# Optimal Workspace Volume of Two Configurations of 3-DOFPKM 

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#### Abstract

Objectives: Parallel Kinematic Machines (PKMs) are closed loop mechanisms. The present study is to find optimal workspace volume of two configurations of translational 3-DOF PKM and optimal location for machining. Methods/ analysis: Kinematic analysis is carried out to find the joint angles of the two configurations of the PKM. Optimal dimensions of two configurations of PKM are obtained considering Workspace Volume Index (WVI) as objective. A source code is developed in MATLAB to find the optimal dimensions using Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO). Velocity analysis is carried out for configuration-I. The optimal location for best machining for configuration-I is identified using condition number of the jacobian matrix of the PKM. Findings: Using kinematic analysis the joint angles of the two configurations of PKM is obtained. Performance index like Workspace Volume Index (WVI) is introduced for the two configurations of PKM. Using WVI as an objective the optimal dimensions of two configurations of PKM are found using GA and PSO. It is observed from the results configuration-II has more WVI compared with configuration-I. Velocity analysis is done for configuration-I of PKM and from this analysis the location of singularities and non-singularities of the PKM was located. It was observed from the results the optimal location for machining is at the middle plane. Application/Improvements: The novelty of the present work lies in finding the optimal WVI of two configurations of 3-DOF PKM using GA and PSO. This PKM could be used as machine tool for 2-D contouring machining and other applications.


Keywords: Optimal Workspace Volume, 3-DOF GA, PKM, PSO

## 1. Introduction

Now a day PKM's attracting more researchers and manufacturers than conventional serial manipulators However, the design of PKM is a difficult task and it needs further research for its wide acceptance in industries. PKMs are finding applications in assembly and manufacturing in the recent past. Most of the research is being carried out on six DOF PKMs. Stewart's ${ }^{1}$ work becomes the initial platform in the area of PKMs. In ${ }^{2}$ analyzed the workspace and Dexterity of Hexaslide Machine tools. However, they suffer from the problem of relatively small workspace,
complex forward kinematics, complicated universal and spherical joints and design difficulties. Hence, researchers are focusing towards lower DOF parallel mechanisms consisting of revolute / prismatic joints since they allow larger range of rotation/translation, respectively and thereby offer larger workspace. It is being claimed that two and three DOF mechanisms offer attractive performance characteristics for manufacturing applications ${ }^{3}$. Translational or rotational moving platform with 3-DOF parallel manipulators have been investigated. Classical $3-R R R$ spherical manipulator was studied in detail ${ }^{4}$. Manipulators having rotating platforms also called as

[^0]rotational parallel manipulators (RPMs) have been proposed ${ }^{5-8}$. PKM's with three translational DOF's have been playing important roles in the industrial applications. DELTA ${ }^{9}$ robot was developed which is belongs to Translational Parallel Manipulators (TPMs). The 3-DOF Translational PKM consisting of 3-UPU (Universal-Prismatic-Universal) joints is designed by Tsai ${ }^{10}$. 3-DOF PRC (Prismatic-Revolute-Cylindrical) Translational Parallel Manipulator which consists of three limbs connected to the moving platform and a fixed base by a Prismatic, Revolute and cylindrical joints was introduced ${ }^{\underline{11}}$. The stiffness and positional errors of the 3-DOF PKM was discussed ${ }^{12-14}$.

The present work focuses on the dimensional design of a two configurations of 3-DOF translational PKM through optimization using GA and PSO. WVI of two configurations are introduced first of its kind. The new results like the variation of Workspace Volume Index (WVI) by increasing the distance of the Z-slider from the origin and Jacobian analysis to find the variation of RCN within the workspace of PKM for configuration-I are studied. Each configuration of PKM understudy has
three independent kinematic chains or limbs (PRRR type) between fixed base or frame and moving tool platform. Each limb consists of two links, namely, arm and forearm interconnected with a revolute joint. Other ends of the arm and forearm are connected to a slider and tool platform, respectively using revolute joint. Actuation of each slider independently drives the respective chain thereby positions the tool platform with same orientation at the desired location within the workspace.

