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Construction of a Gauge Symmetry in *N* - 2 Level of Open String

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Abstract

Objectives: We construct new gauge symmetry in second level $N = 2$ of open string and predict that it corresponds to a new gauge particle with same properties as photon. **Methods:** In literature, using a special state called spurious state has known for describing gauge symmetry in first excited level. We generalize this prescription into second excited level $N = 2$. Since this method is based on the fact that adding spurious state to the spectrum leads no physical consequences, it allows us to attribute gauge invariance property to the $N = 2$ level of spectrum such that we can call this freedom gauge symmetry. **Findings:** In first excited state of string theory, photons associates to a symmetry called gauge symmetry. We generalize this situation to second level of spectrum. This leads us to predict that there exists a new particle which has spin 2 and act like photon i.e. it carries electromagnetic force. This new particle lives in a high energy scale which makes its detection not easy to experimentalists. We also propose that one can generalize this procedure to higher excited levels of string theory to obtain more new particles which live in open string theory energy scale. **Applications/Improvements:** But if someday we detect it, we can use it as a new alternative way for producing electromagnetic waves which includes a large number of applications for it for example, in telecommunication industry, medical sciences and etc.

Keywords: Field Theory, Gauge Symmetry, Gauge Particles, Massive Levels, Open String, Spurious State

1. Introduction

Gauge symmetries, In particular, have attracted considerable amount of interest due to the central role they play in our most successful theories of the fundamental constituents of nature. The standard model of elementary particle physics, for instance, is formulated in terms of gauge symmetry, and so are its most discussed extensions.

The notion of spontaneously broken gauge symmetry is also widely regarded as playing an important role in interpretation of creation of particle masses in standard model of elementary particles by Higgs mechanism

The present paper focuses on a particular aspect of gauge symmetries, namely, the concept of gauge symmetry in string theory. In ordinary quantum field theory, the fundamental constituents of nature are localized points which travel in a 4-dimensional space-time and correspond to usual elementary particles. But the situation in string theory is slightly different such that it describes a world which its fundamental objects are one dimensional string. There are two types of strings: open string which corresponds to gauge theories and closed string which describe gravity. All usual elementary particles are created and annihilated by oscillation modes of these strings. But detection of these strings is still beyond the scope of those energies which we are dealing with in laboratories so in our reachable energies we see the effective theories of these strings and the strings themselves look like point particles at this scale. This description of nature has a main advantage say despite the fact that there exist many particles in the nature, but we have only one string. This

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proposes string theory can be regarded as a nice starting point for unified theories of basic interactions.

Yang-Mills gauge theories that arise in first excited state in open string spectrum are similar to those gauge theories in standard model and are discussed in literature for example[1-3]. In this paper we construct gauge symmetry in second level of the spectrum.

The paper is organized as follows: Section 2 recalls some basic features of nonlinear sigma model in string theory and appearance of massive states (like $N = 2$) in high energies and Section 3 discusses the construction of a gauge symmetry in open string spectrum. The paper closes in Section 4 with a brief concluding remark.

2. Nonlinear Sigma Model

In this section, in order to explore the effect of massive state in string theory we will give a brief introduction to nonlinear sigma model. (for further reading see[1,2,4]). We start with Polyakov action with a general metric $G_{\mu\nu}(X)$ for space-time and with string worldsheet coordinates $(\sigma^0 = \tau, \sigma^1 = \sigma)$ in D dimensional flat space-time[1]:

where α' is the universal $Regge\ slope$, $\gamma\ \gamma$ is the worldsheet metric, X^μ's are string space-time coor dinates, $\mu = (0,1,\cdots,D-1)$, $a = 0,1$ and $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.

First note that replacing Minkowskian space-time metric in Polyakov action by $G_{\mu\nu}(X)$ means that we are going to move from string theory to something like quantum excitations of space-time in general relativity. So we can expand action (1) in terms of flat Minkowskian metric, $G_{\mu\nu}(X) = \eta_{\mu\nu} + h_{\mu\nu}(X)$

We can choose any background metric to expand about. The Minkowski metric is just a standard choice because we know how to calculate with respect to it. But there are also other interesting backgrounds say black holes which one can use them.

