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Investigations on Array Pattern Synthesis using Nature Inspired Metaheuristic Algorithms

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Abstract

Linear and circular antenna arrays need to be properly designed for optimization of electromagnetic problem. The design parameters are decided by the specific beam width using firefly algorithm, desired radiation patterns are to be studied for the reduction of beam width and side lobe level simultaneously based on multi objective approach. PSO is used for comparison based on simulation examples. Linear antennas are designed to obtain the source locations in position only synthesis based on FA. Patterns are synthesized using circular arrays to achieve minimum sidelobe level and high directivity with constraint of beam width. Performance evaluation of bat, cuckoo search algorithms is level as compared to FA. This method is extended to concentric circular arrays with dipoles as radiating elements, FA is found to be most suitable for development of linear and circular antenna arrays. Procedure of thinning helps in reducing the cost of array designs. This method will find extensive usage in design of array pattern.

Keywords: Particle Swarm Optimization, Firefly Algorithm, Cuckoo Search Algorithm, Circular Antenna, Bat Algorithm, Sidelobe Level, Beamwidth

1. Introduction

The concept of array synthesis has a predominant role in wireless communications for which the reason being the requirement of large directive antennas. It is evident from literature survey and mathematical analysis that high directivity is achieved with antenna having large dimensions, which has a direct impact on the operating wavelength. This is undesired as most of the communication using wireless technology is frequency dependent and any change in the operating frequency of the antenna would not serve for the application of its existence[1,2]. An antenna array exists in different geometrical forms considering the arrangement of the elements of the array. In linear arrays all the elements are arranged on a straight line whereas in circular and planar arrays the elements are distributed with a specific geometrical shape but fixed to a common plane. The circular arrays are popular since the revolution of the wireless technology and their capability of scanning with little effort. The analysis of circular array is readily extended to concentric ring arrays and cylindrical arrays that assume typical properties with excellent directive nature[3].

Initially, analytical methods were used to synthesize radiation pattern with low SLL. The first optimum antenna array distribution is the binomial distribution proposed by Stone. Later, Dolph mapped the Chebyshev polynomial onto the array factor polynomial to get all the sidelobes at an equal level. The resulting array factor polynomial coefficients represent the Dolph–Chebyshev

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amplitude distribution[4]. This amplitude taper is optimum in that specifying the maximum sidelobe level results in the smallest beamwidth, or specifying the beamwidth results in the lowest possible maximum sidelobe level. Several numerical techniques are proposed and applied for such array synthesis applications. It can be inferred from the literature that, these numerical methods are treated as highly complex and suffer from inability to handle multi modal problems[5].

From the discussion above, no sufficient work reported on pattern synthesis to produce narrow beams with low sidelobes. Therefore, an attempt is made in the present work to investigate possible methods of pattern synthesis for the antenna arrays mentioned above. Some of the metaheuristic algorithms namely Firefly Algorithm (FA)[6], Bat and Cuckoo Search (CS) algorithms are very popular in electromagnetic. They have proved themselves in handling multi objective problems. Hence, these algorithms are applied to several synthesis problems.

2. Nature Inspired Metaheuristic Algorithms

Since antenna array design problems are generally in a multimodal form, it is at high possibility that most of the classical optimization algorithms are caught up in a local solution. Evolutionary and metaheuristic algorithms can sub-optimally solve a multimodal numerical optimization problem defined by using an objective function without needing the derivative of the relevant problem[7]. Thus, in the present work, application of heuristic algorithms like Firefly, Bat and Cuckoo Search algorithms are considered.

2.1 Firefly Algorithm

Firefly Algorithm (FA) is the one of the latest swarm intelligence metaheuristics. In which the search algorithm is inspired by the flashing behavior of fireflies and the phenomenon of bioluminescent communication. The flashing light helps fireflies for finding mates, attracting their potential prey and protecting themselves from their predators. The main algorithm's principle is that each firefly moves towards brighter and more attractive locations by the flashing light intensity that associated with the objective function of problem is considered. The basic steps of the FA can be summarized as the pseudo code as shown in Figure. 1.

```
Begin
Initialize algorithm parameters max generation, α, β_o, γ
Create initial population of n fireflies x= (x_1, x_2, x_3… x_D)^T within
D dimensional search space
Objective function f(x), x= (x_1, x_2, x_3,………, x_D)^T
Determine I_i at x_i determined by f(x_i)
    While (t < maxgeneration)
    For i=1 to n (all n fireflies)
        For j=1 to n (all n fireflies)
         if (I_j > I_i)
          Move firefly i towards j in D dimension
           end if
            Evaluate new solutions and update light intensity
          end for j
         end for i
       Rank the fireflies and find the current best
    end while
Post process results and visualization
End procedure
```

Figure 1. Pseudo code of the Firefly algorithm.

```
Begin
Initialize algorithm parameters max iteration, A_i, r_i, Q_i.
Create initial bat population (x_1, x_2, x_3… x_K)^T and initial velocities v_i
Objective function f (x_i), x_i= (x_1, x_2, x_3,………, x_K)^T
Determine pulse frequency f_i at x_i
  While (t < max iteration)
    Generate new solutions by adjusting frequency, updating
      velocities and ocations
       if (rand > r_i)
        Select a solution among best solutions
         Create local solution around selected best solution
          end if
           Generate new solutions by flying randomly
          if (rand < A_i & f (x_i) < f (X_*(current best solution))
         Accept the new solutions
        Increase r_i and reduce A_i
       end if
     Rank the bats and find the current best
    end while
  Post process results and visualization
End procedure
```

Figure 2. Pseudo code of the Bat algorithm.