This paper is divided into four sections. The second section discusses about inverse kinematics. The third section discusses about Works space volume index for two configurations of PKM. In the fourth section optimal design of two configurations of PKM is presented and finally in the fifth section results are discussed.

## 2. Kinematic Analysis

The kinematic sketch of the two configurations of translational PKM under study is shown in Figure 1. For these two configurations the origin of the fixed reference frame XYZ is located at point O . The mobile platform is


Figure 1. Schematic of 3-DOF Translational PKM and its Kinematic sketch of (a) configuration-I and (b) configuration-II.
symbolically represented by a square, whose side length $2 L_{\mathrm{p}}$ is defined by points $\mathrm{B}_{1}, \mathrm{~B}_{2}$, and $\mathrm{B}_{3}$. The sliders $\mathrm{A}_{i}$, for $i$ $=1$ to 3 , move on the guide rods or lead screws laid along/ parallel to X -, Y-, and Z- axes, respectively. The three revolute joint axes in each limb are located at points $A_{i}, M_{i}$, and $B_{i}$, respectively, and are parallel to the respective prismatic joint axes. Point $P$ represents the centre of the tool platform. The difference between two configurations of PKM is that the configuration-I of PKM shown in Figure 1 a. the Y-slider is placed at the origin whereas the con-figuration-II of PKM shown in Figure 1 b. the Y-slider is placed at a distance $e_{z}$ from the origin.

### 2.1 Workspace volume of the two configurations of PKM

Actuation of $i^{\text {th }}$ slider along its axis, while holding the other two sliders, moves the tool platform along a line parallel to the respective axis. Thus, the Workspace of the PKM is the product of three slider stroke lengths i.e., $S L_{X}$, $S L_{Y}$, and $S L_{Z}$ which is represented in Figure 2.
$W S V=S L_{X} S L_{Y} S L_{Z}$


Figure 2. Kinematic sketch (a) configuration-Iand (b) configuration-II of 3-DOF PKM with workspace.

The set of two joint angles for each limb, namely, $\theta_{i}$ and $\phi_{i}$, for $i=1$ to 3 , define the possible postures of each
limb which are found for the two configurations of PKM as follows:

### 2.2 Inverse Kinematics of configuration- I of PKM

The coordinates of P from Figure 3 a are expressed in terms of $y$ and $z$ for the first chain as

$$
\begin{align*}
& y=L_{1} \cos \theta_{1}+L_{2} \cos \phi_{1}+L_{p}  \tag{2}\\
& z=L_{1} \sin \theta_{1}+L_{2} \sin \phi_{1} \tag{3}
\end{align*}
$$

From equations (2) and (3) the joint angles of the first chain obtained as

$$
\begin{align*}
& \varphi_{1}=\cos ^{-1}\left[\frac{\left\{\left(y-L_{p}\right)^{2}+z^{2}\right\}-\left(L_{1}^{2}+L_{2}^{2}\right)}{\left(2 L_{1} L_{2}\right)}\right]  \tag{4}\\
& \theta_{1}=\tan ^{-1}\left[\frac{z}{y-L_{p}}\right]-\tan ^{-1}\left[\frac{L_{2} \sin \phi_{1}}{L_{1}+L_{2} \cos \phi_{1}}\right] \tag{5}
\end{align*}
$$

The coordinates of P from Figure 3 b . are expressed in terms of x and z for the second chain as

$$
\begin{align*}
& x=L_{1} \sin \theta_{2}+L_{2} \sin \phi_{2}+L_{p}  \tag{6}\\
& z=L_{1} \cos \theta_{2}+L_{2} \cos \phi_{2} \tag{7}
\end{align*}
$$

