Numerical Study of Flow Past Three Rectangular Rods at Unequal Gap Spacing

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Abstract:

Objectives: To examine the effect of unequal gap spacing for flow past over three rectangular rods a fixed Re (Reynolds number) and Aspect Ratio (AR). Methodology: A two dimensional (2-D) numerical study of flow over three rectangular rods is conducted by using Single Relaxation Time Lattice Boltzmann Method (SRT-LBM). The gap spacing (g) between the rods is varied from 0.5 to 7 with fixed Reynolds number i.e., Re = 150 (Re = $.U_{\infty}d/v$, where U_{∞} is uniform velocity, d is size of an object and v is the kinematic viscosity) and aspect ratio AR = 0.25 (where, AR = 1/w, 1 is the length and w is the width of the rods). Different flow structures, power spectra analysis of lift coefficients and force statistics are studied for this analysis at small, moderate and large gap spacing. Findings: The present study revealed that the gap spacing between the rods has a great impact on the flow behaviour. Therefore, different flow patterns are observed and some of them are outlined as, i). Shear layer reattachment (SLR), ii). Semi developed single bluff body (SDSBB), iii). Fully developed vortex shedding (FDVS), iv). Semi developed irregular vortex shedding, v) Fully generated two row single bluff body, vi). Semi developed irregular single bluff body (SDISBB) and vii). Critical flow (CF) patterns. The critical flow behaviour is observed at the gap spacing $(g_1, g_2) = (5, 2.5)$, (here, g_1 is the gap between upstream rods to middle one and g_2 is the gap between middle rods to downstream rectangular rod). In force statistics analysis, the values of C_{dmean} and Strouhal number are computed at different spacing ratios. The value of mean drags coefficients for all three rods at $g_1 = 3$ is approximately same, but at $g_1 = 1$ and 5, the maximum value of C_{dmean} is observed for C2 as compared to C1 and C3, that is 6.91. Furthermore, Strouhal number values are much less than the single rod (SR) value for all three chosen small, moderate and large gap spacing values. The maximum value of S_t is examined at $(g_1, g_2) = (1, 1)$ i.e 0.19.

Keywords: Drag Coefficient; Rectangular Rods, Single Relaxation Time Lattice Boltzmann Method, Vortex Shedding Suppression

Nomenclature

Cd	Drag coefficient
Cl	Lift coefficient
C _{dmean}	Mean drag coefficient
C _{drms}	Root-mean-square value of drag coefficient
с	Lattice speed
c _s	Speed of sound
e	Velocities direction
f	Vortex shedding frequency
h _i	Particle distribution function
h _i ^(eq)	Equilibrium distribution function
g	Gap spacing

- H Height of the computational domain
- L_u Upstream position
- L_d Downstream position
- Q Number of particles
- Re Reynolds number
- S. Strouhal number
- U Uniform inflow velocity

Greek Symbols

- v Kinematic viscosity
- ρ Fluid density
- ω_i Weighting coefficients
 - τ Relaxation time

