Optimal Designing of Concrete Gravity Dam using Particle Swarm Optimization Algorithm (PSO)

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Abstract

Background/Objectives: Designing and constructing concrete gravity dams must be in a way that not only realize sustained conditions, but also impose the minimum production costs. The major imposed cost in such dams is expenses of excessive use of concrete. Optimizing this cost requires cross-section optimization. **Methods/Statistical Analysis:** Dams geometrical configuration show that the cross-section area directly depends on the bottom width and upstream slope. Thus, area optimization only requires optimizing bottom width and upstream slope. Forces imposed on dam, especially seismic forces are nonlinear; on the other hand, sustained conditions including sustainability versus overturning, slip, and cracks caused by fatigue in normal and earthquake conditions are nonlinear, too. In such problems where objective function and constraints are nonlinear, applying conventional optimization methods cannot be responsive. Evolutionary algorithms are efficient in solving these problems. This research tries to study dam weight problem through using Particle Swarm Optimization Algorithm (PSO), which is an evolutionary algorithm based on birds' searching. **Results:** Comparing the numbers obtained in this method with numbers suggested by conventional methods showed that PSO effectively designs concrete weighting dams and optimizes their dimensions. **Conclusion/Application:** Moreover, some functions were also proposed for optimum designing of weight-concrete dam.

Keywords: Concrete Weight Dam, Optimization, Dam Sustaining, Particle Swarm Optimization Algorithm (PSO)

1. Introduction

Concrete dams classify into four categories: arch, double arch, lacy, and gravity dams. In gravity dams, dam's gravity power is the resistive force against all destructive factors; while, in arch dams, destructive forces transfer to dam bases. Destructive forces in dams include hydrostatic forces resulted from upstream water pressure, lifting power, precipitation, wind, and ice forces, as well as seismic dynamic force. These forces must be restrained by dam weight in gravity dams in which dam weight is the major resistance factor against these forces. Implementation

costs of these dams are higher than other dams regarding concrete higher prices than other materials. Therefore, in the case that optimize dam cross section according to satisfying sustaining conditions such that less concrete is consumed, it strongly influences dam construction costs and subsequently project cost effectiveness. In this problem, dam weight is objective function; however, it requires optimizing bottom width and upstream slope to be optimized. Sustaining conditions are the very constraints of this problem containing resistance against slip, resistance against overturn, and resistance against crack caused by fatigue, by regarding and disregarding region

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seismicity rate. Obviously, these constraints are nonlinear. Common mathematical algorithms are incapable of solving such complicated problems. Therefore, it is necessary to use evolutionary algorithms for solving. Particle Swarm Optimization (PSO) algorithm is one of these efficient algorithms in solving complicated problems.

Ling Wang and Qie He¹ tried to solve engineering problems by using an evolutionary Particle Swarm Optimization algorithm (PSO). They concluded that applying a penalty function for controlling population rate and convergence is difficult in some comparative issues. They used two simultaneous populations for evolutionary discovery and penalty factors rather than applying a penalty function. Moreover, penalty factors automatically enhances by discovery evolution. Schutte et al² used particle swarm algorithm in biomechanics studies. Biomechanics studies use optimization for solving system recognition problems, predicting human movement, and or body organs and or other internal forces that cannot be directly measured. They expressed that biomechanics optimization usually contains some local minimum answers. This research compares three optimization algorithms with PSO in which PSO outperformed. Heirany and Ghaemian³ studied the behavior of concrete weight dam under seismic loading. They analyzed the effect of concrete non-linear parameters on weight dams through using limited element approach. Concrete physical characteristics including elasticity module, tensile strength, and energy were particular for failure. Two various models were used for dam foundation response to seismic vibrations. Tried to optimally design dam reservoirs through using PSO algorithm and showed that regarding nonlinear objective function and dams' reservoirs designing constraints, PSO algorithm successfully and rapidly converged to the comprehensive optimum response. extracted Dez dam reservoir's stirring curve by using PSO algorithm and demonstrated that this algorithm achieves an acceptable response in optimizing complex problems. Optimized cross-sections in concrete gravity dams through genetic algorithm and artificial neural networks and concluded that regarding problem complexity, artificial neural networks and genetic algorithm can effectively applied in estimating optimal cross section of concrete gravity dams. studied PSO optimization algorithm in water optimal distribution and delivery of irrigation networks and concluded that the developed PSO model for one of irrigation canals in Varamin, Iran, provided better results than SA model simply optimized canal duration and dimension. optimized South Pars flood control system by using PSO algorithm and its two introduced models, and by comparing it with the project consultant proposed system showed that this model achieved better results reducing implementation costs. Salmasi⁴ designed concrete gravity dam through genetic algorithm. He computed optimal dam's cross section parameters (including downstream bottom width, upstream bottom width, bottom total width and crest width) by assuming crest width as water height percent. Rania et al⁵ compared genetic algorithm and PSO algorithm in eight criteria problems and showed that genetic algorithm leads to optimal solution in discrete problems; whereas, in continuous searching spaces, PSO algorithm gets the answer in a shorter time with higher accuracy. This difference attributes to genetic algorithm discreteness and PSO continuity.