2.2 Bat Algorithm

The Bat Algorithm is a bio-inspired algorithm developed by Xin-She Yang in 2010. It is based on the echolocation behavior of micro bats with varying pulse emission rates and loudness[8,9]. It is the first algorithm of its kind to use frequency tuning. All the bats use an echolocation to sense the distance. Each virtual bat flies randomly with a velocity v_i at position (solution) x_i with a varying frequency or wavelength and loudness A_i. As it searches and finds

Begin

Objective function f(x), x= $(X_1,X_2, X_3,\dots\dots\dots, X_K)^T$
Initialize population of n host nests X_i (1,2,……..P)
While (t < Max iteration) or (stop criterion)
Obtain a cuckoo randomly by Lévy flight behavior
Select a nest randomly among the host nests say j
Calculate fitness, F_j
if ($F_i < F_j$)
Replace j by new solution;
else j be the solution
end
Abandon a fraction probability P_a of the worst nests
Build new ones at new location via Lévy flights
Keep the best nests with quality solutions
Rank the solutions and find the current best nest
end
Post process results and visualization
End procedure

Figure 3. Pseudo code of the Cuckoo Search (CS) algorithm.

its prey, it changes frequency, loudness and pulse emission rate r. Search is intensified by a local random walk. Loudness varies from a large to a minimum constant value.

2.3 Cuckoo Search Algorithm

Cuckoo Search (CS) is an optimization algorithm developed by Xin-she Yang and Suash Deb in 2009. The algorithm is inspired by the reproduction strategy of cuckoos. Specific egg laying and breeding of cuckoos is the basis of this novel optimization algorithm. In addition, this algorithm is enhanced by the so called Lévy flights, rather than by simple isotropic random walks[10]. The basic steps of the CS can be summarized as the pseudo code as shown in Figure. 3. Once the algorithm is run, the best nest is chosen as the optimum variable.

3. Linear Array

The linear array is one of the most commonly used array configureuration in many practical applications owing to its simplicity and beam shaping property. In a conventional linear array, in general the elements are uniformly spaced and uniformly excited. A broadside linear array of N equispaced isotropic elements with asymmetric excitation positioned along Z-axis is shown in Figure 4.Mathematically; the array factor can be stated as[11]

$$E(\theta)=\sum_{n-1}^{N} A_n \exp\left[j(n-1)\left(kd\ \cos\theta+\beta\right)\right] \qquad (1)$$

Here,

A_n = Amplitude excitation of the n-th element of the array
$k = 2\pi/\lambda$=Wave number
d = Spacing between the elements=$\lambda/2$
θ = Angle of incidence of a plane wave
λ = Signal wavelength
$\beta = -kd\cos\theta_d$
θ_d = Scan angle
N = Number of elements in the array.

Progressive phase β is zero as pattern maxima is directed towards $\theta_d = 90^0$ for broadside linear array. However, in scanned array pattern maxima is oriented at an angle θ_d. Normalized absolute far-field in dB can be expressed as

$$p(\theta) = 20\log_{10}\left[\frac{|E(\theta)|}{\left||E(\theta)|_{max}\right|}\right] \qquad (2)$$

Fitness Function

For a given length of array, both the Half Power Beamwidth (HPBW) and First Null Beamwidth (FNBW) increases as the SLL is lowered. The objective of optimization is to minimize the maximum sidelobe level of the array pattern by adjusting the control parameters while HPBW and FNBW both are kept within some specified constraints. In order to have a tradeoff between the array parameters, the fitness can be calculated using the following equation

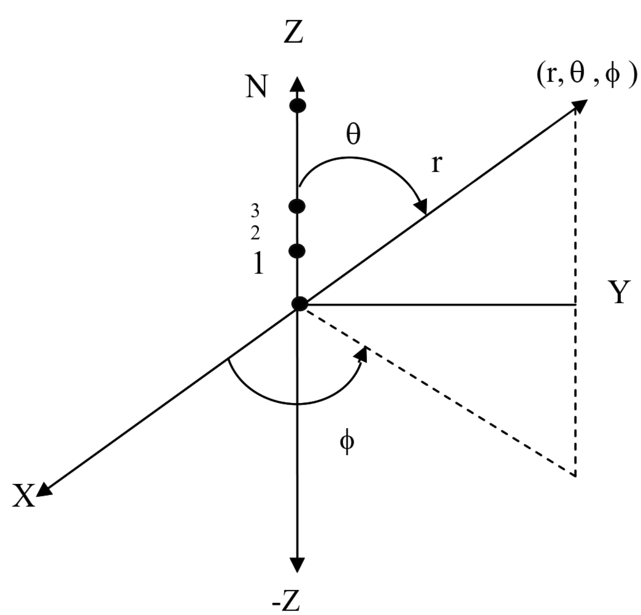

Figure 4. Geometry of broadside linear array.

$$\text{Fitness} = \begin{cases} Max \dfrac{\left(20\log_{10}\left(\left|E(\theta)\right|\right)\right)}{\left(\max\left|E(\theta_d)\right|\right)} & H \le H_u \ and \ F \le F_u \\ 10^4 & else \end{cases} \quad (3)$$

Where H and F are the half power and first null beamwidth of the pattern produced by the array considered for optimization. H_u and F_u are the resultant values obtained with the uniformly excited array of similar size. Firefly Algorithm (FA) and Particle swarm optimization (PSO) are used to obtain the radiation pattern with minimum SideLobe Level (SLL) for specified Half Power Beamwidth (HPBW) and First Null Beamwidth (FNBW). The current amplitude used to excite the array elements for N=40 using each algorithm is presented in Table 1. The obtained power pattern from the FA algorithm is shown in Figure[11]. 5, along with the uniform excitation pattern and PSO optimized array respectively. The result obtained from FA versus PSO result is tabulated in Table 2 for 40-element array and also values of uniform linear array are given. It is seen from Figure.5, that FA offers 3.35dB and 0.78dB SLL reduction compared to uniform linear array and PSO optimized array.