1. Introduction

Fluid flow around more than one objects have attracted much attention in recent times due to its immense contribution in the field of engineering and industries. Its applications are in off-shore structures, heat exchangers, sea-crossing bridges, micro-electro-mechanical devices (MEMS) and other solar equipment. When flow interacts with these structures, it gives rise to complex evolvement of the vortices that ultimately produced drag and lift forces, that create noise and flow induced vibration. As a result damaging of structures occurs. In this regard, many experimental and numerical works has been done to overcome these noises using square/circular rods. These rods are placed in line, side-by-side and in staggered arrangements. As compared to circular or square rods, less work has been done on the rectangular rods. Rectangular rods behave differently as compared to circular rods. In case of rectangular rods, due to large circulation regions, broad wakes appear and separation point is fixed at the front leading edge. Many experimental and numerical studies are available in literature for flow past over a circular, square and rectangular objects. In [1] conducted an experimental study on the vortex shedding frequency of the various single rectangular rods at $70 \le \text{Re} \le 20,000$. The experiment revealed that the vortices reattached twice on the wall of rod at Re = 150-800 as the length to width ratio raises beyond 2. An experimental study of large eddy simulation (LES) for flow past rectangular-section performed by [2] revealed different flow characteristics at side ratio 0.4d to 4d and $Re = 10^5$. It is observed that flow showed local instabilities, which can lead to global instabilities. In [3] conducted 2-D experimental study of the turbulence behavior near wake flow behind the rectangular rod using hot wire anemometer. At different aspect ratio (AR), time averaged and fluctuating velocities are measured. As the aspect ratio increases, turbulence decreases at near wake flow. In [4] numerically investigated the flow around a rectangular rod using Finite Difference Method (FDM) at the range of parameters, $100 \le \text{Re} \le 120$ and $0.4d \le AR \le 8d$. The critical flow pattern was observed at AR = 2.8d and Re = 500-1200. Similar numerical simulation was performed by [5], in which laminar and turbulent flow around a rectangular rod was studied. As the side ratio changes from 2.1d to 6d, vortices alternatively reattached at downstream location of channel. In [6] numerically investigated the flow around a rectangular rod at different aspect ratios at Re = 400-30,000. A

change in the drag coefficients was found at AR = 0-1. The maximum value of the C_{dmean} was observed at AR = 0.6d. In [7] performed numerical investigation of unsteady flow around rectangular rod at low Reynolds numbers using Third-order and second-order Crank-Nicolson scheme by taking AR = 0.4d to 8d and Re = 100-200. Change in Strouhal number (S_t) was observed at Re = 200 and AR = 0.5d. In [8] performed three dimensional (3-D) unsteady flow around a rectangular rod at AR = 5:1, Re = 26,400, low Mach number ($M_{\infty} = 0.1$) and zero degree of attack. Detached Eddy Simulation (DES) was used for turbulence modelling. Result evaluated using hybrid mesh, attracted much attention toward the span-wise extension of the computational domain. To discretize the inviscid fluxes in governing equation, second order Central Difference Scheme (CDS) was applied. Results followed from DES exhibited that hybrid method is well applicable to all problems in which large value of Re is involved. In [9] numerically investigated the flow around rectangular rod using Incompressible lattice Boltzmann method (ILBM) with AR = 0.15d-4d and Re = 100 - 250. Drastic change in the drag coefficients was observed at AR = 0.15d-2d. Zhu Huang et al., [10] numerically simulated the flow around two rectangular rods in tandem at Re = 200 with AR = 1: 2 using Multi-Domain Pseudo-Spectral method (MPS). Flow is assumed to be laminar and uniform. They concluded that at the critical gap spacing of g = 5, drag coefficients approached its maximum value. Also, at the critical gap spacing vortices behind the two rods were shedding in an anti-phase mode. In[11] conducted an experimental study on the flow around two circular rods arranged tandemly. He noticed critical gap spacing at g = 3.5 with Re = 22000. In [12] experimentally exhibited flow around two inline circular rods and examined critical g at 3.5 with negative drag coefficients. In [13] experimentally study the flow around three inline circular rod sat 10900 \leq Re \leq 39200. It was examined that the drag coefficient is high for upstream rod as compared to downstream and middle stream rod has less value of the drag coefficient as compared to other two rods. In [14] numerically simulated the flow around two and three circular rods arranged inline at Re = 100 and 200. For the inline rods, at Re = 100 steady wake patterns observed and sparse Karman street formed at Re = 200 for g = 2. Islam et al. [15] numerically examined the flow around three square rods with varied gap spacing and critical flow pattern was observed at g = 3. All above mentioned studies revealed that less much concentration is paid for flow past three or

more than three rectangular rods at various gap spacing to study flow structure mechanism. Therefore, in present study, we examined the effect of unequal gap spacing for flow past over three rectangular rods a fixed Re and AR.