2. Materials and Methods

2.1 Particle Swarm Optimization (PSO)

Kennedy and Ober heart firstly introduced this method in 1995. Bird flock and fish in food searching and or escaping from predators are inspiration sources of this method. The algorithm assumes each point in searching space as a particle. Particles form by an early random population and initiate searching through moving in searching upper space. This algorithm is similar to all other evolutionary algorithms such as genetic algorithm in creating the first population with this difference that each particle has an individual intelligence and total population has a collective intelligence, too. Hence, each particle memorizes the best position (Pbest) during probing; on the other hand, total population memorizes the best-experienced position by population (Gbest) through sharing information. The next path selects by particle regarding particle rate in previous state and best-experienced individual and social position by particle. The schematic view of this movement is as follows:

Particles' movement relations are as follows:

$$V_{t+1}^{i} = wV_{t}^{i} + c1*rand*(Pbest - X_{t}^{i}) + c2*rand(Gbest - X_{t}^{i})$$
 Relation (1)

 V_{t+1}^i : Particle speed in time series t+1.

 V_{4}^{i} : Particle speed in time series t.

w : Weight inertia factor.

 $X_{t+1}^{i} = X_{t}^{i} + V_{t+1}^{i}$

*c*1 : Cognitive critical index.

Relation (2)



Figure 1. Schematic view of movement in PSO algorithm.

*c*2 : Social critical index.

rand : Random numbers with normal distribution between zero and one.

Pbest : The best experienced position by particle. *Gbest* : The best experienced position by social. X_{i+1}^{i}, X_{i}^{i} : Particle position in time series t and t+1.

Each particle characterized with speed and position. Particle position in later time series equals with particle position in current time series plus particle speed. Particle speed is scalar sum of three components including speed in previous time series, personal experience component, as well as social experience component.

2.2 POS Parameters

2.2.1 Inertia Weight Factor (W)

If the coefficient is 1 in normal condition, then particle's movement speed excessively increases such that finding optimal solution will be difficult at this speed exceed optimal points. In order to control velocity and avoid its explosion, a coefficient is used as inertia weight coefficient. This value was constant in the original version; however, recent studies revealed that linear reduction of W from 0.9 to 0.2 responses better within algorithm iterations. This indicates high velocity at the beginning of probing and low velocity approaching optimal answer. Early versions of PSO applied Vmax parameter (maximum speed) to remove this problem such that the algorithm hinders exceeding the velocity and enables better searching.

2.2.2 Cognitive and Social Factors (C1, C2)

These factors are used to value individual and social experiences. Kennedy suggested value 2 for both in the original PSO version; while, recent studies recommend 1.494. regards 1.5 to 2 for C1 and 2 to 2.5 for C2 more efficient.