Table 1. Amplitude excitation coefficients for N=40 elements using Firefly Algorithm

PSO				FA			
N	Amplitude	N	Amplitude	N	Amplitude	N	Amplitude
1	0.8898	21	0.6596	1	0.9382	21	0.8788
2	0.3119	22	0.3922	2	0.9785	22	0.5630
3	0.5313	23	0.7593	3	0.0734	23	0.5424
4	0.4398	24	0.6564	4	0.5673	24	0.9208
5	0.4765	25	0.6567	5	0.4138	25	0.5795
6	0.2170	26	0.2487	6	0.2434	26	0.4672
7	0.3817	27	0.3362	7	0.5492	27	0.3296
8	0.5382	28	0.6616	8	0.3453	28	0.4355
9	0.5343	29	0.4940	9	0.2928	29	0.3686
10	0.4783	30	0.5866	10	0.3834	30	0.7704
11	0.2125	31	0.3691	11	0.5021	31	0.1796
12	0.5185	32	0.2083	12	0.6620	32	0.6566
13	0.4475	33	0.3053	13	0.4664	33	0.5307
14	0.6804	34	0.2405	14	0.7485	34	0.3398
15	0.4676	35	0.3817	15	0.4821	35	0.6048
16	0.2211	36	0.3941	16	0.5300	36	0.2353
17	0.4582	37	0.4690	17	0.8328	37	0.2288
18	0.2442	38	0.4963	18	0.2089	38	0.3874
19	0.5072	39	0.7402	19	0.5946	39	0.9325
20	0.5506	40	0.5460	20	0.2453	40	0.8936

4. Linear Aperiodic Array

A broadside linear array of N elements, unequally spaced with starting element at origin, is shown in Figure.6. The radiating elements in the array of present interest are considered to be point sources.

Mathematically, the array factor can be characterized as[12]

$$E(\theta) = \sum_{n=0}^{N-1} A_n \exp\left[jkx_n \cos\theta\right] \quad (4)$$

Fitness Function

Two different design examples are considered for the synthesis of a linear aperiodic array using the proposed method i.e. Position Only Synthesis and Position-Amplitude Synthesis. To accomplish these two designs, the technique

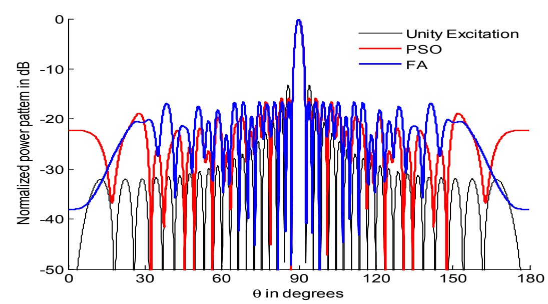

Figure 5. Normalized Power Patterns for an array of N = 40 using Unity Excitation and Firefly Algorithm.

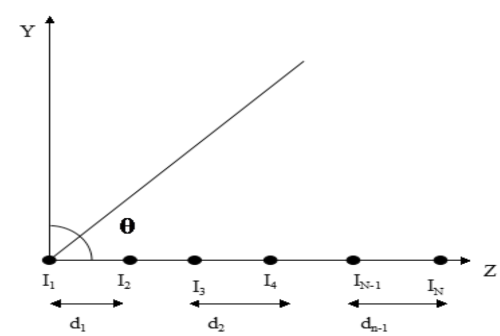

Figure 6. Array of nonuniformly spaced isotropic point sources with asymmetric amplitude distribution.

Table 2. Results for linear array with unity excitation, unequal excitation using PSO and FA

N	Unity Excitation			PSO			FA		
	SLL (dB)	HPBW (deg)	FNBW (deg)	SLL (dB)	HPBW (deg)	FNBW (deg)	SLL (dB)	HPBW (deg)	FNBW (deg)
40	-13.23	2.58	5.68	-15.80	2.58	5.67	-16.58	2.58	5.68

of Firefly Algorithm (FA) is used. The goal of optimization is to minimize the maximum sidelobe level of the array pattern by adjusting the element parameters of the array subject to given design specifications and constraints.

Case 1: Position Only Synthesis

The objective of first design is to minimize the sidelobe level by adjusting element positions.

Thus, the fitness function is defined with the evaluation of maximum sidelobe level as

$$Fitness1 = f(\bar{\rho}) = Max_{\theta \in S} \left| \frac{P(\theta)}{P(\theta_o)} \right| \tag{5}$$

Here, S is the space spanned by the angle θ excluding the main lobe and $\bar{\rho}$ represents the parameter vector. Therefore, $\bar{\rho}$ can be given by

$$\bar{\rho} = \{d_n\} \qquad 0 \le n \le (N-1) \tag{6}$$

The objective function described above minimizes all the sidelobe levels and maximizes the power in the main lobe located at $\theta = \theta_o$.

Case 2: Position-Amplitude Synthesis

The second design objective is to further reduce the SLL for fixed beamwidth by optimizing the positions along with the amplitude excitation coefficients. The beamwidth obtained in the first problem is taken as the design constraint for the second problem.

