ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, Vol 10(31), DOI: 10.17485/ijst/2017/v10i31/116145, August 2017 ISSN (Online) : 0974-5645

Reliability and Availability Evaluation of a Series-Parallel System Subject to Random Failure

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Abstract

Objective: To compute the reliability/availability of casting system, a part of steel industry with a view to increase its productivity. **Methods/Analysis:** The reliability of casting system of a steel industry, consisting of four units, has been computed using Supplementary Variable Technique (SVT) by keeping the failure rates constant and varying the repair rates. From the transition diagram of the casting system, Chapman-Kolmogorov differential equations have been developed using mnemonic rule which are further solved using Lagrange's method. The transient state availability of the system has been computed by Runge-Kutta fourth order method using MATLAB. Mean Time Between Failures (MTBF) has been calculated numerically. **Findings:** Besides increasing the plant availability and hence production, the findings of this paper may help in maintenance planning and scheduling of the said system. **Novelty/Improvement:** Reliability of the casting system of a steel industry has not been discussed so far.

Keywords: Lagrange's Method, Runge-Kutta, Supplementary Variable Technique, MATLAB, MTBF

1. Introduction

The ultimate aim of any manufacturing enterprise is to increase production and hence profitability, which, in turn, can be achieved only when the plant runs for longer duration of time without much interruptions. Modern day plants consist of complex systems with some of their units as standby using perfect switching. To evaluate the performance of a system, knowledge of the factors which affect it is required. Reliability assessment is an integral part of performance analysis particularly in process industries.

These issues have been addressed by many researchers using different techniques. [1]used supplementary variable technique to obtain Laplace transforms of various state probabilities to investigate the mathematical model of a system. [2] carried out the reliability analysis of the crushing system of a sugar mill. The problem was formulated using supplementary variable technique and solved using Lagrange's method. [3]considered a system with N operating units and M warm standby units having 'R' repair facilities. Supplementary variable technique was used to formulate the problem and solved using Lagrange's method. [4]analyzed the reliability of computer network system by supplementary variable technique and concluded that steady state transition probability degrades slowly with time. [5]discussed the reliability of an N-unit series repairable system and derived system availability, the idle probability of the repairman and the rate of service for customers using a supplementary variable technique and Laplace transform. [6]described the availability of combed sliver production system, a part of yarn production plant. The problem was formulated using supplementary variable technique and probability consideration. [7]studied about cost estimation of nuclear power generation plant and used supplementary variable

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technique for mathematical formulation of the model. Laplace transform was utilized to solve the mathematical equations. [8]computed the reliability of poly-tube manufacturing plant using supplementary variable technique. [9]obtained the integro-differential equations governing the behaviour of the system by using supplementary variables method, probability arguments and limiting transitions. [10]worked on coherent systems and series connection of k-out-of-n standby subsystems with exponentially distributed component lifetimes and analyzed system reliability, mean time to failure, and steady-state availability as a function of the component failure rates. [11]presented a novel method for availability analysis of an engineering system incorporating waiting time to repair by supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula.

Transient state availability has been evaluated by many researchers using different numerical methods. [12]implemented an implicit Runge-Kutta method to solve systems of nonlinear equations. [13]compared three numerical methods for reliability calculations i.e. Markov, third order implicit Runge-Kutta method and acyclic Markov chain evaluator algorithm. [14]presented two different methods i.e. LUD (Lower Upper Decomposition) and Runge-Kutta to calculate the steady-state probabilities and frequencies of two different engineering models. [15]computed reliability, availability and mean time before failure of a plastic-pipe manufacturing plant. The differential equations were solved numerically using Runge-Kutta fourth order method. [16]discussed a procedure for finding the formula of 5th order Runge-Kutta method and then applied it to find the numerical solutions of ordinary differential equations. [17]assessed the availability of crank-case manufacturing system using Lagrange method and Runge-Kutta method to solve the partial and ordinary differential equations respectively. [18]suggested a Runge-Kutta method based on the sparse matrix storage scheme to numerically solve and analyze the reliability model. [19]dealt with the numerical solution of initial value problems (IVPs), for systems of ordinary differential equations (ODEs), by an explicit fourth-order Runge–Kutta method. [20]presented a modified Runge-Kutta algorithm which yields a conservative estimate (overestimate) of the crack size for fatigue crack growth even for large integration step sizes. [21]constructed an explicit Runge-Kutta method for solving directly fourth-order ordinary differential equations (ODEs) and denoted it as (RKFD).

Looking at the previous work in the related field, it has been observed that reliability/availability of the casting system of a steel industry has never been worked upon. Hence, in a way, this study is first of its kind. In order to compute the reliability of the said system, we have considered that the system is subjected to constant failure rate and variable repair rate. Supplementary variable technique has been used for reliability modelling of the system. Availability has also been calculated by taking constant failure and repair rates using Runge-Kutta method. Finally, criticality analysis of all the systems has been done to get some idea of the maintenance priority.