From equations (6) and (7) the joint angles of the second chain obtained as

$$
\begin{align*}
& \varphi_{2}=\cos ^{-1}\left[\frac{\left\{\left(x-L_{p}\right)^{2}+z^{2}\right\}-\left(L_{1}^{2}+L_{2}^{2}\right)}{\left(2 L_{1} L_{2}\right)}\right]  \tag{8}\\
& \theta_{2}=\tan ^{-1}\left[\frac{z}{x-L_{p}}\right]-\tan ^{-1}\left[\frac{L_{2} \sin \phi_{2}}{L_{1}+L_{2} \cos \phi_{2}}\right] \tag{9}
\end{align*}
$$

The coordinates of P from Figure 3 c . are expressed in terms of $x$ and $y$ for the third chain as

$$
\begin{align*}
& x=D_{1}+L_{1} \cos \theta_{3}+L_{2} \cos \phi_{3}-L_{p}  \tag{10}\\
& y=D_{2}+L_{1} \sin \theta_{3}+L_{2} \sin \phi_{3} \tag{11}
\end{align*}
$$

From equations (10) and (11) the joint angles of the third chain obtained as

$$
\begin{equation*}
\varphi_{3}=\cos ^{-1}\left[\frac{\left.\left\{\left(x+L_{p}-D_{1}\right)^{2}+\left(y-D_{2}\right)^{2}\right)\right\}-\left(L_{1}^{2}+L_{2}^{2}\right)}{\left(2 L_{1} L_{2}\right)}\right] \tag{12}
\end{equation*}
$$


(a)

$$
\begin{equation*}
\theta_{3}=\tan ^{-1}\left[\frac{\left(y-D_{2}\right)}{\left(x+L_{p}-D_{1}\right)}\right]-\tan ^{-1}\left[\frac{L_{2} \sin \phi_{3}}{L_{1}+L_{2} \cos \phi_{3}}\right] \tag{13}
\end{equation*}
$$

Figure 3. Schematic diagrams of (a) first limb, (b) second limb, and (c) Third limb of configuration-I of 3-DOF PKM.

### 2.3 Inverse Kinematics of configuration- II of PKM

According to Figure 4a, for the first chain, the set of actuated joints are:

$$
\begin{align*}
& \varphi_{1}=-\cos ^{-1}\left[\frac{\left\{\left(y-L_{p}\right)^{2}+z^{2}\right\}-\left(L_{1}^{2}+L_{2}^{2}\right)}{\left(2 L_{1} L_{2}\right)}\right]  \tag{14}\\
& \theta_{1}=\tan ^{-1}\left[\frac{z}{y-L_{p}}\right]-\tan ^{-1}\left[\frac{L_{2} \sin \phi_{1}}{L_{1}+L_{2} \cos \phi_{1}}\right] \tag{15}
\end{align*}
$$

From Figure 4 b.for the second chains the set of actuated joints are

$$
\begin{align*}
& \varphi_{2}=-\cos ^{-1}\left[\frac{\left\{\left(x-L_{p}\right)^{2}+\left(z-e_{z}\right)^{2}\right\}-\left(L_{1}^{2}+L_{2}^{2}\right)}{\left(2 L_{1} L_{2}\right)}\right]  \tag{16}\\
& \theta_{2}=\tan ^{-1}\left[\frac{z-e_{z}}{x-L_{p}}\right]-\tan ^{-1}\left[\frac{L_{2} \sin \phi_{2}}{L_{1}+L_{2} \cos \phi_{2}}\right] \tag{17}
\end{align*}
$$

From Figure 4 c . for the third chain the set of actuated joints are

$$
\begin{align*}
& \varphi_{3}=\cos ^{-1}\left[\frac{\left.\left\{\left(x+L_{p}-D_{1}\right)^{2}+\left(y-D_{2}\right)^{2}\right)\right\}-\left(L_{1}^{2}+L_{2}^{2}\right)}{\left(2 L_{1} L_{2}\right)}\right]  \tag{18}\\
& \theta_{3}=\tan ^{-1}\left[\frac{\left(y-D_{2}\right)}{\left(x+L_{p}-D_{1}\right)}\right]-\tan ^{-1}\left[\frac{L_{2} \sin \phi_{3}}{L_{1}+L_{2} \cos \phi_{3}}\right] \tag{19}
\end{align*}
$$



Figure 4. Schematic diagrams of (a) first limb, (b) second limb, and (c) Third limb of configuration-II of 3-DOF PKM.