By considering all the above, the fitness function is given by

$$Fitness = Max_{\theta \in S} \left| \frac{P(\theta)}{P(\theta_o)} \right| \quad subjectto\ FNBW \le FNBW_d \tag{7}$$

$$\bar{\rho} = \{d_n, A_n\} \qquad 0 \le n \le (N-1) \tag{8}$$

Where, $FNBW_0$ is first null beamwidth of the pattern produced by Position-Amplitude Synthesis. $FNBW_d$ is the value obtained with Position Only Synthesis for the array of similar size. FA is applied for the synthesis of linear array using Position Only Synthesis and the Position-Amplitude Synthesis. The synthesis is carried out for N=40 elements in both cases. In Position Only Synthesis, the excitation amplitudes are fixed as A_n=1 and the positions of the elements are optimized by the FA. In Position-Amplitude Synthesis, both the positions as well as the excitation amplitudes are determined simultaneously using FA. The optimum element positions in Position Only Synthesis and optimum element positions as well as amplitude coefficients in Position-Amplitude Synthesis derived using[12,13] FA are presented in Figure. 7 and corresponding power patterns are given in Figure. 8. The lowest sidelobe level achieved and the null-to-null beamwidth (FNBW) of the array for the number of elements N=40 are given in Table 3. The lowest sidelobe level

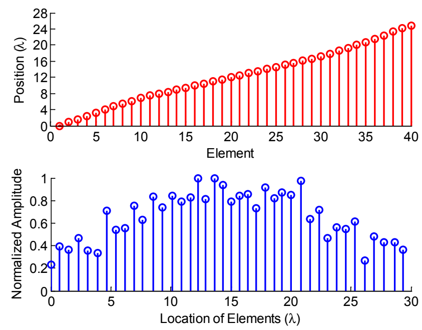

Figure 7. Element Positions obtained in Position Only Synthesis and Positions, Amplitudes obtained in Position-Amplitude Synthesis using FA.

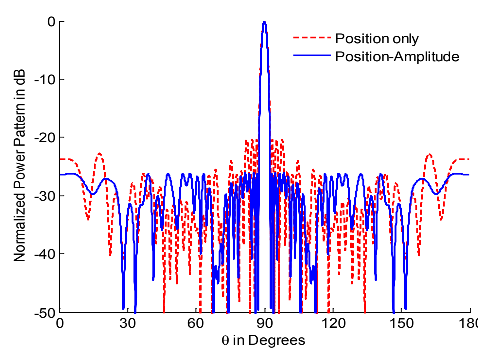

Figure 8. Resultant Normalized Power Patterns for N=40.

Table 3. SLL and FNBW obtained from Position Only Synthesis and Position-Amplitude Synthesis using Firefly Algorithm (FA)

Element Number	Position Only Synthesis		Position-Amplitude Synthesis	
	SLL(dB)	FNBW(deg)	SLL(dB)	FNBW(deg)
40	-20.69	5.27	-26.15	5.26

of the FA-based Position-Amplitude Synthesis is lower by about 5 dB when compared to Position Only Synthesis without deteriorating the beamwidth.

5. Circular Array

The elements are nonuniformly spaced on a circle of radius 'r' in the Y-Z plane. The elements are assumed to be isotropic sources so that the radiation pattern of the array can be described by its array factor. The geometry of an N

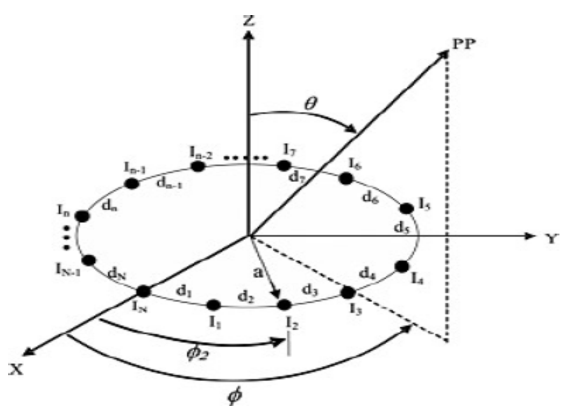

Figure 9. Typical geometry of isotropic circular array.

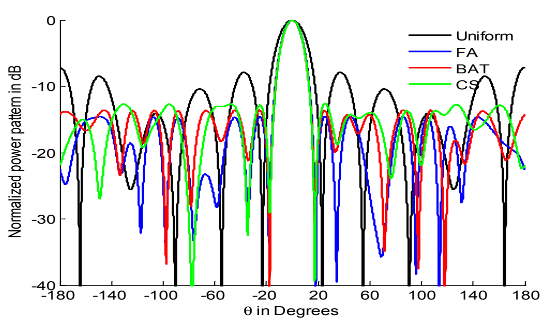

Figure 10. Normalized Power Patterns of 12 elements Circular Array.

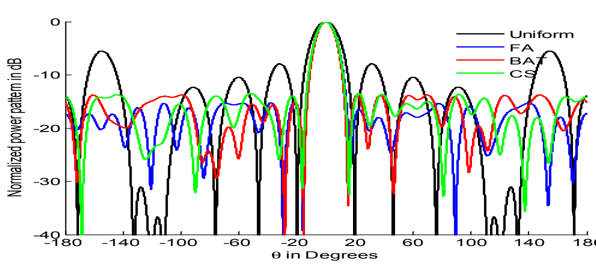

Figure 11. Normalized Power Patterns of 14 elements Circular Array.

element circular antenna[14,15] has been shown in figure. 11. The array factor in the Y-Z plane can be written as

$$A_F(\theta) = \sum_{n=1}^{N} A_n \exp\left[jka \cos\left(\theta - \phi_n + \alpha_n\right)\right] \tag{9}$$

where,

$$ka = \sum_{i=}^{N} d_i \tag{10}$$

$$\phi_n = \frac{\left(\pi \sum_{i=}^{n} d_i\right)}{\left(\sum_{i=}^{N} d_i\right)} \tag{11}$$

$$\alpha_n = -ka\, cas\left(\theta_o - \phi_n\right) \tag{12}$$

A= $[A_1, A_2 \ldots A_n \ldots A_N]$, A_n represents the excitation amplitude of the n-th element of the array, d= $[d_1, d_2 \ldots d_n \ldots d_N]$, d_n represents the distance from element n to (n+1). Excitation current phases are fixed at zero degree.