This paper is arranged as follows. Introduction is discussed in Section 1, numerical method is described in Section 2, Section 3 consists of problem statement, code validation study, effect of computational domain and impact of uniform inflow velocity will be discussed in Section 4. Results and discussions will be presented in Section 5. Finally, conclusion is drawn in Section 6.

2. Lattice Boltzmann Method

The numerical technique Single Relaxation Time-Lattice Boltzmann Method (SRT-LBM) for the particle distribution function h is applied to solve present problem. The lattice structure D2Q9, in which nine particles are involved is used here. The general form of LBE equation is given as

$$h_{i}(x+e_{i},t+1)-h_{i}(x,t) = -\frac{\left[h_{i}(x,t)-h_{i}^{(eq)}(x,t)\right]}{\tau}$$
(1)

where h_i is the particle distribution function and $h_i^{(eq)}$ is the particle equilibrium distribution function, e_i are the direction vectors and \mathbf{x} is the position vector.The equilibrium distribution function is calculated as (Wolf-Gladow [16])

$$h_{i}^{(eq)} = \rho \omega_{i} \left(1 + 3(e_{i}.u) + 4.5(e_{i}.u)^{2} - 1.5u^{2} \right)$$
(2)

The kinematic viscosity is to be calculated through the relation $v = \frac{2\tau - 1}{6}$. Eq (1) can be solved through BGK

approximation(Bhatnagar-Gross-Krook (BGK) [17]) i.e. through streaming and collision steps. During collision mass and momentum are conserved. In streaming particle at each node moved to its neighboring nodes at respective velocities(Wolf-Gladow [16]).

$$h_{i}^{*}(x,t) = h_{i}(x,t) - \left[\frac{h_{i}(x,t) - h_{i}^{*}(x,t)}{\tau}\right]$$
 (3)

Where h_i^* are the intermediate function.

$$h_{i}(x+e_{i},t+1)=h_{i}^{*}(x,t)$$
 (4)



Figure 1. D2Q9 particles velocity model.

The density (ρ) , velocity (\mathbf{u}) and pressure (p) at each computational node are calculated as

$$\rho = \sum g_i \qquad i = 0, \dots, 8$$
$$\boldsymbol{u} = \frac{1}{\rho} \sum g_i \boldsymbol{e}_i \qquad i = 1, \dots, 8$$
$$\boldsymbol{p} = \rho c_s^2$$

where c_s is the speed of sound $(=1/\sqrt{3})$.

3. Problem Description and Boundary Conditions

A two-dimensional (2-D) numerical study constitutes of three rectangular rods of width 'w' and diameter'd' between 2-D plane channel *is carried out* with upstream distance $L_u = 6d$, downstream distance $L_d = 25d$ and the height of the computational domain H = 11d. The vital parameter is spacing ratio between the rods and its value varies from g = 0.5 to 7 with fixed Re = 150 and AR = 0.25.C1, C2, C3 are upstream, downstream, and middle stream rods, respectively. The uniform inflow velocity ($\mathbf{u} = U_{\infty}$, $\mathbf{v} = 0$) is applied at the inlet and convective boundary condition $(\partial \mathbf{u}/\partial \mathbf{x} = \partial \mathbf{v}/\partial \mathbf{x} = 0)$ is applied at the outlet of the channel. The no-slip ($\mathbf{u} = \mathbf{v} = 0$) boundary conditions are applied





Fig. 2: A schematic diagram for flow past three rectangular rods

at the surface of the rod, as well on the top and bottom wall of the channel (Cheng et al. [18]). The fluid forces are calculated using momentum exchange method (Dazchi et al. [19]).

4. Code Validation Study, Effect of Computational Domain and Impact of Uniform Inflow Velocity

4.1. Code Validation Study

To check the validity of present code, the physical parameters such as C_{dmean} and S_t of single rectangular rod at Re = 80-200 are calculated and compared with the values of C_{dmean} and S_t are available in literature both experimentally and numerically. The values of C_{dmean} at Re = 150 found good agreement with the numerical work of Norberg [6] and experimental work of Okajima [1]. While, the values of the Strouhal number at Re = 140-160 found excellent agreement with experimental and numerical work of Okajima [1], Norberg [6], Robichaux [20], and Gera [21] etc, that assure the validity of present code, (see Figure 3).