2.2.3 Rand

This parameter is used in order to prevent being trapped in local optimal points and to comprehensively probe searching space. It operates like mutation operator in genetic algorithm.

PSO algorithm steps are as follows:

- 1. Initiate.
- 2. Determining C1, C2, W factors and stop condition (population number and iteration number).
- 3. Initial population.
- 4. Calculating objective function fitness for population items.
- 5. Selecting the most appropriate particle and placing it in Gbest memory.
- 6. Determining W value.
- Updating speed and position using relations (1) and (2).
- 8. Comparing Pbest and Gbest fitness and replacing if one fitness is better than the other.
- 9. Investigating complete condition, if it dissatisfies, return to step 4.
- 10. Print Gbest as optimal solution.
- 11. Finish.

According to nonlinear and critical constraints, this research used some penalties when obtained answer does not meet constraints, meaning that finite value replaces computed value such that algorithm is unable to approach that point in later steps.

2.3 Statement of the Problem

Figure 2 represents concrete gravity dam's type cross.



Figure 2. Concrete Gravity Dam Cross-Section and Imposed orces.

Imposed forces over concrete gravity dam cross-section are as follows:

A. Hydrostatic power of upstream water (F1): it is calculated as follows:

$$F_1 = \frac{1}{2} \gamma_w h^2$$

B. Dam weight force (W1,W2,W3): The forces compute by transforming dam's cross-section into three geometrical shapes and doing computations for width unit. Relations based on m, b, h, T, and Fb variables are as follows:

$$W_1 = \frac{1}{2} \gamma_c.m.b.h$$
$$W_2 = \gamma_c.T.(F_b + h)$$
$$W_3 = \gamma_c.\frac{h.(b - m.b - T)^2}{2.(b - m.b)}$$

Fb: Free heightb: bottom widthm: a ratio of upstream bottom widthT: dam crest widthΓc: concrete specific weighth:upstream water height

C. Water weight force over dam upstream slope (F2):

$$F_2 = \frac{1}{2}m.b.h$$

D. Lifting power (U): it is of dam's destructive powers obtained from following relation:

$$U = \frac{1}{2} \gamma_w.b.h$$

E. Force resulted from upstream precipitation: since the small size, it is not incorporated in calculations.

F. Wind, waves and ice forces: as these forces' values vary depending on climate, and are smaller than other forces, they are ignored in computations.

G. Seismic dynamic force (Fe1, Fe2, Fe3, Fe): it imposes on dams in two ways: one as the factor of dam body gravity forces with a value equals the product of earthquake factor (a) by weight value, which obtains as follows:

$$Fe_i = \alpha . W_i$$

And, the other is earthquake force in dam reservoir imposed nonlinearity and its value in height is upstream water height:

$$P_e = 0.73(\frac{90 - \theta^{\circ}}{\theta^{\circ}}).\alpha.\gamma_w.h$$
$$F_e = 0.726.P_e.h$$
$$M_e = 0.229.P_e.h^2$$

Pe: Earthquake hydrodynamic pressure in height h

a: earthquake factor which is almost 0.15 to 0.2 in most Iranian dams

Fe: earthquake power in dam reservoir

Me: Earthquake hydrodynamic force moment Fw: water specific weight

2.4 Sustainment Criteria

It is clear that not all aforementioned forces imposed simultaneously; thus, two modes are studied: the first is the full mode in which water is at normal balance and water hydrostatic force and uplifting force are imposed. Sustainability is considered for normal and imposed seismic statuses. The second mode is the empty one in which resistance constraints against fatigue is also added.

2.4.1 Sustainability versus Slip

There are three criteria for assessing sustainability versus slipping all of which assumed as problem constraints.