Fitness Function

The uniform circular array is of high sidelobe geometry (approximately 8 dB below the main lobe). The first and most important parameter in pattern synthesis of antenna array is the Sidelobe Level (SLL) that is desired to be as low as possible. So, the objective of the work is to minimize the maximum sidelobe level of the array pattern by adjusting the amplitudes and positions of the elements while First Null Beamwidth (FNBW) is kept within some specified constraints. Thus the following fitness function is used.

$$Fitness = Max_{\theta \in S} \left| \frac{P(\theta)}{P(\theta_o)} \right| \quad subject to F \le F_u \tag{13}$$

Here

S is the space spanned by the angle θ excluding the main lobe.

F is the first null beamwidth of the pattern produced by the array considered for optimization.

F_u is the resultant values obtained with the uniform circular array.

The amplitude excitations and positions of the elements of circular array are optimized using Firefly, Bat, and Cuckoo Search Algorithms to produce a radiation pattern with optimal performance. The SLL and FNBW obtained for uniform circular arrays are given in Table 4 and the value of FNBW is taken as design constraint. The optimized amplitudes and element positions for 12-element and 14-element array are presented in Tables 5-6. The patterns are numerically computed and obtained pat-

Table 4. Parameters obtained for Uniform Circular Antenna Arrays

N	FNBW(deg)	SLL(dB)
12	46.26	-7.9
14	39.65	-7.9

Table 5. Optimized Amplitudes and Positions for N=12 element Circular Array using FA, Bat and CS Algorithms

N	FA		Bat		CS	
	A_n	$x_n(\lambda)$	A_n	$x_n(\lambda)$	A_n	$x_n(\lambda)$
1	0.6548	0.6548	0.3587	0.8415	0.8549	0.9566
2	1.6520	0.9972	0.5581	1.7427	0.2510	1.4867
3	2.3373	0.6853	0.2025	2.3868	0.4827	2.1941
4	3.0128	0.6755	0.7279	3.0624	0.4492	2.8912
5	3.9651	0.9523	0.9301	4.0444	0.5136	3.6620
6	4.6276	0.6625	0.9754	4.7949	1.0000	4.4747
7	5.3135	0.6859	0.8398	5.4234	0.7582	5.0552
8	6.3021	0.9886	0.5608	6.3791	0.4575	6.0298
9	6.9943	0.6922	0.3334	7.1462	0.5802	6.7582
10	7.7005	0.7062	0.5494	7.8358	0.8127	7.5695
11	8.6569	0.9564	0.3632	8.8358	0.7740	8.5695
12	9.3303	0.6734	1.0000	9.5458	0.9517	9.1284

terns are compared with that of uniform circular array also. The resultant patterns are presented in Figures 10-11. The array performances obtained using FA, Bat and CS are determined in terms of maximum SLL, FNBW, ADR, Directivity and Circumference are presented in Table 7. Results shows that the pattern generated by using Bat algorithm has better reduced beamwidth than the pattern produced by Firefly and Cuckoo Search algorithms. But the pattern produced by Firefly algorithm has low sidelobes compared to that of the patterns produced by the other two algorithms. It is also evident from the results that the length of the array (circumference) is also less for the optimized element positions produced by the Cuckoo Search algorithm when compared to that of the other two algorithms.

Table 6. Optimized Amplitudes and Positions for N=14 element Circular Array using FA, Bat and CS Algorithms

N	FA		Bat		CS	
	A_n	$x_n(\lambda)$	A_n	$x_n(\lambda)$	A_n	$x_n(\lambda)$
1	0.8359	0.5848	0.4957	0.5524	0.8503	0.6659
2	0.5603	1.5810	0.6058	1.4372	0.4421	1.6659
3	0.2965	2.2520	0.6281	2.1236	0.9331	2.2592
4	0.4513	3.0706	0.0852	2.7814	0.2722	2.9707
5	0.3150	3.9862	0.4039	3.3072	0.6230	3.5520
6	0.7424	4.8427	0.3537	4.1394	0.6167	4.4055
7	1.0000	5.3589	0.4292	4.5586	0.7647	5.0652
8	0.5325	5.9438	1.0000	5.0337	0.8024	5.5846
9	0.7782	6.9438	0.1720	5.9664	0.5906	6.5718
10	0.2804	7.6491	0.7580	6.7951	0.1440	7.3623
11	0.3617	8.3107	0.3355	7.5726	0.0770	8.1700
12	0.3911	9.0349	0.6332	8.0937	0.8697	8.8398
13	0.3953	9.8622	0.5340	9.0937	0.7989	9.8334
14	0.8314	10.6251	0.9264	9.6397	1.0000	10.3943

6. Concentric Circular Array

The elements are arranged in planar circular arrays containing concentric rings, which differ in radius and number of elements. If all the elements are assumed to be isotropic sources, the radiation pattern of this array can be written in terms of its array factor only. Concentric Circular Array (CCA) which uses dipoles as radiating elements is considered in this paper. A planar array of M concentric circular rings with centre element is shown in Figure. 12 When the actual elements are dipoles, the total field can be formed by multiplying the array factor of the isotropic sources with the field of a single element.

The far-field radiation pattern of the concentric dipole ring array with single element at the centre can be given by

$$E_d(\theta) = E_p(\theta) * A_F(\theta) \quad (14)$$

where, $E_p(\theta)$ gives the element pattern while $A_F(\theta)$ is the array factor of the CCA [14-15].