4.2. Effect of Computational Domain

Different numerical simulations have been performed for flow around single rectangle rod at different values of L_u , L_d and H. The corresponding results obtained in the form of mean drag force and Strouhal number. First, varying L_u from 5d to 7d, keeping L_d and H fixed we noticed that values of the mean drag forces decreases while values



Figure 3. Comparison of S_t and C_{dmean} at different Reynolds number.

of the S_t also showed deterioration. In comparison with literature, S_t showed good experimental and numerical agreement with Norberg [6] and Sohanker et al. [7]. Secondly, we kept upstream distance and height of the computational domain constant while changing downstream location from 20d to 30d. We observed that mean drag forces showed minor difference in its values and Strouhal number remain the same. Their values are in good agreement with the Norberg, [6] and Sohanker et al. [7]. In third case, we kept L_u and L_d uniform and only made change in H from 9d to 12d. We came to know that C_{dmean} and S_t both have a decreasing behavior. Also, found great conformation with the work of Norberg [6] and Sohanker et al. [7]. Therefore, to proceed present work, we selected L_u=6d, L_d=25d and H = 11d, see Table 1.

4.3. Impact of Uniform Inflow Velocity

The uniform inflow velocity played a significant role in the flow structure phenomena. It has the power to delay

Cases	C _{dmean}	S _t	
Norberg [6]	-	0.150	
Okajima [1]	1.492	0.141	
Gera et al. [16]	1.411	0.141	
Sohanker et al. [7]	1.408	0.161	
Upstream Location (L _u)			
$L_{u} = 5d; L_{d} = 25d; H = 11d$	1.5410	0.1571	
$L_{u}^{u} = 6d; L_{d}^{u} = 25d; H = 11d$	1.5390	0.1560	
$L_{u} = 7d; L_{d} = 25d; H = 11d$	1.5320	0.1520	
Downstream Location (L _d)			
$L_{u} = 6d; L_{d} = 20d; H = 11d$	1.5320	0.1571	
$L_{u}^{u} = 6d; L_{d}^{u} = 25d; H = 11d$	1.5301	0.1571	
$L_u = 6d; L_d = 30d; H = 11d$	1.5210	0.1571	
Height of Computational Domain (H)			
$L_{u} = 6d; L_{d} = 25d; H = 9d$	1.5440	0.1615	
$L_{u} = 6d; L_{d} = 25d; H = 11$	1.5420	0.1571	
$L_u = 6d; L_d = 25d; H = 12d$	1.5401	0.1527	

Table 1.	Computational domain independence study
	at Re = 150.

or fast the vortex shedding phenomena. It can be easily noticed that, at $U_{\infty}=0.0268$ and $U_{\infty}=0.031$ the vortex shedding is very delayed as compared to other values of the uniform inflow velocity. At $U_{\infty}=0.05834$ the vortex shedding phenomena is very fast and easily disturbs the flow. So, selected $U_{\infty}=0.04385$ to solve this problem.



Figure 4. Comparison of different values of uniform inflow velocity.

5. Results and Discussions

To study the flow behavior over rectangular rods at unequal gap spacing by taking $g_{1=}$ 1, 3 and 5 and g_{2} = 0.5-7 with fixed Re = 150 and AR = 0.25, obtained results

are classified into three different flow regimes, i) Small wake regime, ii) Moderate wake regime, iii) Large wake regime.