This paper is divided in 5 sections. Present section is introductory in nature. Section 2 consists of brief description of the system, various notations and assumptions used in the analysis. Mathematical modelling of the system is done in Section 3. Chapman-Kolmogorov equations of the casting system are developed using SVT assuming constant failure and variable repair rates. The equations have also been developed keeping both, failure and repair rates constant. In Section 4, for analyzing the transient state availability, the system of differential equations is solved using Runge-Kutta fourth order method with the help of MATLAB software package and the effects of failure and repair rates of various combinations of different subsystems on the casting system have been evaluated. MTBF has been calculated using Simpson's 3/8 rule, at the end of each row in Table 1-6 to give an insight of the maintenance time available. Section 5 gives us the conclusion of the analysis done in previous section.

2. System Description, Various Notations And Assumptions

The aim of casting system is to produce bloom and slabs as the final product with molten metal as the input. This system consists of four principal subsystems namely, transfer laddle, continuous casting machine, shot blasting machine and grinding machine. All the units are subject to major failure except grinding machine, which seldom fails and is also supported by stand-by unit with perfect switching. Hence it has not been considered for analysis. Figure 1 gives us the flow chart of casting process.

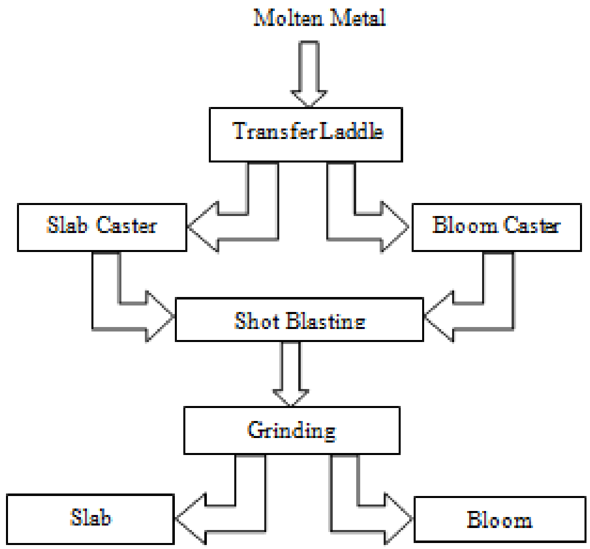

Figure 1. Flow chart of the casting process.

2.1 System Description

- Sub-system A (Transfer Laddle)**:** It is basically a bucket which transfers molten metal from one place to another. Flux can be easily extracted by adjusting these laddles. This subsystem is having two units in parallel. System keeps on working at reduced capacity on failure of one unit and fails completely only when both units fail.
- Sub-system B (Continuous Casting Machine): Molten steel is obtained by opening the nozzle of the teeming laddle which is then made to flow through a shroud into the tundish. The function of the shroud is to protect the steel from coming in contact with the atmosphere. The tundish acts as a reservoir in sequencing and maintains a supply of molten steel to the caster even when the laddle is being changed. It is having two units in parallel. If one unit fails, system keeps on working at reduced capacity and complete failure occurs only when both units fail.
- Sub-system C (Shot Blasting): Its function is removal of the unwanted surface oxide layer from the slab by charging shots on the surface with sufficient velocity. It consists of two units. If one unit fails, system's capacity reduces. Major failure occurs when both units fail.

2.2 Notations

A, B, C Indicate that the respective subsystems are working at full capacity

a, b, c Indicate that the respective subsystems are in failed state

A', B', C' Indicate that the respective subsystems are working at reduced capacity

$\alpha_i (i=1\ to\ 3)$ Indicate the failure rates of subsystems A, B and C respectively

$\beta_i (i=1\ to\ 3)$ Indicate the repair rates of subsystem A, B and C respectively

P_0(t) Denotes the probability that at time't', all the units are working

P_i(x,t) Denotes the probability that at time't', the system is in state *i* and having an elapsed repair time x

2.3 Assumptions

Present analysis is based on following assumptions:

- Failure and repair rates are constant and independent of each other.
- In case of assessment of availability using SVT, repair rates are considered variable and failure rates as constant.
- A repaired unit is as good as new.
- Service and repair/maintenance and replacement facilities are always available.
- There are no simultaneous failures.
- System may work at reduced capacity.

3. Mathematical Formulation of the System

To determine the reliability of the said system, we develop Chapman-Kolmogorov differential equations by applying SVT. Probability considerations, using mnemonic rule, give us the following set of differential equations associated with the transition diagram Figure 2 of the system at time (t+Δt):

$$P_0(t+\Delta t)=\left[1-\alpha_1\Delta t-\alpha_2\Delta t-\alpha_3\Delta t\right]P_0(t)+\int\beta_1P_1(x,t)\,dx\Delta t$$
$$+\int\beta_2P_4(x,t)\,dx\Delta t+\int\beta_3P_6(x,t)\,dx\Delta t$$

$$P_0(t+\Delta t)-P_0(t)=-\left[\alpha_1\Delta t+\alpha_2\Delta t+\alpha_3\Delta t\right]P_0(t)+\int\beta_1P_1(x,t)\,dx\Delta t$$
$$+\int\beta_2P_4(x,t)\,dx\Delta t+\int\beta_3P_6(x,t)\,dx\Delta t$$