A. Confident slip coefficient: is the ratio of total horizontal forces to total vertical forces, which must be smaller than static friction coefficient. If this coefficient is less than 0.65 regarding seismic force and 0.85 disregarding seismic force, dam is sustainable.

$$f = \frac{\sum F_H}{\sum F_V} \le f'$$

 ΣF_{H} : Total horizontal forces ΣF_{V} : Total vertical forces f': Static friction coefficient

B. Confidence coefficient against slip: this method considers shear forces along increased sustainability. The coefficient is 1 to 1.5.

$$SF = \frac{f' \cdot \sum F_V}{\sum F_H} \le 1 - 1.5$$

C. Confidence coefficient against shear-friction: shear forces enters in computations. The allowed shear stress value is 7 to 14 kg/cm (0.25 shear resistance). Static

friction coefficient is in the range of 0.65-0.75. according USBR standard, this value disregarding seismic force is larger than 4 and regarding this power must be larger than 1.5 in order to avoid any financial and life damages.

$$SFF = \frac{f' \cdot \sum F_V + b.\sigma}{\sum F_H}$$

 $\boldsymbol{\sigma}$: Allowed shear tension at shear surface

2.4.2 Sustainability against Vertical Fatigueless or Tension in Dam Body

Sustainability against fatigue requires positive upstream and downstream vertical fatigue in empty and full situations.

$$\sigma_u = \frac{\sum F_v}{b} - \frac{6\sum M_o}{b^2}$$
$$\sigma_d = \frac{\sum F_v}{b} + \frac{6\sum M_o}{b^2}$$

 σ_{μ} : Upstream vertical fatigue at dam body

 σ_d : Downstream vertical fatigue at dam body

 ΣM_{o} : Total moments of forces imposed on dam in relation to surface center

2.4.3 Sustainability versus Overturn

If total resistant moments to dam toe are about 1.5 to 1.7 times of total overturning moments at the same point, dam remains sustainable.

$$\sum F_{(over)} = \frac{\sum M_{E-\text{Resistant}}}{\sum M_{E-\text{overturn}}}$$

2.5 Problem Solution

The problem solved by a program written in MATLAB software with its objective function and all constraints determined. Crest width (T) is 1.5 and free height (fb) is 4% of upstream water depth. Height values are 10, 15, 20, 25, 30, 35, and 40 m and earthquake coefficient value assumed 0.1, 0.15, and 0.2. m varies within 0.14 to 0.3 through adding 0.1 in each iteration and the program computes the variable bottom width (b) value. The problem contains 1155 different modes. The program was solved in two modes, one regarding sustainability versus overturn, and one disregarding this factor. In order to better compare this method with other common problem solving methods, obtained solutions are compared with conventional methods.

Al PSO algorithm parameters are as follows: w value of 1 in first iteration linearly changes to 0.2 in last iteration. Cognitive and social factors values (C1 and C2) are 1.494. Scope searching space is assumed 4 to 50 m for bottom width; further, maximum velocity factor (Vmax) is used to prevent explosive speed. This value equals 0.1 times of upper and lower limits difference in searching space. The numbers of iterations and particles (population) are 32 and 50, respectively. Higher values show no significant difference in solution. According to increased implementation time because of increasing iterations numbers and population, there is no need to increase these values.

3 Discussion and Result

3.1 Optimization Problem

In order to assess PSO problem-solving capability, firstly, dam designing problem solved in restricted condition. Dam was designed in four different heights of 15, 20, 25, and 30 and seismic coefficients of 0.1 and 0.2. Population of this example is 100 particles and iterations are 50 times. A sample of problem solving is shown in Table 1.

Table 1 shows that Particle Swarm Optimization algorithm (PSO) calculates bottom width, dam weight, and upstream bottom width ratio for four heights of 15, 20, 25, and 30 and two seismic coefficients of 0.1 and 0.2. Problem is solved in this condition in 11.4 seconds indicating algorithm high rate in problem solving.

3.2 Studying the Numbers of Population and Iterations

The second step in studying this algorithm is obtaining the best numbers of particles' population and the best iteration numbers. Algorithm efficiency was verified through six different population and seven different iterations at constant height and seismic coefficient of 16 m and 0.15, respectively.