5.1. Vorticity Contour Visualization

5.1.1 Small Wake Regime

This flow regime consists of $g_1 = 1$ and varying g_2 from 0.5 to 7. At small gap spacing when $g_1 = 1$ and $1 \le g_2 \le 1.5$, if $g_2 = 1$ as soon as the flow interacts with the downstream rod (C1), immediately gets attached to middle stream (C2) and upstream rod (C3) treating three as one rod. From the boundary of the C3, flow rolls up and generate vortices in an alternate fashion, we assign a name shear layer reattachment single bluff body (SLRSBB) flow pattern in (Figure 5(a)). This flow pattern is also observed experimentally by Abdollah et al. (2008) for Re = 8600 and 17400 and numerically by Shams et al. (2012) for Re = 100 - 250 for different aspect ratios in case of single rectangular cylinder. Similar behavior is observed for $g_2 = 1.5$ but number of vortices decreases quite sharply. At $g_1 = 1$ and $2 \le g_2 \le 2.5$, the flow arrived at C1 and vortices are completely generated in first gap between C1 and C2, but partial generation of vortices take place in second gap spacing between C2 and C3. After that vortices detached from C3 and move in an alternate fashion. That is why, it named as semi generated single bluff body (SGSBB) flow pattern in (Figure 5(b,c)). By increasing the gap spacing, keeping $g_1 = 1$ and $3 \le g_2 \le 5$, showed pattern is slightly different from what we observed. So, the main difference is that vortices increase in numbers and due to increase in gap, complete form of shed vortices appear.

The power spectra analysis of lift coefficients for three rods with varying gap spacing are given in graphs of S_t. Here C1, C2 and C3 are Showing vortex shedding frequency of upstream, middle stream and upstream rod. For shear layer reattachment single bluff body and semi generated single bluff body, the sharp peaks of the Strouhal number has been observed. We also observed that by increasing gap spacing from $g_2 = 1$ to 4, middle stream and upstream rods affected quite badly. Some multi peaks has been observed. By comparing their magnitude, major distortion has been observed at C3as shown in (Figure 6(a-f)). The maximum magnitude of St obtained at $g_{1=}$ 1and $g_2 = 2.5$ for all three selected rods in small wake regime clear from Figure 6(f).



Figure 5. Vorticity contour visualization for (a) Shear layer reattachment single bluff body, (b, c) Semi developed single bluff body.



Figure 6(a-f). Power spectra analysis of (a-c) Shear layer reattachment single bluff body, (d-f) Semi developed single bluff body.

5.1.2. Moderate Wake Regime

Due to enough spacing in first gap, at $g_1 = 3$ and $g_2 = 1$ vortices completely generated in first gap between C1 and C2. But partial generation of vortices takes place in second gap between C2 and C3. As soon as the vortices separated from C3, they immediately arranged themselves in an alternate way. Such behavior of flow is called as semi developed vortex shedding (SDIVS) flow pattern (see Figure 7(a) with respect to second gap. By increasing gap spacing up-to some extent means $g_2 = 1.5$, flow pattern shows similarity with the semi developed vortex shedding flow pattern, but major change occurred at C3. Flow becomes unsteady or irregular as it passes through C3 for a little while. After wards continue its motion in a typical way means alternatively. (see Figure 7(b)). At $g_1 = 3$ and $g_2 = 2$ and 3, vortices fully formed in first and second gap. But abruptly changes its shape as its goes through C3. They transformed themselves immediately in to two parallel rows for a short while, later move alternatively. This flow pattern due to its detreating behavior called as fully generated two row single bluff body (FGTRSBB) flow pattern (Figure 7(c, d)). At $g_1 = 3$ and $g_2 = 2.5$, 4, 5, the vortices completely generated in between the two gaps and may differ from each other by shape, size and in numbers. This flow pattern is categorized as fully developed vortex shedding (FDVS) flow pattern (Figure 7(e)). The Power spectra analysis of moderate wake regime show that semi developed vortex shedding, sharp peak has been noted. But for semi developed irregular vortex shedding flow pattern, flow changes its behavior from steady to unsteady as it emerged through C3. That is why C3 in the graph has noisier and disrupt behavior (see Figure 8(c, f, I, l, o). But overall sharp peak has been clearly seen. For fully generated two row single bluff body flow pattern, power spectrum analysis observed some harmonic peaks (see Figure 8). Since the vortices



Figure 7. Vorticity contour visualization for (a, b) Semi developed irregular vortex shedding, (c, d) Fully developed two row single bluff body and (e) Fully developed vortex shedding.