Dividing both sides by Δt, we get

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t}=-\left[\alpha_1+\alpha_2+\alpha_3\right]P_0(t)+$$

$$\int \beta_1 P_1(x,t)\,dx + \int \beta_2 P_4(x,t)\,dx + \int \beta_3 P_6(x,t)\,dx$$

$$\left[\frac{\partial}{\partial t} + L_0\right] P_0(t) = M_0(t) \quad (1)$$

Similarly, we can write the differential equations for other states as follows:

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_1(x)\right] P_1(x,t) = M_1(x,t) \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_2(x)\right] P_2(x,t) = M_2(x,t) \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_3(x)\right] P_3(x,t) = M_3(x,t) \quad (4)$$

$$\left|\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_4(x)\right| P_4(x,t) = M_4(x,t) \quad (5)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_5(x)\right] P_5(x,t) = M_5(x,t) \quad (6)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_6(x)\right] P_6(x,t) = M_6(x,t) \quad (7)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + L_7(x)\right] P_7(x,t) = M_7(x,t) \quad (8)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_1(x)\right] P_j(x,t) = 0; \quad j = 9,10,11,17 \quad (9)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2(x)\right] P_k(x,t) = 0; \quad k = 12,13,14,18 \quad (10)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_3(x)\right] P_l(x,t) = 0; \quad l = 8,15,16,19 \quad (11)$$

Where,

$$L_0 = \sum_{i=1}^{3} \alpha_i$$

$$L_1(x) = \sum_{i=1}^{3} \alpha_i + \beta_1(x)$$

$$L_2(x) = \sum_{i=1}^{3} \alpha_i + \beta_1(x) + \beta_3(x)$$

$$L_3(x) = \sum_{i=1}^{3} \alpha_i + \beta_1(x) + \beta_2(x)$$

$$L_4(x) = \sum_{i=1}^{3} \alpha_i + \beta_2(x)$$

$$L_5(x) = \sum_{i=1}^{3} \alpha_i + \beta_2(x) + \beta_3(x)$$

$$L_6(x) = \sum_{i=1}^{3} \alpha_i + \beta_3(x)$$

$$L_7(x) = \sum_{i=1}^{3} \alpha_i + \beta_1(x) + \beta_2(x) + \beta_3(x)$$

$$M_0(t) = \int P_1(x,t)\beta_1(x)\,dx + \int P_4(x,t)\beta_2(x)\,dx + \int P_6(x,t)\beta_3(x)\,dx$$

$$M_1(x,t) = \alpha_1 P_0(t) + \int P_2(x,t)\beta_3(x)\,dx + \int P_3(x,t)\beta_2(x)\,dx + \int P_{10}(x,t)\beta_1(x)\,dx$$

$$M_2(x,t) = \alpha_3 P_1(t) + \alpha_1 P_6(t) + \int P_7(x,t)\beta_2(x)\,dx + \int P_8(x,t)\beta_3(x)\,dx + \int P_9(x,t)\beta_1(x)\,dx$$

$$M_3(x,t) = \alpha_2 P_1(t) + \alpha_1 P_4(t) + \int P_7(x,t)\beta_3(x)\,dx + \int P_{11}(x,t)\beta_1(x)\,dx + \int P_{12}(x,t)\beta_2(x)\,dx$$

$$M_4(x,t) = \alpha_2 P_0(t) + \int P_3(x,t)\beta_1(x)\,dx + \int P_5(x,t)\beta_3(x)\,dx + \int P_{13}(x,t)\beta_2(x)\,dx$$

$$M_5(x,t) = \alpha_3 P_4(t) + \alpha_2 P_6(t) + \int P_7(x,t)\beta_1(x)\,dx + \int P_{14}(x,t)\beta_2(x)\,dx + \int P_{15}(x,t)\beta_3(x)\,dx$$

$$M_6(x,t) = \alpha_3 P_0(t) + \int P_2(x,t)\beta_1(x)\,dx + \int P_5(x,t)\beta_2(x)\,dx + \int P_{16}(x,t)\beta_3(x)\,dx$$

$$M_7(x,t) = \alpha_2 P_2(t) + \alpha_3 P_3(t) + \alpha_1 P_5(t) + \int P_{17}(x,t)\beta_1(x)\,dx + \int P_{18}(x,t)\beta_2(x)\,dx + \int P_{19}(x,t)\beta_3(x)\,dx$$

- Initial Conditions

$$P_0(0) = 1$$

$$P_i(x,0) = 0 \qquad (i = 1,2,3\ldots\ldots\ldots\ldots.19)$$

- Boundary Conditions

$$P_1(0,t) = \alpha_1 P_0(t),\ P_2(0,t) = \int \alpha_3 P_1(x,t)\,dx + \int \alpha_1 P_6(x,t)\,dx,$$
$$P_3(0,t) = \int \alpha_1 P_4(x,t)\,dx + \int \alpha_2 P_1(x,t)\,dx,\ P_4(0,t) = \alpha_4 P_0(t),$$
$$P_5(0,t) = \int \alpha_3 P_4(x,t)\,dx + \int \alpha_2 P_6(x,t)\,dx,\ P_6(0,t) = \alpha_3 P_0(t),$$