Now remind general background of mass less closed string states contains a graviton $G_{\mu\nu}(X)$, a dilaton $\Phi(X)$ and an antisymmetric tensor $B_{\mu\nu}(X)$. So we can write nonlinear sigma model action as:

where $R^{(2)}$ is the worldsheet $2d$ scalar curvature of worldsheet metric γ_{ab} .

Now recall that stress tensor obtain by varying Polyakov action with respect to the metric say:

$$T^{\alpha\beta} \propto \frac{\delta S}{\delta g_{\alpha\beta}},$$

so Weyl rescaling (as a symmetry of (1)) of metric gives the trace of T and then we have to demand Weyl anomaly vanishes:

$$< T^\alpha{}_\alpha >= 0.$$

In general background, sigma model is not conformally invariant instead we have:

$$T^\alpha{}_\alpha = -\frac{1}{2\alpha'}\beta^G_{\mu\nu} g^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu$$
$$= -\frac{1}{2\alpha'}\beta^G_{\mu\nu}\varepsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu - \frac{1}{2}\beta^\Phi R^{(2)}.$$

Where one can obtain beta functions after cumbersome calculations as follows (see[4]):

$$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} + 2\alpha' \nabla_\mu \nabla_\nu \Phi - \frac{\alpha'}{4} H_{\mu\lambda\omega} H_\nu{}^{\lambda\omega} + O(\alpha'^2),$$
$$\beta^B_{\mu\nu} = \frac{\alpha'}{2}\nabla^\omega H_{\omega\mu\nu} + \alpha' \nabla^\omega \Phi H_{\omega\mu\nu} + O(\alpha'),$$
$$\beta^\Phi = \frac{D-26}{6} - \frac{\alpha'}{2}\nabla^2\Phi + \alpha' \nabla^\omega \Phi \nabla_\omega \Phi - \frac{\alpha'}{24} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + O(\alpha'^2),$$

where $H_{\mu\nu\lambda}$ if the field strength of $B_{\mu\nu}$: $H = dB$:

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + perm.$$

One can regard these as loop corrections for beta functions in sigma model such that Weyl invariance condition leads to:

$$\beta_\Phi = \beta_G = \beta_B = 0.$$

So we can assume that these equation of motions come from varying and extremizing:

$$S = \frac{1}{2\kappa_0^2 \int d^2 X}\sqrt{G}\, e^{2\Phi}\left(R - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu \Phi \partial^\mu \Phi + O(\alpha')\right),$$

where κ_0 is a constant should be determined and all other geometric contents like $\sqrt{G}$ and R live on the target space. We conclude that up to a scale factor this is similar to Einstein-Hilbert action plus massless contents matter in string spectrum. So when energy is not low enough one should add massive terms to this action. In next section we construct such massive states in second level of open string spectrum by using gauge invariance property.

3. Gauge Symmetry and Spectrum

Now we are in a position to generalize the approach given in[5,6] for mass less state $N = 1$ to $N = 2$ level and find content(s) which live in this level.

The mass-shell condition for physical states[1,2] $(L_0 - \alpha)|Phys\rangle = 0$ is equivalent to:

$$M^2 = -p^2 = \frac{1}{\alpha'(N-a)},$$

where N N is the number operator which is defined such that count excitation modes $N \equiv \sum_{n=1}^{\infty} \alpha_{-n} . \alpha_n$ and α 's are mode expantions.

We see ground state $N = 0$ has a unique form say all oscillators lie in the Fock Vaccum, so it is given by $|0;k\rangle$. The constraint (12) leads to:

$$M^2 = -k^2 = -\frac{a}{\alpha'}.$$

It has negative mass-squared so we conclude that this is a tachyonic state which describes a particle which travels faster than the speed of light. So the bosonic string theory is not an appropriate quantum theory, because the energy of vacuum corresponds to a complex number so it becomes unstable. In analogous with ordinary quantum field theory, the existence of tachyonic states leads us to expand around a local extreme of the potential function, and show that we selected the wrong vacuum. This problem can remove by regarding tachyon free superstring theories which we will not discuss about them in this paper. For more about superstrings see references. Let us continue without worrying about this tachyonic state.