Figure 8(a-o). Power spectra analysis for (a, f) Semi developed irregular vortex shedding, (g- l) Fully developed two row single bluff body and (m-o) Fully developed vortex shedding.

fully generated through C1 so sharp peak for C1 is noted. Since C2 under gone complete vortex shedding with merging of vortices at downstream location, that why some secondary peaks with primary frequency has been observed. Initially two peaks have been noted, later converted themselves in to single sharp peak. Similarly, for fully developed two row single bluff body flow pattern, a strong distortion found at C3 because from the C3 flow strongly changed its behavior. For fully developed vortex shedding, power spectra analysis exhibited sharp peak but distortion has been there, due to change in strength, size and shape of the vortices, (see Figure 8(a-o)). In comparison of Strouhal number magnitude, for moderate wake regime, the maximum magnitude will be found for first cylinder at $g_1 = 3$ and $g_2 = 4$ for fully developed single bluff body flow.

5.1.3. Large Wake Regime

When $g_1 = 5$ and $1 \le g_2 \le 1.5$, there is enough space, therefore, vortices formed completely. But in the second gap, due to small space they could not fully evolved. From C_3 to onward, flow strongly became unsteady. This flow pattern is called semi developed irregular single bluff body (SDISBB) flow pattern in (Figure 9(a)). This flow pattern also has been observed for small wake regime. At $g_2 = 1.5$ similar pattern has been noted, but strength of vortices improved. At $g_1 = 5$ and $g_2 = 2.5$ and6, here two extremes of gap spacing have been observed. Flow became highly irregular as it passed from C3 and scattered in different direction. We called this pattern as critical flow (CF) pattern see (Figure 9(b)).Igarashi [24] experimentally investigated the critical gap spacing (critical flow pattern) with similar flow and force coefficients characteristics for flow past two tandem circular cylinders at g = 3.5 and Re $= 2.2 \times 10^4$. Kim et al. [25] experimentally observed that g = 2.5 is the critical gap spacing for flow past two tandem square cylinders (AR = 1). At $g_1 = 5$ and $2 \le g_2 \le 5$, flow showed almost similar behavior with some minor differences. At C3, flow became irregular for short period. Only size, shape and number of vortices changes. We assign a name as fully developed irregular single bluff body (FDVS) flow pattern as shown in (Figure 9(c)). This flow pattern also examined for moderate wake regime at $g_2 = 2.5$, 4, 5. The power spectrum analysis of large wake regime of different flow pattern showed distortion. Irregularities, multiple peaks and in some cases secondary peaks also observed. Multi-peaks represent the irregular behavior. While, secondary peaks and distortion enable us to conclude that critical flow pattern has been observed at certain points in (Figure 10(a-i)).

5.2. Force Statistics

To study the effect of unequal gap spacing, we calculated the physical parameters, such as C_{dmean} and Strouhal numbers against g_2 by fixing g_1 . The change in the mean drag coefficients by keeping $g_1 = 1$ and $1 \le g_2 \le 6$ for three rods, showed that at $g_1 = 1$, the value of Cd1 synchronizes with the C_{dmean} of single rod (SC) as shown in (Figure 11(a)). At the end, showed slight change in behavior at $g_2 = 6$, having value 2. 1035. The Cd2 showed sudden



Figure 9. Vorticity contour visualization (a) Semi developed irregular single bluff body, (b) Critical flow pattern, (c) Fully developed vortex shedding



Figure 10(a-i). Power spectra analysis for (a-c) Semi developed irregular single bluff body, (d-f) Critical flow pattern, (g-i) Fully developed vortex shedding.

