figure 2 and 3 that during the first 4 sec, GA-NDO-SMC and GA-SMC methods results in same control input and states responses, hence proving the retention of the nominal control performance. It can also be observed that in the presence of mismatched disturbance, the GA-SMC method does not succeed in driving the state x_1 to the desired equilibrium, which shows that GA-SMC is vulnerable to mismatched disturbance. GA-NDO-SMC methodology can finally suppress the mismatched disturbance. Also, chattering in the control input is reduced in GA-NDO-SMC in comparison to chattering in the control input in GA-SMC.

The control parameters for SMC as obtained through GA by optimizing the fitness function (9) are:

Switching function coefficient $\lambda = 2$

Switching gain $\eta = 5$

The control parameters for NDOSMC as obtained through GA by optimizing the fitness function (9) are:

Switching function coefficient $\lambda = 14.4708$

Switching gain $\eta = 6.2875$

Observer gain $m = 12.4352$

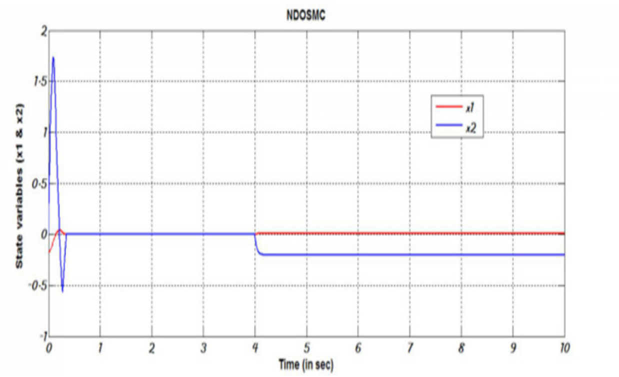

Figure 3. State variables of Inverted Pendulum for GA-NDO-SMC.

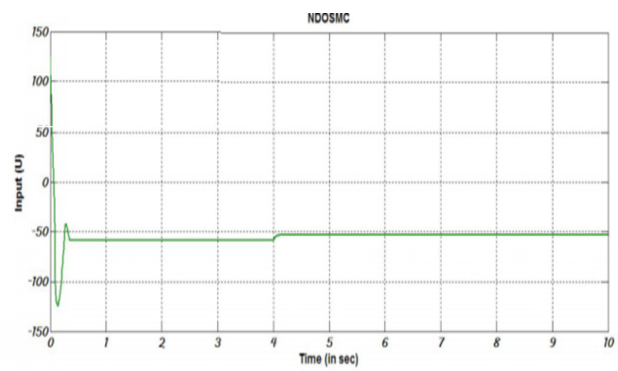

Figure 3. Control Input of Inverted Pendulum for GA-NDO-SMC.

4. Conclusion

In this paper, optimized sliding mode control with nonlinear disturbance observer has been used to reduce the effect of mismatched disturbances in inverted pendulum. It is shown in the simulation results that GA-NDO-SMC method suppresses the mismatched disturbance while retaining the nominal performance. Also, chattering in control input is reduced in GA-NDO-SMC in comparison to chattering in control input in GA-SMC.

5. References

1. Soltanpour MR, Shafiei SE. Robust Back stepping Control of Robot Manipulator in Task Space with uncertainties in Kinematics and Dynamics. Electronics and Electrical Engineering ELEKTRIKA IR ELEKTROTECHNIKA. 2009; 96(8):76-80.
2. Islam S, Xiaoping P. Liu, and El Saddik A. Adaptive sliding mode control of unmanned four rotor flying vehicle. International Journal of Robotics and Automation. 2015; 30(2).
3. Camacho EF, Bordons C. Springer: Model Predictive Control. 2004.
4. Errouissi R, Ouhrouche M, Chen WH, Trzynadlowski AM. Robust nonlinear predictive controller for permanent magnet synchronous motors with optimized cost function. IEEE Transactions on Industrial Electronics. 2012 Jul; 59(7):2849-58. Crossref.
5. Perruquetti W, Barbot JP. Sliding mode control in Engineering. New York, USA: Automation and Control Engineering. 2002 Jan; p. 432.
6. Choi HH. Sliding-mode output feedback control design. IEEE Transactions on Industrial Electronics. 2008 Nov; 55(11):4047-54. Crossref.
7. Choi HH. LMI-based sliding surface design for integral sliding model control of mismatched uncertain systems. IEEE Transactions on Automatic Control. 2007 Apr; 52(4):736-42. Crossref.
8. Barmish B, Leitmann G. On ultimate boundedness control of uncertain systems in the absence of matching condition. IEEE Transactions on Automatic Control. 1982 Feb; 27(1):153-8. Crossref.
9. Feng Y, Yu X, Man Z. Non-singular terminal sliding mode control of rigid manipulators. Automatica. 2002 Dec; 38(12):2159-67.
10. Fulwani D, Bandyopadhyay B, Fridman L. Non-linear sliding surface towards high performance robust control. IET Control Theory and Applications. 2012; 6(2):235-42. Crossref.
11. Yang J, Li S and Yu X. Sliding-Mode Control for Systems with Mismatched Uncertainties via a Disturbance Observer. IEEE Transactions on Industrial Electronics. 2013; 60(1):160-9. Crossref.
12. Li S, Yang J, Chen WH, Chen X. Boca Raton, FL, USA: CRC Press Inc.: Disturbance Observer based Control - Methods and Applications. 2014.
13. Chen PC, Chen CW, Chiang WL. Linear matrix inequality conditions of nonlinear systems by genetic algorithm-based $H\infty$ adaptive fuzzy sliding mode controller. Journal of Vibration and Control. 2011; 17(2):163-73. Crossref.
14. Rani MR, Selamat H, Zamzuri H. Multiobjective optimization for PID controller tuning using the global ranking genetic algorithm. International Journal of Innovative Computing, Information and Control. 2012 Jan; 8 (1):269-84.
15. Douiri MR, Cherkaoui M, Assadki A. Genetic algorithms based fuzzy speed controllers for indirect field oriented control of induction motor drive. International Journal of Circuits, Systems and Signal Processing. 2012; 6(1):21-8.
16. Yeh FK. Attitude controller design of mini unmanned aerial vehicles using fuzzy sliding-mode control degraded by white noise interference. IET Control Theory and Applications. 2012 Jun; 6(9):1205-12. Crossref.
17. Pang HP, Yang Q. Optimal Sliding Mode Control for a Class of Uncertain Nonlinear Systems Based on Feedback Linearization. Robust Control Theory and Applications, Prof. AndrzejBartoszewicz. 2011; p. 141-62.