### 2.4 Velocity analysis of configuration-I

By differentiating Eqs. (2) and (3)

$$
\begin{align*}
& \dot{y}=-L_{1} \sin \theta_{1} \dot{\theta}_{1}-L_{2} \sin \phi_{1} \dot{\phi}_{1}  \tag{20}\\
& \dot{z}=L_{1} \cos \theta_{1} \dot{\theta}_{1}+L_{2} \cos \phi_{1} \dot{\phi}_{1} \tag{21}
\end{align*}
$$

Eliminating $\dot{\phi}_{1}$ from Eqs. (20) and

$$
\begin{equation*}
\dot{y} \cos \phi_{1}+\dot{z} \sin \phi_{1}=\sin \left(\phi_{1}-\theta_{1}\right) \dot{\theta}_{1} L_{1} \tag{21}
\end{equation*}
$$

By differentiating Eqs. (6) and (7)

$$
\dot{z}=-L_{1} \sin \theta_{2} \dot{\theta}_{2}-L_{2} \sin \phi_{2} \dot{\phi}_{2}
$$

$$
\begin{equation*}
\dot{x}=L_{1} \cos \theta_{2} \dot{\theta}_{2}+L_{2} \cos \phi_{2} \dot{\phi}_{2} \tag{24}
\end{equation*}
$$

Eliminating $\dot{\phi}_{2}$ from Eqs. (23) and (24)

$$
\begin{equation*}
\dot{z} \cos \phi_{2}+\dot{x} \sin \phi_{2}=\sin \left(\phi_{2}-\theta_{2}\right) \dot{\theta}_{2} L_{1} \tag{25}
\end{equation*}
$$

By differentiating Eqs. (10) and (11)

$$
\begin{align*}
& \dot{x}=-L_{1} \sin \theta_{3} \dot{\theta}_{3}-L_{2} \sin \phi_{3} \dot{\phi}_{3}  \tag{26}\\
& \dot{y}=L_{1} \cos \theta_{3} \dot{\theta}_{3}+L_{2} \cos \phi_{3} \dot{\phi}_{3} \tag{27}
\end{align*}
$$

Eliminating $\dot{\phi}_{3}$ from Eqs. (26) and (27)
$\dot{x} \cos \phi_{3}+\dot{y} \sin \phi_{3}=\sin \left(\phi_{3}-\theta_{3}\right) \dot{\theta}_{3} L_{1}$
Eqs. (22), (25) and (28) can be arranged as

$$
\left[\begin{array}{ccc}
0 & \cos \phi_{1} & \sin \phi_{1}  \tag{29}\\
\sin \phi_{2} & 0 & \cos \phi_{2} \\
\cos \phi_{3} & \sin \phi_{3} & 0
\end{array}\right]\left\{\begin{array}{l}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{array}\right\}=\left[\begin{array}{ccc}
L_{1} \sin \left(\phi_{1}-\theta_{1}\right) & 0 & 0 \\
0 & L_{1} \sin \left(\phi_{2}-\theta_{2}\right) & 0 \\
0 & 0 & L_{1} \sin \left(\phi_{3}-\theta_{3}\right)
\end{array}\right]\left\{\begin{array}{l}
\dot{\theta}_{1} \\
\dot{\theta}_{2} \\
\dot{\theta}_{3}
\end{array}\right\}
$$