jump at $g_2 = 1.5$ and reached at its maximum value of 2.9841. Afterwards, Cd2 decreased quiet sharply and found minimum value at $g_2 = 4$. Cd3 showed very unusual

behavior as compared to Cd1 and Cd2. Cd3 reached its maximum value at $g_2 = 1.5$ that is 3.9064. But as soon as it reached its maximum value, one can observe sudden



Figure 11(a-c). Comparison of C_{dmean} for (a) Small wake regime, (b) moderate wake regime and (c) large wake regime at different gap spacing.

decline for the remaining gap spacing. In comparison with SC, Cd2 and Cd3 have largest value at certain points. Other than this all Cdmean of C1, C2, C3 have smallest values than single rod. The change in the mean drag coefficients for $g_1 = 3$, revealed that all the three rods have some sort of overlapping of the values of the C_{dmean} that is clearly observable in the graph. Largest value of the $\mathrm{C}_{_{\mathrm{dmean}}}$ has been noted twice i.e. $(g_1, g_2) = (3, 2)$ and $(g_1, g_2) = (3, 3)$ 3) and smallest value has been observed at $g_2 = 2.5$. In comparison with single rod, all the values of C_{dmean} of three rectangular rods went through major decline as shown in (Figure 11(b)). From the graph of mean drag coefficients of $g_1 = 5$, one can notice that drag coefficients found its maximum value at $g_2 = 1.5$ and it is approximately 6.8985 for C2. It is because due to sufficient gap spacing. effect of C1 on C2 reduces After that, declined in the values is quite sharp, but still larger than the SC. While C1 and C2 have smallest value of $\mathrm{C}_{\mathrm{dmean}}$ but still much larger than the single rod in (Figure 11(c)). The comparison of the St with single rod at $g_1 = 1$, exhibited that the S_t of C1, C2and C3 have small values with minor differences in between them. But all the values of Strouhal number for three rods are less than the single rod (see Figure 12(a)). While, at g_1 = 3, all the S_t of rods have special kind of synchronization in the values. Still there are less than the S_t of the SC shown in Figure 12(b). When we further increase the gap spacing at $g_1 = 5$, we got that all the S_t of three rods are so small than the S_t of single rod as shown in Figure 12(c). All the values of the S_t of C1, C2 and C3 coincide with each other with minor differences.

6. Conclusions

A numerical investigation of the flow around three rectangular rods using SRT-LBM with Re = 150, AR = 0.25 and varying the gap spacing from $g_1 = 1$, 3 and 5 and $g_2 = 0.5$ to 7 revealed different results. Some are outlines as

- Seven different flow patterns has been observed. i). Shear layer reattachment single bluff body, ii). Semi developed single bluff body, iii). Fully developed vortex shedding, iv). Semi developed irregular vortex shedding, v) Fully generated two row single bluff body, vi). Semi developed irregular single bluff body, vii). Critical flow for small, moderate and large wake regime.
- The critical flow pattern has been observed when $(g_1, g_2) = (5, 2.5)$ and $(g_1, g_2) = (5, 6)$, due to sufficiently

large gap spacing flow is highly unsteady and become critical.

- The power spectra analysis showed that by increasing the gap spacing flow becomes unsteady and irregular, that is why multipeak has been observed. The amplitude of the values of the Strouhal number varied with the increase in the gap spacing. At critical flow, Strouhal number has primary and secondary peaks which depicts the unsteadiness in the flow.
- The change in the mean drag coefficient showed that maximum value of C_{dmean} is observed at $(g_1, g_2) = (1, 1.5)$ that is 6.9 and after that showed C_{dmean} decreases at the remaining gap spacing for small wake region. While at $g_1 = 3$, mixed trend of the values of the C_{dmean} with varying gap spacing has been observed. When $g_1 = 5$, all the values of the C_{dmean} showed increasing and decreasing behavior and larger than the single rod value.
- All the values of the Strouhal numbers are synchronized with each other but all are less than the Strouhal numbers of the single rods for all three chosen small moderate and large gap spacing. The maximum value of St is examined at $(g_1, g_2) = (1, 1)$ i.e., 0.19.

7. References

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