3.2.1 Studying the Best Number of Population Using Bottom Width ratio to Height (b/h)

Bottom width ratio to height diagram is the first diagram analyzed for estimating the best number of population and iteration. Considering that this ratio is the recommended parameter of designing a gravity-concrete dam in conventional methods and references, it is used for getting the best implementation mode. Figure 3 shows this

enter the number of population =100							
enter the number of iteration= 50							
Height (m)	Seismic coefficient	Upstream bottom width m	Bottom width (m) B	Bottom width to height b/h	Dam weight (Ton/m)		
15	0.1	0.3	14.98	1	275.61		
15	0.2	0.29	17.43	1.16	319.15		
20	0.1	0.21	20.3	1.01	493.45		
20	0.2	0.28	24.27	1.21	588.34		
25	0.1	0.27	25.2	1.01	763.34		
25	0.2	0.3	29.17	1.17	881.95		
30	0.1	0.29	30.53	1.02	1107.17		
30	0.2	0.29	35.89	1.2	1299.56		
Elapsed	time	is	11.38804	seconds.			

 Table 1.
 Program implementation results



Figure 3. Changing bottom width ratio to height in relation to iteration and population numbers prioritizing population number.

ratio in multiple execution states (various iterations and population) prioritizing population number.

Diagram reveals that the best number of population for executing this program is 250 particles.

3.2.2 Studying the Best Iteration Numbers Using Bottom Width Ratio to Height

The best iteration number is also estimated by this bottom width ratio to height (b/h). The parameter for different population and iterations prioritizing iteration number is illustrated in figure 3.

Figure 4 shows that the best iteration number for implementing this program is 250 particles. Thus, the best

iteration and population numbers are 250 particles and iterations, respectively.

3.3 Program Implementation for Different Heights and Seismic Factors

Considering that objective function and problem constraints are nonlinear, it requires complicated computations for solving concrete gravity dam optimization. These calculations are too heavy for a particular height and seismic coefficient. The estimations of the implemented program were for seven different heights and three various seismic coefficients. However, 250 iterations and 250 populations also made the problem more difficult; particle swarm optimization algorithm solved this problem for 21 different states in less than 7 min. It demonstrates that this algorithm is highly efficient in solving difficult and challenging problems.

3.4 Comparing Computation Weight by Conventional Methods with Particle Swarm Optimization Algorithm

In order to verify PSO efficiency, calculated values were compared with values suggested in Water transferring constructs. The book recommended a table 2 for designing gravity concrete dam.

3.4.1 Diagram of Comparing Bottom Width in Conventional Method and PSO

The values of bottom width computed by conventional method and PSO are illustrated in diagram in order to better compare these two approaches.

Above figure indicates that PSO algorithm achieved better bottom width in all states (excluding for height 40 m and zero seismic factor which is discussed in the following), satisfying all constraints, than conventional methods.

Table 2.Bottom width value to height accordingto conventional method (Beiramie, 1997)

	a=0	a=0.1	a=0.2
М	0.28	0.15	0.3
b/h	0.92	1.04	1.27

Only at height 40 m and seismic factor zero, the calculated weight by conventional method is less than PSO value. Therefore, suggested conventional values were placed in the nine constraints, which demonstrated that the proposed bottom width by conventional method is 36.8 m, which is basically unable to satisfy problem constraints; thus, dam built with 36.8 bottom width is unsustainable. However, PSO value (37.14 m) meets constraints.

3.4.2 Comparing Conventional Method Weight with PSO Weight

Figure 5 shows comparing conventional method weight with PSO weight.

As figure 6 presents, PSO algorithm optimizes dam weight in all states.

3.4.3 Deriving Bottom Width Function to Height

According to bottom width computed by PSO algorithm, it is possible to apply a function to measure bottom width rather than suggesting one number for bottom width ratio to height in conventional methods. Thus, PSO values for various seismic factors were charted in Excel software and fitness function was obtained. Figures 6, 7, and 8 shows functions of different seismic coefficients.