Table 7. Comparison of results obtained with the three algorithms

N	Algorithms	Max SLL(dB)	FNBW(deg)	Directivity(dB)	ADR	Circumference(λ)
14	FA	-15.26	33.88	11.95	3.56	10.63
	BAT	-13.77	30.65	11.41	11.73	9.64
	CS	-13.46	32.08	11.64	12.99	10.39
16	FA	-13.77	27.49	14.17	2.85	12.00
	BAT	-12.69	26.42	14.51	3.23	11.67
	CS	-12.27	31.00	13.03	8.04	10.93

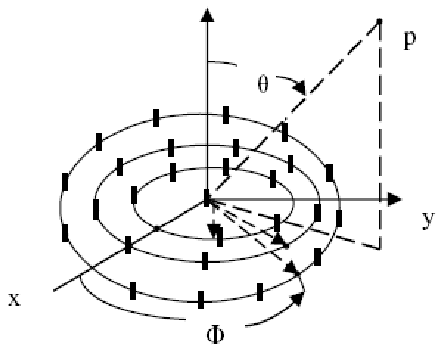

Figure 12. Geometry of a concentric circular array (CCA).

The dipole element pattern is given by [16]

$$E_d(\theta) = \left[\frac{\cos(kL\cos\theta) - \cos kL}{\sin\theta}\right] \quad (15)$$

$$A_F(\theta) = 1 + \sum_{m=1}^{M}\sum_{n=1}^{N_m} A_{mn} \exp\left[jkr_m \cos(\theta - \phi_{mn})\right. \quad (16)$$

$$\left.\sin\theta_o \cos(\phi_o - \phi_{mn})\right)$$

$$N_m = \frac{2\pi r_m}{d_m}, \quad r_m = \frac{m\lambda}{2}, \; d_m = \frac{\lambda}{2} \quad (17)$$

$$\phi_{mn} = 2n\pi / N_m \quad (18)$$

Here,

N_m=Number of elements in the m-th ring
r_m=Radius of the m-th ring
d_m= Inter-element spacing of m-th ring
k = 2 π /λ= Wave number
θ = Elevation angle
λ = Signal wavelength
ϕ = Aϕzimuth angle
L= Length of the dipole

Where θ_o, ϕ_o represents direction at which main beam achieves its maximum. Since the number of equally spaced elements must be an integer, the value in eq. (16) must be rounded up or down. To keep d ≥ λ/2, the digits to the right of the decimal point are dropped.

The Normalized power patterns in dB can be expressed as

$$E_n(\theta) = 20\log_{10}\left[\frac{|E_d(\theta)|}{\left\|E_d(\theta)\right|_{\max}|}\right] \quad (19)$$

6.1 Concept of Thinning

Thinning means turning off some elements in a uniformly spaced or periodic array to create a desired amplitude density across the aperture. An element connected to the feed network is 'ON' and an element connected to a matched load or dummy load is 'OFF'. It is possible to lower sidelobes by turning off selected elements in the uniform array. Some elements within a ring are turned off or effectively removed from the ring in order to modify the current density on the aperture. The goal is to minimize the maximum sidelobe level by creating a low sidelobe density taper on the array aperture[17].

Fitness Function

Four design examples are considered for thinning a concentric circular array. The goal of optimization task in the four antenna designs is to minimize the maximum SLL subjected to numerous design constraints. Uniform amplitude excitation is used in the first problem where remaining three are nonuniformly excited. The first three designs are synthesized for different values of null-to-null beamwidth as optimization constraint. However, last design instance is synthesized for a fixed level of thinning percentage and null-to-null beamwidth. By considering all the above, the fitness functions are formulated as follows.

$$Fitness1 = \begin{cases} Max(SLL) & subject\ to\ F_o \le F_d \\ 10^2 & otherwise \end{cases} \quad (20)$$

$$Fitness2 = \begin{cases} Max(SLL) + (Th_o - Th_d)^2 & subject\ to\ F_o \le F_d \\ 10^2 & otherwise \end{cases} \quad (21)$$

F_o and F_d are the obtained and desired values of the first null beamwidth. Th_o and Th_d are the obtained and desired values of percentage of thinning. Thus, for the design of thinned CCA with minimum SLL, the optimization task is to search for the excitation amplitudes that are turned ON or OFF that accomplish the above two fitness functions.

Example 1: Optimizes the position of the turned on elements of the uniformly excited array while null-to-null beamwidth is kept within the beamwidth of fully populated array.

Example 2: Calculates the nonuniform excitations as well as turned on elements for the same beamwidth.

Example 3: The same case is again repeated in the third example for a wider null-to-null beam i.e. 10^0 more than FNBW of a fully populated array.

Example 4: The synthesis of CCA with pre fixed thinning percentage is carried out in the last instance while null-to-null beamwidth is not exceeded the beamwidth of fully populated array. Desired value of thinning is fixed at 50%.

FA is applied to find the optimal set of ON and OFF elements that will generate pencil beam. Four instantiations of the design problem are evaluated for 7-ring CCA.

The thinning configuration found for the point sources was then applied to an array of dipoles. The optimal uniform, nonuniform excitations and distribution of turned 'ON' and turned 'OFF' elements obtained in four cases for 7-ring CCA are presented in Tables (8-14) and corresponding patterns are presented in Figures. (13-16).

The results obtained from the four design instances for 7-ring CCA are presented in Table 15. It is found from the results that use of nonuniform amplitude excitations in the thinned array reduces the SLL more effectively. It is also observed that increasing null-to-null beamwidth can significantly enhance the sidelobe level.