Eq. (29) is written as

$$
\begin{align*}
& J_{t} \dot{X}=J_{a} \dot{\theta}  \tag{30}\\
& \dot{\theta}=J_{a}^{-1} J_{t} \dot{X}  \tag{31}\\
& \dot{\theta}=J \dot{X} \tag{32}
\end{align*}
$$

where,
$J=J_{a}^{-1} J_{t}$ Is the Jacobian matrix of the PKM
$\dot{X}=\left[\begin{array}{l}\dot{x} \\ \dot{y} \\ \dot{z}\end{array}\right]$ Is position of the end-effector of the PKM
$\dot{\theta}=\left[\begin{array}{c}\dot{\theta}_{1} \\ \dot{\theta}_{2} \\ \dot{\theta}_{3}\end{array}\right]$ Are angular velocities of links of the PKM
$\mathrm{J}_{\mathrm{t}}=\left[\begin{array}{ccc}0 & \cos \phi_{1} & \sin \phi_{1} \\ \sin \phi_{2} & 0 & \cos \phi_{2} \\ \cos \phi_{3} & \sin \phi_{3} & 0\end{array}\right]$
$\mathrm{J}_{\mathrm{a}}=\left[\begin{array}{ccc}\mathrm{L}_{1} \sin \left(\varphi_{1}-\theta_{1}\right) & 0 & 0 \\ 0 & \mathrm{~L}_{1} \sin \left(\varphi_{2}-\theta_{2}\right) & 0 \\ 0 & 0 & \mathrm{~L}_{1} \sin \left(\varphi_{3}-\theta_{3}\right)\end{array}\right]$

## 3. Performance Index

Performance of the PKM depends on its dimensions. The combinations of these links and link lengths play a very important role in optimal design15-17. The performance index used for design optimization is:

$$
w S V=S L_{X} S L_{Y} S L_{Z}
$$

where, $S L_{X}, S L_{Y}$, and $S L_{Z}$ represents the stroke lengths of the PKM sliders. Besides, the PKM size is considered as the volume of the smallest possible rectangular parallelepiped containing both the PKM and workspace, which can be computed easily for a given set of PKM parameters. Thus, $W V I$ is found as

$$
\begin{equation*}
W V I=\frac{W S V}{P K M s i z e} \tag{33}
\end{equation*}
$$

## 4. Design Optimization

The design problem can be expressed in standard optimization problem which includes design variables, constraints and objectives ${ }^{18}$. An attempt has been made to find the optimal dimensions that provide the optimal performance characteristics. Thus the objective of the optimization problem is Maximum WVI.

### 4.1 Design Variables

The design variables considered in optimization are presented in Table 1. Another variable is considered for configuration-II i.e. the distance of the Y -slider from the origin i.e. $e_{z}$.

### 4.2 Design constraints

To find the optimal dimensions of the two configurations of PKM's under study, the design constraints considered are:
(i) The tool center point ' P ' shall reach the entire workspace or typical grid points within the workspace.
(ii) Included angle between arm and forearm (a) in each limb shall be between $10^{\circ}$ and $170^{\circ}$.
(iii) $L_{2}-L_{1} \leq 0$
(iv) $X I+S_{X} \leq D_{1}$
(v) $X I+S_{X} \leq D_{1}$

### 4.3 Genetic Algorithm (GA)

Genetic Algorithm is depending on the concept of natural genetics and natural selection. The basic elements of natural genetics are reproduction, crossover and, mutation are used in the genetic search procedure and the advantage of using GA is that it can find global optimum solution with a high probability in most cases.

### 4.4 Particle Swarm Optimization (PSO)

In this birds or fishes are assumed as particles. In these particles flew to reach the optimum solutions by updating their current optimum solutions. PSO does not possess GA operators like cross over and mutation.