$P_7(0,t)=\int\alpha_2 P_2(x,t)dx+\int\alpha_1 P_5(x,t)dx+\int\alpha_3 P_3(x,t)dx,$

$P_8(0,t)=\int\alpha_3 P_2(x,t)dx,\ P_9(0,t)=\int\alpha_1 P_2(x,t)dx,$

$P_{10}(0,t)=\int\alpha_1 P_1(x,t)dx,\ P_{11}(0,t)=\int\alpha_1 P_3(x,t)dx,$

$P_{12}(0,t)=\int\alpha_2 P_3(x,t)dx,\ P_{13}(0,t)=\int\alpha_2 P_4(x,t)dx,$

$P_{14}(0,t)=\int\alpha_2 P_5(x,t)dx,\ P_{15}(0,t)=\int\alpha_3 P_5(x,t)dx,$

$P_{16}(0,t)=\int\alpha_3 P_6(x,t)dx,\ P_{17}(0,t)=\int\alpha_1 P_7(x,t)dx,$

$P_{18}(0,t)=\int\alpha_2 P_7(x,t)dx,\ P_{19}(0,t)=\int\alpha_3 P_7(x,t)dx.$

Set of differential equations from (1) to (11) along with initial conditions and boundary conditions is called Chapman-Kolmogorov differential difference equations. Equation (1) is a linear differential equation of first order and Equations (2) to (11) are linear partial differential equations of first order (Lagrange's type). All these equations have been solved using Lagrange's method. The probabilities of each state and expression of availability has been derived as follows:

$$P_0(t)=e^{-L_0 t}\left[1+\int M_0(t)e^{L_0 t}dt\right]$$

$$P_1(x,t)=e^{-\int L_1(x)dx}\left[\int M_1(x,t)e^{\int L_1(x)dx}dx+\alpha_1 P_0(t-x)\right]$$