Table 3 provides the difference between conventional method and PSO algorithm in introducing the value of suggested bottom width for concrete gravity dam. Conventional method uses a constant value of bottom



Figure 4. Changing of bottom width ratio to height in relation to iteration and population number prioritizing iteration number.



Figure 5. Comparing computed bottom width by conventional method and PSO algorithm.



Figure 6. Comparing unit weight of computed length by conventional and PSO methods.



Figure 7. (a) Fitness function of bottom width to height with 0.1 sesimic factor.

Seismic factor	Bottom width to height	Bottom width to height in PSO				
	in conventional method	algorithm				
a=0.1	1.04h	$0.000367h^2 + 1.01169h - 0.33357$				
a=0.15	1.155h	$0.00045h^2 + 1.08919h - 0.24857$				
a=0.2	1.27 h	$-0.00033h^2 + 1.22024h - 0.7$				





Figure 7. (b) Fitness function of bottom width to height with 0.15 sesimic factor.



Figure 8. Fitness function of bottom width to height with 0.15 seismic factor.

width ratio to height. In other word, there is a linear relation seen between bottom width and height; whereas, this relation in PSO method is nonlinear and quadratic.

Bottom width value in PSO computations is a quadratic function; while, conventional method introduces a linear function.

4. Conclusion

Dam optimization is one goal of dam designers. Dam designing must be such that not only satisfies sustaining condition, but also leads to minimum costs. This issue is multiplied in gravity dams due to high costs of materials



Figure 9. Fitness function of bottom width to height with 0.2 seismic factor.

and excessive concrete volumes. Objective function in dam weight optimization problem is dam cross-section and bottom width at foundation (base) and upstream slope are the variables. Constraints are establishing dam sustaining criteria. Regarding constraints nonlinearity, Particle Swarm Optimization (PSO) algorithm was used for optimization. This research used 1155 states of program implementation in MATLAB software the results of which are as follows:

- PSO algorithm is easily programmed and implemented; in addition, it simply responses to nonlinear complicated problems and finds optimal solution.
- Results revealed that PSO solutions are different from suggested methods, which can easily optimize dam weight in spite of nonlinear constraints. This difference increases where seismic factor and upstream slope enhance, such that PSO algorithm estimates less weight than conventional method.
- PSO values apply a function instead of a number for measuring bottom width. This function introduces a quadratic relation between bottom width and height per triple seismic factors. These functions can decisively be used for designing concrete gravity dams. The functions are as follows:
- for 0.1 seismic factor $b = 0.000367h^2 + 1.01169h 0.33357$

- for 0.15 seismic factor $b = 0.00045h^2 + 1.08919h 0.24857$
- for 0.2 seismic factor $b = -0.00033h^2 + 1.22024h 0.7$

The aforementioned functions with high fitness values can be appropriate alternatives for conventional methods.

5. Reference

- He Q, Wang L. An effective co-evolutionary particle swarm optimization for constrained engineering design problems. Engineering Applications of Artificial Intelligence. 2006; 20:89–99.
- Schutte JF, Koh B, Reinbolt J, Benjamin J, Raphael T, Alan D. Evaluation of a Particle Swarm Algorithm for biomechanical optimization. Journal of Biomechanical Engineering. 2005; 127(3):465–74.
- 3. Heirany Z, Ghaemian M. Effect of concrete nonlinear parameters on the seismic response of concrete gravity dams. World Academy of Science, Engineering and Technology. 2011; 5:638–41.
- Salmasi F. Design of dam by Genetic Algorithms. World Academy of Science, Engineering and Technology. 2011; 5(8):682–7.
- Rainia H, Cohaim B, Weck O, Venter G. Comparative study of Particle Swarm Optimization Ad the Genetic Algorithm. American Institute of Aeronautics and Astronautics (AIAA). 2004; 152–60.