Table 1. Variables considered for optimization of configurations I and II of PKM

| Design Variables | Configuration-I <br> symbol | Configuration-II <br> symbol |
| :--- | :---: | :---: |
| Length of the arm | $\mathrm{L}_{1}$ | $\mathrm{~L}_{1}$ |
| Length of the <br> forearm | $\mathrm{L}_{2}$ | $\mathrm{~L}_{2}$ |
| Distance of the <br> Z-slider from the <br> origin | $\mathrm{D}_{1}$ | $\mathrm{D}_{1}$ |
| Offset of the Z-slider <br> from the X-axis | $\mathrm{D}_{2}$ | $\mathrm{D}_{2}$ |
| Starting point of the <br> X-slider | XI | XI |
| Starting point of <br> the Y-slider | YI | YI |
| Starting point of <br> the Z-slider | ZI | ZI |
| Distance of the <br> Y-slider from <br> the origin | - | $\mathrm{e}_{\mathrm{z}}$ |

## 5. Results and Discussion

A source code is developed to study the effect of variation in dimensions upon performance of configuration-I of a PKM. The performance index i.e. Workspace Volume Index (WVI) is computed with stroke lengths $\mathrm{SL}_{\mathrm{x}}=0.4$ $\mathrm{m}, \mathrm{SL}_{\mathrm{Y}}=0.4 \mathrm{~m}$ and $\mathrm{SL}_{\mathrm{Z}}=0.3 \mathrm{~m}$ i.e. the workspace would be a rectangular parallelepiped of $0.4 \mathrm{~m} \times 0.4 \mathrm{~m} \times 0.3 \mathrm{~m}$ The
program developed is executed in MATLAB. The program is run by varying the distance of the Z-slider from the origin i.e. $D_{1}$, arm length $\left(L_{1}\right)$ and forearm length $\left(L_{2}\right)$ of the PKM.

Following parameters are considered to simulate PKM:
$\mathrm{L}_{1}=0.533 \mathrm{~m}, \mathrm{~L}_{2}=0.508 \mathrm{~m}, \mathrm{D}_{2}=0.227 \mathrm{~m}, \mathrm{XI}=0.221 \mathrm{~m}$, $\mathrm{YI}=0.242 \mathrm{~m}, \mathrm{ZI}=-0.557 \mathrm{~m}^{2}$ and $\mathrm{L}_{\mathrm{p}}=0.1 \mathrm{~m}$.

For fixed value of $\mathrm{L}_{2}, \mathrm{D}_{2}, \mathrm{XI}$, YI and ZI the variation of Workspace Volume Index (WVI)with the parameters $\mathrm{L}_{1}$ and $\mathrm{D}_{1}$ are shown in Figure 5 a. Similarly, for fixed value of $\mathrm{L}_{1}, \mathrm{D}_{2}, \mathrm{XI}, \mathrm{YI}$ and ZI the variation of WVI with the parameters $L_{1}$ and $D_{1}$ are shown in Figure 5 b. The parameters are chosen here in such a way that that they should reach the workspace of the PKM. It is observed from Figure 5 a, the maximum WVI is 0.0525 is achievable with $L_{1}=0.51 \mathrm{~m}$ and $\mathrm{D}_{1}=1.22 \mathrm{~m}$ and from Figure 5 b , the maximum WVI is 0.0523 is achievable with $\mathrm{L}_{2}$ $=0.49 \mathrm{~m}$ and $\mathrm{D}_{1}=1.94 \mathrm{~m}$. It is evident from the results that the variation of Workspace Volume Index (WVI) is effected by increasing the distance of the Z-slider from the origin i.e. $\mathrm{D}_{1}$.


Figure 5. Variation of Workspace Volume Index (WVI) with the parameters (a) $L_{1}$ and $D_{1}$ And (b) $L_{2}$ and $D_{1}$ for configuration-I of PKM.

### 5.1 Optimization of PKM

Optimization of two configurations of PKM is done by considering $\mathrm{WVI}_{\text {max }}$ as objective using GA and PSO. The prescribed workspace for optimal design of PKM understudy is considered as a rectangular parallelepiped of dimensions of 0.4 mx 0.4 mx x 0.3 m (i.e., $\mathrm{SL}_{\mathrm{x}}=0.4 \mathrm{~m}$, $\mathrm{SL}_{\mathrm{y}}$ $\left.=0.4 \mathrm{~m}, \mathrm{SL}_{\mathrm{Z}}=0.3 \mathrm{~m}\right)$. The platform length parameter $\left(\mathrm{L}_{\mathrm{p}}\right)$ is considered as 0.1 m .