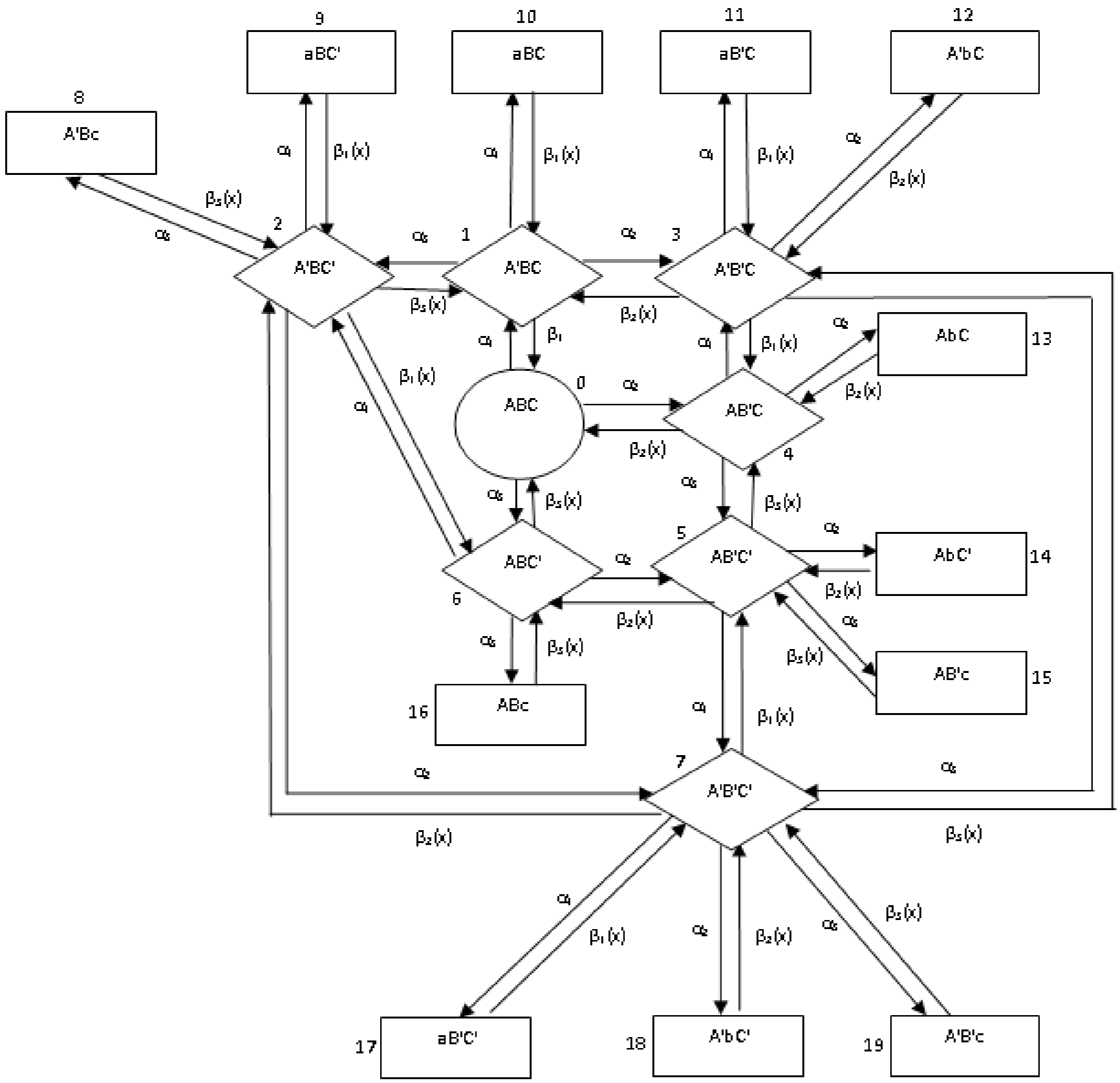

Figure 2. Transition diagram of casting system.

$$P_2(x,t)=e^{-\int L_2(x)dx}\left[\int M_2(x,t)e^{\int L_2(x)dx}dx+\int \alpha_3 P_1(x,t-x)dx+\int \alpha_1 P_6(x,t-x)dx\right]$$

$$P_3(x,t)=e^{-\int L_3(x)dx}\left[\int M_3(x,t)e^{\int L_3(x)dx}dx+\int \alpha_2 P_1(x,t-x)dx+\int \alpha_1 P_4(x,t-x)dx\right]$$

$$P_4(x,t)=e^{-\int L_4(x)dx}\left[\int M_4(x,t)e^{\int L_4(x)dx}dx+\alpha_2 P_0(t-x)\right]$$

$$P_5(x,t)=e^{-\int L_5(x)dx}\left[\int M_5(x,t)e^{\int L_5(x)dx}dx+\int \alpha_3 P_4(x,t-x)dx+\int \alpha_2 P_6(x,t-x)dx\right]$$

$$P_6(x,t)=e^{-\int L_6(x)dx}\left[\int M_6(x,t)e^{\int L_6(x)dx}dx+\alpha_3 P_0(t-x)\right]$$

$$P_7(x,t)=e^{-\int L_7(x)dx}\left[\int M_7(x,t)e^{\int L_7(x)dx}dx+\right.$$

$$\left.\int \alpha_2 P_2(x,t-x)dx+\int \alpha_3 P_3(x,t-x)dx+\int \alpha_1 P_5(x,t-x)dx\right]$$

$$P_8(x,t)=e^{-\int \beta_3(x)dx}\int \alpha_3 P_2(x,t-x)dx$$

$$P_9(x,t)=e^{-\int \beta_1(x)dx}\int \alpha_1 P_2(x,t-x)dx$$

$$P_{10}(x,t)=e^{-\int \beta_1(x)dx}\int \alpha_1 P_1(x,t-x)dx$$

$$P_{11}(x,t)=e^{-\int \beta_1(x)dx}\int \alpha_1 P_3(x,t-x)dx$$

$$P_{12}(x,t)=e^{-\int \beta_2(x)dx}\int \alpha_2 P_3(x,t-x)dx$$

$$P_{13}(x,t)=e^{-\int \beta_2(x)dx}\int \alpha_2 P_4(x,t-x)dx$$

$$P_{14}(x,t)=e^{-\int \beta_2(x)dx}\int \alpha_2 P_5(x,t-x)dx$$

$$P_{15}(x,t)=e^{-\int \beta_3(x)dx}\int \alpha_3 P_5(x,t-x)dx$$

$$P_{16}(x,t)=e^{-\int \beta_3(x)dx}\int \alpha_3 P_6(x,t-x)dx$$

$$P_{17}(x,t)=e^{-\int \beta_1(x)dx}\int \alpha_1 P_7(x,t-x)dx$$

$$P_{18}(x,t)=e^{-\int \beta_2(x)dx}\int \alpha_2 P_7(x,t-x)dx$$

$$P_{19}(x,t)=e^{-\int \beta_3(x)dx}\int \alpha_3 P_7(x,t-x)dx$$

Finally, the expression of time dependent availability A(t) is obtained by summation of probabilities of all the working states and reduced capacity states, i.e.

$$A(t)=P_0(t)+\int\sum_{i=1}^{7}P_i(x,t)\,dx \tag{12}$$

Availability expression of the casting system as given by equation (12) can be solved using constant failure rates and variable repair rates from the concerned plant.