First excited state $N = 1$: We try to obtain $N = 1$ by exciting the vaccum state once say, $\alpha^{\mu}_{(-1)}|k;0\rangle$. There is a freedom for us to define a polarization vector ζ^{μ}. So the most general form for this level is given by:

$$|\zeta;k\rangle \equiv \zeta.\alpha_{(-1)}|0;k\rangle.$$

Let us study the mass-shell constraint for this state:

$$(L_0 - a)|\zeta;k\rangle = 0 \rightarrow M^2 = 1/a'(1-a).$$

Furthermore, by using Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12(m^3-m)\delta_{m+n,0}},$$

we can investigate the consequence of physical condition:

$$L_m|Phys\rangle = 0 \; for \; m > 0.$$

For $m = 1$ we obtain:

$$L_1|\zeta;k\rangle = \sqrt{(2\alpha')}(k.\alpha_1)(\zeta.\alpha_{(-1)}) = \sqrt{(2\alpha')}(k.\zeta)|0;k\rangle,$$

so by considering physical state constraint $L_1|\zeta;k\rangle = 0$, momentum and polarization vector are orthogonal i.e. we have:

$$k.\zeta = 0.$$

By ζ *-function regularization* mechanism one can show that $\sum_{n=1}^{\infty} n = -\frac{1}{12}$, so we obtain the critical dimension of space-time in bosonic string theory $D = 26$. In other words, if this condition is violated is some special manner the cancellation of conformal anomaly will not occur because it holds only when we have the equivalence between quantization in two different gauges say, conformal and light-cone gauges. We know that quantization in conformal gauge can be defined if we introduce Faddeev-Popov ghost fields[7] for world-sheet in order to fix the gauge in conformal field theory.

Let us begin with first excited level. In this case we can define a state $|\psi\rangle \equiv L_{(-1)}|0;k\rangle$ which has some interesting properties: First, this state is any arbitrary physical state because one can show that

$$\langle phys|\psi\rangle = \langle\psi|phys\rangle^* = \langle 0;k|L_1|phys\rangle = 0$$

. Second, one can also check that states satisfy $L_1|\psi\rangle = 2L_0|0;k\rangle = 2\alpha' k^2|0;k\rangle$ and finally it can be written in the form:

$$|\psi\rangle = \sqrt{(2\alpha')}|k;k\rangle.$$

In literature this state is known as *spurious state.* If we demand spurious state be a physical state then it should orthogonal to itself i.e. it must be a zero-norm state. Now we want count the number of independent contents in polarization vector in first excited level. As we mentioned earlier, the state $|\zeta;k\rangle$ undergoes two constraints: first, $k^2 = m^2 = 0$, and second, it should satisfy the orthogonal condition (18). Each of these removes one degree of freedom so we are left with $D-2$ independent components for polarization vector. This situation describes a mass less spin 1 particle with polarization vector ζ^μ , such that we can translate it to the Yang-Mills theory language and write the action for this configuration.

Since the spurious state $|\psi\rangle$ is a physical and zero-norm state, we conclude that adding it to any arbitrary physical state leads no physical effects. So we can define (see[5,6]):

$$|phys\rangle \sim |phys\rangle + \lambda|\psi\rangle,$$

where λ is a gauge parameter. For a physical state $|Phys\rangle = |\zeta;k\rangle$, and by considering (19) we can rewrite the relation (20) in terms of polarization vector:

This is nothing but $U(1)$ gauge symmetry in momentum space, and so in first excited level we have 24 masless physical states of a $U(1)$ Maxwell field in a 26 -dimensional space-time.