For executing PSO algorithm the following data is taken

Population size $=25$

Cognitive coefficient $\mathrm{C}_{1}=2$
Social coefficient $\mathrm{C}_{2}=2$
Initial inertia weight $\mathrm{WI}=0.9$
Final inertia weight $\mathrm{w}_{\mathrm{f}}=0.4$
The best dimensions of the two configurations of PKM and its corresponding $\mathrm{WVI}_{\text {max }}$ are shown in Tables 2 and 3 using GA and PSO. Figure 6 shows the convergence of GA and PSO for configuration-I am shown in Figure 6. It is observed that the Table 2 and Table $4, \mathrm{WVI}_{\text {max }}$ is maximum for configuration-II. The convergence processes of PSO for various inputs like $L_{1}$ and $L_{2}$ of PKM are shown in Figure 7. The resultant Machine volume for the obtained optimal dimensions of configurations I and II using GA are

Machine Volume (MV) $0.6857 \mathrm{~m}^{3}$ of Configuration-I $=0.6857 \mathrm{~m}^{3}$

Workspace Volume (WSV) of Configuration-I = $0.0480 \mathrm{~m}^{3}$

Therefore, WVI $=0.0480 / 0.6857=0.070$
Machine Volume (MV) $0.6857 \mathrm{~m}^{3}$ of Configuration-II $=0.4968 \mathrm{~m}^{3}$

Workspace Volume (WSV) of Configuration-I = $0.0480 \mathrm{~m}^{3}$

Therefore, WVI $=0.0480 / 0.4968=0.0966$


Figure 6. The convergence process of (a) GA and (b) PSO optimization.


Figure 7. The convergence process of PSO optimization for inputs of L1 and L2 of PKM.

Table 2. Best solution and the corresponding design variables of PKM using GA

|  | Configuration-I <br> WVI $_{\text {max }}(\mathbf{0 . 0 7 0})$ | Configuration-II <br> WVI $_{\text {max }}(0.0966)$ |
| :--- | :---: | :---: |
| $\mathrm{L}_{1}(\mathrm{~m})$ | 0.482 | 0.40 |
| $\mathrm{~L}_{2}(\mathrm{~m})$ | 0.431 | 0.35 |
| $\mathrm{D}(\mathrm{m})$ | 0.947 | 0.90 |
| $\mathrm{D}_{2}(\mathrm{~m})$ | 0.113 | 0.20 |
| $\mathrm{XI}(\mathrm{m})$ | 0.101 | 0.2 |
| $\mathrm{YI}(\mathrm{m})$ | 0.100 | 0.2 |
| $\mathrm{ZI}(\mathrm{m})$ | -0.484 | 0.10 |
| $\mathrm{e}_{\mathrm{z}}$ | - | -0.5 |

Table 3. Best solution and the corresponding design variables of PKM using PSO

|  | Configuration-I <br> WVI $_{\text {max }}$ <br> $(\mathbf{0 . 0 7 0})$ | Configuration-II <br> WVI $_{\text {max }}$ <br> $(\mathbf{0 . 0 9 2 1})$ |
| :--- | :---: | :---: |
| $\mathrm{L}_{1}(\mathrm{~m})$ | 0.449 | 0.443 |
| $\mathrm{~L}_{2}(\mathrm{~m})$ | 0.441 | 0.322 |
| $\mathrm{D}^{(\mathrm{m})}$ | 0.946 | 0.894 |
| $\mathrm{D}_{2}(\mathrm{~m})$ | 0.0401 | 0.166 |
| $\mathrm{XI}(\mathrm{m})$ | 0.100 | 0.244 |
| $\mathrm{YI}(\mathrm{m})$ | 0.100 | 0.270 |
| $\mathrm{ZI}(\mathrm{m})$ | -0.451 | -0.170 |
| $\mathrm{e}_{\mathrm{z}}$ | - | -0.594 |