3.1 Availability of the system when both failure as well as repair rates are constant

As the mechanical components are mainly subjected to random failures and the repair time is also not consistent, therefore, failure and repair rates may be considered constant. In this case, the system of Equations (1) to (11) can be represented as follows:

$$P_0(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i\right]=P_1(t)\beta_1+P_4(t)\beta_2+P_6(t)\beta_3 \tag{13}$$

$$P_1(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\beta_1\right]=P_{10}(t)\beta_1+P_3(t)\beta_2+P_2(t)\beta_3+P_0(t)\alpha_1 \tag{14}$$

$$P_2(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\beta_1+\beta_3\right]=P_9(t)\beta_1+P_7(t)\beta_2+P_8(t)\beta_3+P_6(t)\alpha_1+P_1(t)\alpha_3 \tag{15}$$

$$P_3(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\beta_1+\beta_2\right]=P_{11}(t)\beta_1+P_{12}(t)\beta_2+P_7(t)\beta_3+P_4(t)\alpha_1+P_1(t)\alpha_2 \tag{16}$$

$$P_4(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\beta_2\right]=P_3(t)\beta_1+P_{13}(t)\beta_2+P_5(t)\beta_3+P_0(t)\alpha_2 \tag{17}$$

$$P_5(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\beta_2+\beta_3\right]=P_7(t)\beta_1+P_{14}(t)\beta_2+P_{15}(t)\beta_3+P_6(t)\alpha_2+P_4(t)\alpha_3 \tag{18}$$

$$P_6(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\beta_3\right]=P_2(t)\beta_1+P_5(t)\beta_2+P_{16}(t)\beta_3+P_0(t)\alpha_3 \tag{19}$$

$$P_7(t)\left[\frac{\partial}{\partial t}+\sum_{i=1}^{3}\alpha_i+\sum_{j=1}^{3}\beta_j\right]=P_{17}(t)\beta_1+P_{18}(t)\beta_2+P_{19}(t)\beta_3+P_5(t)\alpha_1+P_2(t)\alpha_2+P_3(t)\alpha_3 \tag{20}$$

$$P_i(t)\left[\frac{\partial}{\partial t}+\beta_1\right]=P_j(t)\alpha_1 \tag{21}$$

$for\; i=9, j=2; i=10, j=1; i=11, j=3; i=17, j=7$

$$P_i(t)\left[\frac{\partial}{\partial t}+\beta_2\right]=P_j(t).\alpha_2 \tag{22}$$

$for\; i=12, j=3; i=13, j=4; i=14, j=5; i=18, j=7$

$$P_i(t)\left[\frac{\partial}{\partial t}+\beta_3\right]=P_j(t)\alpha_3 \quad (23)$$

$$for\, i=8, j=2; i=15, j=5; i=16, j=6; i=19, j=7$$

- Initial Conditions:

$$P_i(t)=1 \quad for\ i=0$$
$$=0 \quad for\ i\neq 0$$

To examine the effect of failure and repair rates on the availability in transient state, the system of differential equations (13) to (23) with initial conditions has been solved numerically using Runge-Kutta fourth order method. Analysis has been done for a period of 360 days divided over an interval of 30 days and the data has been tabulated in Tables 1-6. These tables present the effect of failure and repair rates of various subsystems on the reliability of the system. MTBF, which has been computed using Simpson's3/8 rule, with corresponding failure rates, has been given in the last row of each table.

Table 1. Effect of failure rate of transfer laddle (α_1) on availability

Time (days) ↓	α_1 →	0.016	0.018	0.02	0.022	0.024
30		0.9583	0.9519	0.9450	0.9376	0.9299
60		0.9379	0.9291	0.9197	0.9099	0.8997
90		0.9323	0.9230	0.9132	0.9029	0.8922
120		0.9308	0.9214	0.9115	0.9011	0.8904
150		0.9303	0.9209	0.9110	0.9006	0.8899
180		0.9302	0.9208	0.9109	0.9005	0.8897
210		0.9302	0.9208	0.9108	0.9004	0.8897
240		0.9301	0.9208	0.9108	0.9004	0.8897
270		0.9301	0.9207	0.9108	0.9004	0.8897
300		0.9301	0.9207	0.9108	0.9004	0.8897
330		0.9301	0.9207	0.9108	0.9004	0.8897
360		0.9301	0.9207	0.9108	0.9004	0.8897
MTBF		336.92	333.77	333.43	326.93	321.98

Table 2. Effect of failure rate of continuous casting machine (α_2) on availability

Time (days) ↓	α_2 →	0.0083333	0.0097221	0.0111111	0.0124999	0.0138888
30		0.9583	0.9548	0.9509	0.9466	0.9420
60		0.9379	0.9322	0.9260	0.9192	0.9120
90		0.9323	0.9259	0.9189	0.9113	0.9032
120		0.9308	0.9241	0.9169	0.9090	0.9007
150		0.9303	0.9236	0.9163	0.9083	0.8999
180		0.9302	0.9235	0.9161	0.9081	0.8997
210		0.9302	0.9234	0.9160	0.9081	0.8996
240		0.9301	0.9234	0.9160	0.9080	0.8995
270		0.9301	0.9234	0.9160	0.9080	0.8995
300		0.9301	0.9234	0.9160	0.9080	0.8995
330		0.9301	0.9234	0.9160	0.9080	0.8995
360		0.9301	0.9234	0.9160	0.9080	0.8995
MTBF		336.92	334.73	332.33	329.72	326.95