For $N = 2$, first we consider the general form of second level state:

$$|\chi;\xi;k\rangle \equiv \chi_{\mu\nu}\alpha^{\mu}_{(-1)}\alpha^{\nu}_{(-1)}|0;k\rangle + \xi_\mu\alpha^{\mu}_{(-2)}|0;k\rangle,$$

where χ and ξ are polarization rank 2 symmetric tensor and polarization vector, respectively. The mass-shell condition gives:

$$M^2 = \frac{1}{\alpha'}.$$

For $L_1|\chi;\xi;k\rangle = 0$ we obtain:

$$L_1|\chi;\xi;k\rangle = \chi_{\mu\nu}(\alpha^{\mu}_{(-1)}\alpha^{\nu}_{0} + \alpha^{\nu}_{(-1)}\alpha^{\mu}_{0})|0;k\rangle + 2\xi_\mu\alpha^{\mu}_{(-1)}|0;k\rangle$$

and for $L_2|\chi;\xi;k\rangle = 0$ we have:

$$\chi_{\mu\nu}\eta^{\mu\nu}|0;k\rangle + 2\xi_\mu\alpha^{\mu}_{0}|0;k\rangle = 0.$$

We will come back to these physical constraints later.

Now, let us see what spurious state in second level of the spectrum is. The number of zero-norm spurious states increases sharply if we have $a = 1$ and choose the space-time dimension suitably. To see this let us construct zero-norm spurious states of the form:

$$|\psi\rangle = (L_{(-2)} + \gamma L^2_{(-1)}|0;k\rangle.$$

This has zero-norm for a definite γ . The mass-shell constraint becomes:

$$(L_0 + 1)|0;k\rangle = 0.$$

Now we should impose the condition that $|\psi\rangle$ is a physical state, that is, $L_1|\psi\rangle = 0$ and $L_2|\psi\rangle = 0$, since the rest of the constraints $L_m|\psi\rangle = 0$ for $m \geq 3$ are then also satisfied as a consequence of the Virasoro algebra. One can easily show that these lead to $\gamma = \frac{3}{2}$ and $D = 26$. If we start with (36) we have:

<Not Clear>

Now we can add this state to general form of second level state (22) as a gauge transformation:

$$|\chi;\xi;k\rangle \sim |\chi;\xi;k\rangle + \lambda|1/2\eta + (2\alpha'\gamma)kk;\sqrt{}(2\alpha')k;k\rangle$$

So equivalence relation for χ becomes:

$$\chi_{\mu\nu} \sim \chi_{\mu\nu} + \frac{\lambda}{2}\eta_{\mu\nu} + \lambda(3\alpha')k_\mu k_\nu,$$

and for ξ it is:

Finally we demand (22) be physical so by (24) and (25) for spin 2 field χ we should have:

and for vector field we should have:

$$\xi_\mu\alpha^{\mu}_{-1} = \xi_\mu k^\mu = 0.$$

First term in (33) implies that $\xi = 0$ so the second term is automatically satisfied. Hence, as soon as we deal with physical states we should omit vector field ξ and only consider χ . Note also that by using (31) one could choose a gauge in which, but it does not respect to the second condition in (33) because $k^2 \neq 0$ in second level.

4. Conclusion

The aim of this paper has been to clarify how one can obtain a gauge symmetry in second level of open string spectrum. We saw by using general form of spurious state that one can construct a gauge symmetry which acts on a vector field (which can be omitted) and a rank 2 symmetric tensor field X . The latter symmetry transformation is a diffeomorphism gauge invariance. Since X is massive so in low energy limit $(\alpha' \rightarrow 0)$ we can regard it as a correction ($O(\alpha')$) of effective theory. For being more clarify, consider the notion of the effective action for the light fields in string theory. We know that the mass less field content for open string theory is A_μ and all other particles have masses of order of the string scale. The effective action for open string theory is[3]:

$$S_{eff} = -\frac{C}{4\int d^D x} e^{-\Phi} Tr F_{\mu\nu} F^{\mu\nu} + O(\alpha'),$$

where Φ is dilaton, C is dimensionful constant which can be fixed and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We can imagine integrating out the heavy fields that will induce corrections to the action of the light fields. At tree level, this procedure can be implemented by considering the full (on-shell) scattering amplitudes of the light fields from string calculations, expanding them in α' and finding the extra interactions induced on the light fields. Since the amplitudes used are on-shell, the effective action can be calculated up to terms that vanish by using the equations of motion.

Finally we propose that one can generalize this procedure and write spurious state in higher levels of spectrum to obtain more gauge symmetries in open string spectrum.

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