For the above obtained optimal dimensions of the configuration-I of a PKM i.e.
$\mathrm{L}_{1}=0.482 \mathrm{~m}, \mathrm{~L}_{2}=0.431 \mathrm{~m}, \mathrm{D}_{1}=0.947 \mathrm{~m}, \mathrm{D}_{2}=0.113$ $\mathrm{m}, \mathrm{XI}=0.101 \mathrm{~m}, \mathrm{YI}=0.100 \mathrm{~m}, \mathrm{ZI}=-0.484 \mathrm{~m}$ and $L_{\mathrm{P}}=0.1 \mathrm{~m}$.

The effect of variation of Reciprocal Condition Number (RCN) of the Jacobian matrix of the PKM with in the workspace is shown in Figure 8. Tables 3 and 4 show the variation of RCN at different planes. The best location for machining is at the middle plane for the coordinates x $=0.226 \mathrm{~m}, \mathrm{y}=0.225 \mathrm{~m}$ and $\mathrm{z}=-0.634 \mathrm{~m}$ which is far away from singularities.


Figure 8. Variation of RCN at three planes.

Using equation (30) the PKM is simulated for circular trajectory for the above said optimal dimensions and the results of PKM are shown in Figure 9.

The tool trajectory specifications are taken as
Coordinates of the circular trajectory are $\mathrm{XC}=0.3008$ $\mathrm{m}, \mathrm{y}_{\mathrm{c}}=0.3007 \mathrm{~m}$ and $\mathrm{Zc}=-0.6011 \mathrm{~m}$ and radius $\mathrm{r}_{\mathrm{o}}=0.10 \mathrm{~m}$.


Figure 9. Numerical results of the mean angular velocities of the arms for circular trajectory of the PKM.

Table 4. Lowest and highest values of RCN at three planes

| RCN of the Jacobian matrix |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Plane | Minimum <br> RCN | $\mathrm{x}(\mathrm{m})$ | $\mathrm{y}(\mathrm{m})$ | $\mathrm{z}(\mathrm{m})$ | Maximum <br> RCN | $\mathrm{x}(\mathrm{m})$ | $\mathrm{y}(\mathrm{m})$ | $\mathrm{z}(\mathrm{m})$ |
| Top | 0.0012 | 0.351 | 0.3 | -0.484 | 0.5798 | 0.101 | 0.225 | -0.484 |
| Middle | 0.0033 | 0.351 | 0.3 | -0.634 | 0.900 | 0.226 | 0.225 | -0.634 |
| Bottom | 0.0002 | 0.501 | 0.45 | -0.784 | 0.368 | 0.276 | 0.275 | -0.784 |

From Figure 9 the highest value of mean angular velocity of arms is $735.87 \mathrm{rad} / \mathrm{sec}$ which occurs at angle $310^{\circ}$ and RCN is 0.0009 which is close to singularity.

## 6. Conclusions

In this paper, the inverse kinematics of a 3- DOF PKM is presented. For a given coordinates of tool centre point, the joint angles of two links of each chain are found. Velocity analysis is carried for configuration-I of PKM, from these Jacobian matrices of slider and end-effector is obtained. For a set of dimensions the effect of variation in dimensions upon performance of configuration-I of a PKM is carried out. Optimal size of the two configurations of translational PKM based on maximum Workspace Volume Index by using GA and PSO in MATLAB is found. It is observed that configuration-II has maximum Workspace Volume Index compared to configuration-I. For the obtained optimal dimensions of the configura-tion-I the variation of RCN of Jacobian matrix are shown in the region of workspace of the PKM. From these results it is found that the best location for machining is at the middle plane which is far from singularities.

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