Table 8. Distribution of turned on and turned off elements of the uniformly excited thinned 7-ring CCA with FNBW=19.5^0

Ring No.	Amplitude distribution
1	101010
2	100010000110
3	101010001001110100
4	100100100111011111101010
5	101110110100110000000101010101001
6	111011010011001010110110111000100000
7	110011000010100001011101001011101100100011 1

Table 9. Distribution of turned on and turned off elements of the non uniformly excited thinned 7-ring CCA with FNBW=19.5^0

Ring No.	Amplitude distribution
1	001101
2	000111111001
3	100100011100011111
4	101001110110000001110011 0
5	010001111101000100001100101010 1
6	101100101011110101110111100011110011 1
7	111001001110111000011111011101101111101011 1

Table 10. Non uniform amplitude distribution of the thinned 7-ring CCA with FNBW=19.5$_0$

Ring No.	Amplitude distribution
1	0,0,0.6254,0.5897,0,0.7247
2	0,0,0,0.8310,0.6502,0.5612,0.6153,0.7327,0.5691,0,0, 0.5504
3	0.6045,0,0,0.5837,0,0,0,0.6261,0.8971,0.5216,0,0,0,0.7506,0.6584,0.6043, 0.5303,0.6701
4	0.5681,0,0.5241,0,0,0.5924,0.8026,0.5266,0,0.6244,0.6957,0,0,0,0,0,0, 0.6659,0.5315,0.7703,0,0,0.6108,0.5164,0
5	0,0.6858,0,0,0,0.5361,0.8161,0.5607,0.5232,0.7670,0,0.6195, 0,0,0,0.5380, 0,0,0,0,0.5316,0.6668,0,0,0.6500,0,0.6298, 0,0.5960,0,0.5143
6	0.6714,0,0.7372,0.6716,0,0,0.8511,0,0.7103,0,0.5485,0.6936,0.7360,0.7449,0,0.5200,0,0.7224,0.7936,0.5528,0,0.6332, 0.6434,0.6732,0.63 24,0,0,0,0.6441,0.7491,0.7649,0.6477,0,0,0.6285, 0.6769,0.5019
7	0.5618,0.6202,0.5725,0,0,0.7132,0,0,0.6633,0.5608,0.7214,0,0.5539,0.5243, 0.5618,0,0,0,0,0.6603,0.5757,0.5998, 0.6939,0.5399,0,0.6343, 0.5729,0.5796, 0,0.5901,0.5701, 0,0.8347,0.7796,0.5226,0.7002,0.7034,0,0.5536,0,0.8328, 0.6955,0.6850

Table 11. Distribution of turned on and turned off elements of the non uniformly excited thinned 7-ring CCA with FNBW=29.5^0

Ring No.	Amplitude distribution
1	011100
2	110001111011
3	110000011011111001
4	000011010110001011111000 0
5	001110111001100000001001110001 00
6	010001011011111110000101011110111010 01
7	000001101101111010001110011111111111010010 1

Table 13. Distribution of turned on and turned off elements of the non uniformly excited 50% thinned 7-ring CCA with FNBW=19.5^0

Ring No.	Amplitude distribution
1	100110
2	011111100001
3	100000100010101000
4	000011010100100111000011 0
5	001011001010110101010111000001 0
6	101000111000100000000100010000111001 00
7	111111111000100101001110110010000001000001 0

Table 12. Nonuniform amplitude distribution of the thinned 7-ring CCA with FNBW=29.5^0

Ring No.	Amplitude distribution
1	0,0.7009,0.6109,0.5435,0,0
2	0.6086,0.5968,0,0,0,0.6010,0.6226,0.5525,0.6049,0,0.5986,0.5372
3	0.7208,0.5493,0,0,0,0,0,0.6869,0.5372,0,0.7163,0.5974,0.5605,0.5193,0.5904,0,0,0.6079
4	0,0,0,0,0.6347,0.5618,0,0.5630,0,0.5424,0.5047,0,0,0,0.5461,0,0.5952, 0.6077,0.6642,0.5344,0.5586,0,0,0,0
5	0,0,0.7070,0.7854,0.5854,0,0.5258,0.6580,0.7058,0,0,0.7178,0.6871,0,0, 0,0,0,0,0.5220,0,0,0.6040,0. 6789,0.5294, 0,0,0,0.7817,0,0
6	0,0.8284,0,0,0,0.5957,0,0.5691,0.5013,0,0.6526,0.6052,0.6608,0.6064,0.5955,0.5772,0,0,0,0,0.7048,0,0.5563,0, 0.5949,0.8284,0.5396,0.55 25,0,0.5972,0.5976,0.5828,0, 0.6464,0,0,0.5154
7	0,0,0,0,0,0.7135,0.5210,0,0.6111,0.6193,0,0.5522,0.6351,0.6335,0.5517,0,0.6086,0,0,0,0.5145,0.5357,0.6511,0,0, 0.5162,0.5166,0.5359,0. 6228,0.6595,0.6885,0.5203,0.6403,0.6485, 0.5967,0.5878,0,0.5160,0,0,0.5378,0,0.5134

Table 14. Distribution of turned on and turned off elements of the non uniformly excited 50% thinned 7-ring CCA with FNBW=19.5^0