Table 3. Effect of failure rate of shot blasting machine (α_3) on availability

Time (days) ↓ / α_3 →	**0.0059523**	**0.0065476**	**0.0071428**	**0.0077380**	**0.0083333**
30	0.9583	0.9577	0.9570	0.9562	0.9554
60	0.9379	0.9371	0.9362	0.9352	0.9342
90	0.9323	0.9314	0.9305	0.9296	0.9285
120	0.9308	0.9299	0.9290	0.9281	0.9270
150	0.9303	0.9295	0.9286	0.9276	0.9266
180	0.9302	0.9294	0.9285	0.9275	0.9265
210	0.9302	0.9293	0.9284	0.9275	0.9264
240	0.9301	0.9293	0.9284	0.9274	0.9264
270	0.9301	0.9293	0.9284	0.9274	0.9264
300	0.9301	0.9293	0.9284	0.9274	0.9264
330	0.9301	0.9293	0.9284	0.9274	0.9264
360	0.9301	0.9293	0.9284	0.9274	0.9264
MTBF	336.92	336.64	336.33	336.00	335.65

Table 4. Effect of repair rate of transfer laddle (β_1) on availability

Time (days) ↓ / β_1 →	**0.0666666**	**0.0749999**	**0.0833333**	**0.0916666**	**0.10**
30	0.9583	0.9615	0.9643	0.9666	0.9687
60	0.9379	0.9443	0.9493	0.9533	0.9566
90	0.9323	0.9398	0.9454	0.9498	0.9533
120	0.9308	0.9385	0.9443	0.9488	0.9523
150	0.9303	0.9382	0.9440	0.9485	0.9520
180	0.9302	0.9381	0.9439	0.9484	0.9519
210	0.9302	0.9380	0.9439	0.9483	0.9518
240	0.9301	0.9380	0.9439	0.9483	0.9518
270	0.9301	0.9380	0.9438	0.9483	0.9518
300	0.9301	0.9380	0.9438	0.9483	0.9518
330	0.9301	0.9380	0.9438	0.9483	0.9518
360	0.9301	0.9380	0.9438	0.9483	0.9518
MTBF	336.92	339.45	341.35	342.82	343.98

Table 5. Effect of repair rate of continuous casting machine (β_2) on availability

Time (days) ↓ / β_2 →	**0.05**	**0.0567307**	**0.0634615**	**0.0701923**	**0.0769230**
30	0.9583	0.9594	0.9604	0.9612	0.9620
60	0.9379	0.9407	0.9429	0.9447	0.9462
90	0.9323	0.9360	0.9388	0.9410	0.9427
120	0.9308	0.9349	0.9379	0.9402	0.9419
150	0.9303	0.9346	0.9377	0.9400	0.9418
180	0.9302	0.9345	0.9376	0.9400	0.9417
210	0.9302	0.9345	0.9376	0.9399	0.9417
240	0.9301	0.9345	0.9376	0.9399	0.9417
270	0.9301	0.9345	0.9376	0.9399	0.9417
300	0.9301	0.9345	0.9376	0.9399	0.9417
330	0.9301	0.9345	0.9376	0.9399	0.9417
360	0.9301	0.9345	0.9376	0.9399	0.9417
MTBF	336.92	338.26	339.23	339.96	340.54

Table 6. Effect of repair rate of shot blasting machine (β_3) on availability

Time (days) ↓ / β_3 →	0.0833333	0.0982142	0.1130952	0.1279761	0.1428571
30	0.9583	0.9590	0.9594	0.9598	0.9601
60	0.9379	0.9389	0.9396	0.9401	0.9404
90	0.9323	0.9334	0.9341	0.9346	0.9350
120	0.9308	0.9319	0.9326	0.9331	0.9334
150	0.9303	0.9315	0.9322	0.9327	0.9330
180	0.9302	0.9313	0.9320	0.9325	0.9329
210	0.9302	0.9313	0.9320	0.9325	0.9328
240	0.9301	0.9313	0.9320	0.9325	0.9328
270	0.9301	0.9313	0.9320	0.9325	0.9328
300	0.9301	0.9313	0.9320	0.9325	0.9328
330	0.9301	0.9313	0.9320	0.9325	0.9328
360	0.9301	0.9313	0.9320	0.9325	0.9328
MTBF	336.92	337.31	337.54	337.71	337.82

4. Results and Analysis

- Effect of failure rate of transfer laddle (α_1) on system availability:

By varying failure rate α_1 from 0.016, 0.018, 0.02, 0.022 and 0.024 and keeping α_2 = 0.0083333, α_3 = 0.0059523, β_1 = 0.066666, β_2 = 0.05, and β_3 = 0.083333, the availability of the system has been computed and compiled in Table 1, which shows that there is a decrease in availability upto 4.05 percent. Also availability decreases by upto 4.02 percent with the increase in time from 30 to 360 days. MTBF decreases by approximately 15 days with the increase in failure rate from 0.016 to 0.024.

- Effect of failure rate of continuous casting machine (α_2) on system availability:

As presented in Table 2, as failure rate α_2 increases from 0.0083333 to 0.0138888 and the values of α_1, α_3, β_1, β_2 and β_3 are kept at 0.016, 0.0059523, 0.066666, 0.05 and 0.083333 respectively; availability shows a downward trend of maximum 3.06 percent. However availability decreases by upto 4.25 percent as time increases from 30 to 360 days. It is also observed that MTBF also decreases by 10 days as failure rate increases.