Ring No.	Amplitude distribution
1	0.7565,0,0,0.7016,0.5607,0
2	0,0.6843,0.5095,0.5160,0.5217,0.6310,0.6595,0,0,0,0,0.5459
3	0.7502,0,0,0,0,0,0.5105,0,0,0,0.5040,0,0.7328,0,0.7120,0,0,0
4	0,0,0,0,0.6808,0.7355,0,0.6093,0,0.5289,0,0,0.6708,0,0,0.7367,0.5421, 0.5167,0,0,0,0,0.7736,0.5073,0
5	0,0,0.5613,0,0.7138,0.6084,0,0,0.5633,0,0.5541,0,0.6205,0.7338,0,0.5265,0,0.6849,0,0.5406,0,0.6701,0.5905, 0.6039,0,0,0,0,0,0.7130,0
6	0.6826,0,0.6294,0,0,0,0.6105,0.6183,0.5060,0,0,0,0.6979,0,0,0,0,0,0,0,0.5365,0,0,0,0.5266,0,0,0,0,0.5953, 0.5715,0.5381,0,0,0.7678,0,0
7	0.6771,0.6515,0.6016,0.5615,0.5643,0.5107,0.7113,0.5692,0.5249,0,0,0,0.7008,0,0,0.6883,0,0.6943,0,0,0.6405,0.7096,0.53 00,0,0.6114,0.5487,0,0,0.6143,0,0,0,0,0,0,0.5610,0,0,0,0,0,0.6327,0

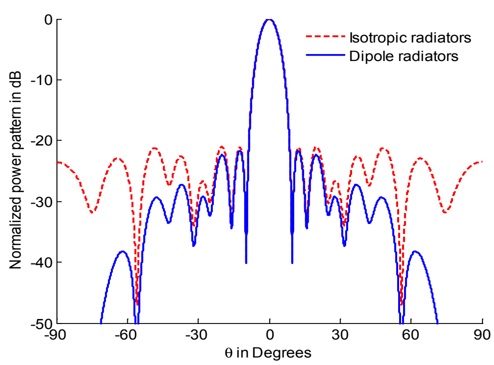

Figure 13. Normalized power patterns of a thinned 7-ring Isotropic and Dipole CCA with FNBW=19.5^0

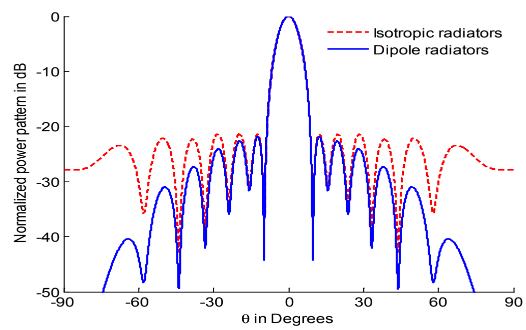

Figure 14. Normalized power patterns of the non uniformly excited thinned 7-ring Isotropic and Dipole CCA with FNBW=19.5^0

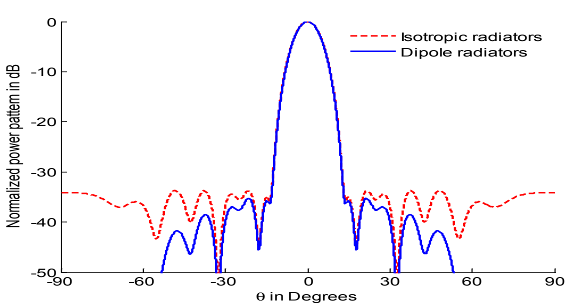

Figure 15. Normalized power patterns of the non uniformly excited thinned 7-ring Isotropic and Dipole CCA with FNBW=27.7^0

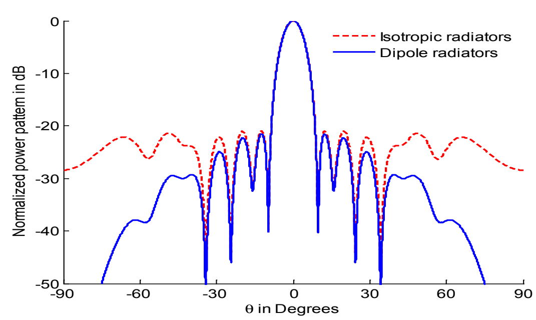

Figure 16. Normalized power patterns of the non uniformly excited 50% thinned 7-ring Isotropic and Dipole CCA with FNBW=19.5^0

Table 15. Results obtained using thinned 7-ring CCA

Design parameters	Example 1	Example 2	Example 3	Example 4
Percentage of thinning	51.44	42.19	46.24	58.38
SLL(dB)	-21.02	-21.39	-33.72	-21.01
FNBW(deg)	19.5^0	19.5^0	29.5^0	19.5^0

7. Conclusions

In order to have a tradeoff between conflicting array parameters like beamwidth and SLL, a swarm based optimization technique, namely Firefly Algorithm (FA) is used to design the linear arrays. The obtained result is compared with the PSO optimized result. FA efficiently computed the nonuniform amplitude excitation distribution for broadside linear arrays. The results reveal that the design of nonuniformly excited linear antenna array with optimized amplitude excitations using FA offers a considerable sidelobe level reduction without deteriorating the beamwidth.

Therefore, the same FA is applied to design nonuniformly spaced linear antenna array with equal and unequal amplitudes. FA searches more effectively for the best amplitude excitations and positions of the elements that produce low sidelobes. The results reveal that the power patterns obtained by Position-Amplitude Synthesis have useful radiation characteristics in terms of SLL and also null-to-null beamwidth when compared to Position Only Synthesis.

It is also evident from the literature survey, performance comparison of Firefly, Bat and Cuckoo Search Algorithms applied to design of the nonuniform circular arrays, has not been reported so far. Thinning of an array to obtain low sidelobes is much simpler than the general problem of non uniform spacing of the elements. FA is applied to find the optimum thinned layout of the concentric circular arrays. Dipoles are used as radiators to include practical element radiation characteristics in the synthesis process. Suitable thinning limits the cost of beam forming network. Fixing the percentage of thinning at a higher value increases the power efficiency of the feeding network with little compromise on the design specifications.

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