- Effect of failure rate of shot blasting machine (α_3) on system availability:

Next, we have studied the effect of failure rate of shot blasting machine on the availability of casting system. The results shown in Table 3 indicate that by varying failure rate α_3=0.0059523, 0.0065476, 0.0071428, 0.0077380 and 0.0083333 and taking α_1 = 0.016, α_2 = 0.008333, β_1 = 0.066666, β_2 = 0.05, β_3 = 0.083333, the availability decreases by 0.38 percent. It is also seen that there is a decrease of 2.90 percent in availability with the increase in time from 30 to 360 days. In this case, little change is observed in MTBF (almost 1 day) with the increase in failure rate.

- Effect of repair rate of transfer laddle (β_1) on system availability:

The results presented in Table 4 indicate the availability of the system when repair rate β_1 of the transfer laddle subsystem is varied from 0.0666666 to 0.10. Taking values of α_1 = 0.016, α_2 = 0.0083333, α_3 = 0.0059523, β_2 = 0.05, β_3 = 0.083333, one can see that availability improves upto 2.17 percent. Whereas, there is a decrease of 1.7-2.8 percent in availability as number of days increase from 30 to 360. MTBF also increases by around 7 days with the increase in repair rate.

- Effect of repair rate of continuous casting machine (β_2) on system availability:

Now, we have studied the effect of repair rate of continuous casting machine on the system availability. As β_2 is varied from 0.05 to 0.0769230 in five steps and the values of failure and repair rates of other subsystems i.e. $\alpha_1, \alpha_2, \alpha_3, \beta_1$ and β_3 are taken as 0.016, 0.0083333, 0.0059523, 0.066666 and 0.083333 respectively, it is observed that availability of the system decreases by 2-2.8 percent with the increase in time from 30 to 360 days. But, it increases by only 1.16 percent as repair rate increases from 0.05 to 0.0769230. Improvement in repair rate results in increase in MTBF of around 4 days as shown in the Table 5.

- Effect of repair rate of shot blasting machine (β_3) on system availability

At last, we have computed the effect of improvement of repair rate of shot blasting machine on the overall system availability as shown in Table 6. We see that as β_3 increases from 0.0833333 to 0.1428571 and the value of failure and repair rates of other subsystems are kept at $\alpha_1 = 0.016$, $\alpha_2 = 0.0083333$, $\alpha_3 = 0.0059523$, $\beta_1 = 0.066666$ and $\beta_2 = 0.05$, availability shows an increase of 0.27 percent. But as the number of days increase from 30 to 360, there is a decrease of around 2.7-2.8 percent in the value of availability. MTBF increases by just one day with the increase in repair rate.

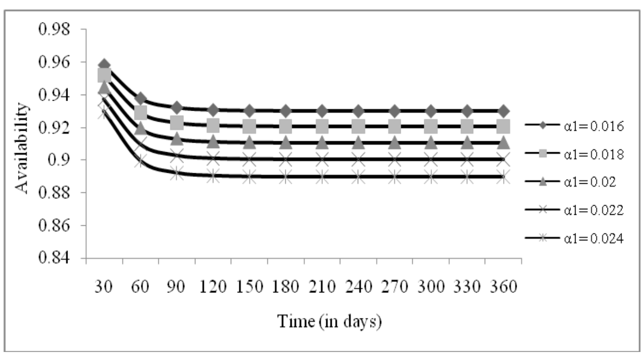

Figure 3. Effect of failure rate of transfer laddle on availability.

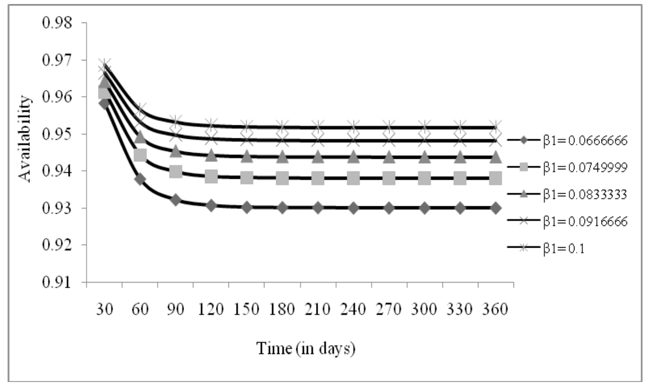

Figure 4. Effect of repair rate of transfer laddle on availability.

5. Conclusion

By comparing the results computed in of Tables 1-6, it reveals that subsystem A i.e. transfer laddle has maximum impact on the availability as well as on MTBF of the system. This phenomenon has also been depicted in the Figure 3 and 4. Second most important subsystem is B i.e. continuous casting machine whereas subsystem C (Shot blasting machine) has least impact on the availability and MTBF of the system. Hence, we infer that, as far as repair/ maintenance work on the basis of failure/repair rates is concerned, the priority should be as follows:

- Transfer laddle
- Continuous casting machine
- Shot blasting machine

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