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A Pre-Compression Algorithm for Optimal Acquisition of Modernized GNSS Signal

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Abstract

The acquisition of GNSS signal is the most challenging part of any GNSS receiver. With the modernized GNSS system having longer code length and wider bandwidth, the size of the data block becomes large. To reduce the computational load of the processor, the reduction in the size of the data block without compromise in the acquisition loop performance provides overall improvement to the system. Here, a model is developed for acquisition of GPS signal L1 and L2C by compressing the input raw GPS signal with sequences of perfect periodic autocorrelation properties such as the Ipatov sequence. A compression matrix of the required dimension is constructed for compressing the input GPS signal. The improvement in the acquisition performance of the model is discussed.

Keywords: Acquisition, Compression, GPS, L2C, Modernised GPS

1. Introduction

The acquisition loop is an important component of a GNSS receiver as it detects the presence of the satellite signal from the received composite GNSS signals. Huge uncertainties in Doppler frequency and code phase make this algorithm computationally intensive. Generally a frequency domain based FFT correlation using blocks of data is used for acquisition in an SDR platform. The block length depends on the code clock and the integration period, for instance, for C/A code acquisition as the code duration is 1msec with a clock frequency of 1.023MHz the minimum length of data block size is 2046 samples. For longer code lengths such as GPS L2C/CM code with duration of 20msecs and a clock frequency of 1.023MHz the data block size is 40920 samples. These large block sizes make the FFT correlation process very intensive. Any pre-processing of the data without degrading the

correlation properties will therefore have a significant impact on the receiver implementation in terms of processing power and latency. Multi-rate algorithms have been proposed to reduce the complexity; however there is degradation in the correlation properties which results in a SNR reduction. Therefore, this paper proposes a compression algorithm that pre-compresses the GNSS signals before the acquisition leading to a significant reduction in the number of computations and latencies. Moreover, the correlation properties are retained without reducing the SNR.

With the compression algorithm, a compression matrix is constructed using the ternary sequences which possess the property of perfect periodic autocorrelation. The size of the matrix is determined by the data size before (N) and after (M) compression. For preserving the correlation properties the values of M has to be co-prime to that of N^4 . Such a Toeplitz structured compression matrix

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A of size $M \times N$ with entries h_i such a way that the matrix A can be written as

$$A = \begin{bmatrix} h_0 & h_1 & h_2 & \dots & h_{N-1} \\ h_{-1} & h_0 & h_1 & \dots & h_{N-2} \\ h_{-2} & h_{-1} & h_0 & \dots & h_{N-3} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ h_{-M+1} & \dots & \dots & h_{N-M-1} & h_{N-M} \end{bmatrix}$$
 (1)

When the compressed sensing is used, the required trade-off between SNR and processing bandwidth are explained in³. The application of compressed sensing for spectrum estimation in cognitive radios is presented in⁴. The synthesis of perfect sequences is explained in⁵. A discussion on various sequences and arrays with perfect periodic correlation are given in⁶. The concept on m-sequences and ternary sequences are explained in⁷.

The paper composition is as follows: Signal acquisition using compressed sensing is explained in Section 1; Modelling of GPS signal compressing is explained in Section 2; SDR based acquisition is analysed in the section 3 and finally Section 4 discusses the experimental results.

Optimisation of GNSS Signal Acquisition using Compressed Sensing

The GNSS signal acquisition is possible at the sub-Nyquist rate by employing Compressed Sensing (CS) methods² to compress the raw GPS signal data using compression matrix. The relationship between sampling frequency and SNR requires optimization while using the compressed sensing for the reasonable estimation of the functions³. The autocorrelation properties can be preserved when the compressed sensing is implemented through linear projection, such as Toeplitz-structured random matrices or linear compression with time shifted random pre-integration⁴.

If Toeplitz matrix is employed for spectrum compression, it leads to residual bias when used for sequences of smaller length due to the autocorrelation of the individual elements. This can be avoided by employing deterministic compression matrices constructed using sequences with perfect periodic correlation properties. A compression matrix *A* formed using the Zadoff-Chu or

Ipatov sequences, which are incoherent with the Fourier basis can be used⁵. The Zadoff-Chu sequence is a polyphase sequence, while the Ipatov sequence is a ternary sequence. Since, the ternary sequences have smaller number of phases and requires less multiplication during implementation, it is preferred here.

When, the energy efficiency of a perfect sequence is calculated it found that it is very close to the square of its mean value and hence it approaches the unity for longer sequence length (N). Luke⁶ has given a list of multivalued perfect sequences with better energy efficiency. From the multi-valued sequence of period $n = q^m-1$, Ipatov sequences of length N can be constructed using polynomial algebraic methods.

$$N = \frac{P^{w(2d+1)} - 1}{P^w - 1} \tag{2}$$

where, P is Prime; d, w positive integer

A typical plot representing the first 100 samples of the L2C code, m-sequence and Ipatov sequence, which are generated using Matlab program, is given in Figure 1.

The binary behavior of the L2C code and the ternary behavior of the m-sequence (m=3) and the Ipatov sequence are clearly visible from the represented plot in Figure 1.

Many sequences families are easily described using the theory of finite fields. The finite fields are also known as *Galois fields*, with p^m elements for any prime p and any positive integer m. The Galois field GF (p^m) with p^m elements is unique. The Galois field with p^m elements can be constructed with a suitable polynomial f(x) of degree m. The polynomial f(x) is selected such a way that f(x) can not be written as product of two polynomials, of degree ≥ 1 .

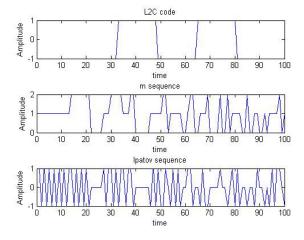


Figure 1. L2C code, m-sequence and Ipatov sequence plot.

For example Galois field GF (3³) can be constructed with $f(x) = x^3 + 2x + 1$, which is easily an irreducible polynomial over GF (3)⁷.

To construct ternary sequences of higher length, two ternary sequences of relatively prime length can be periodically multiplied that gives more ternary sequences but this reduces the energy efficiency. When an m-sequence (b_n) is generated using a primitive polynomial y(x), the non zero elements are represented as $b_n = a^u$ when $b_n \neq 0$, where α is primitive element and u is an integer.

Now, the perfect ternary sequence $a_n = (-1)^n c_n$ can be constructed with period N, such a way,

$$c_n = \begin{cases} 0, & \text{if } b_n = 0\\ \left(-1\right)^u, & \text{if } b_n = \alpha^u \end{cases}$$
 (3)

The correlation properties possessed by a ternary sequence (p = 3) is better than the compatible binary sequence (p = 2). The ternary sequences with better autocorrelation properties can be constructed from the maximum length sequences (m-sequence), which are deterministic and easy to generate in hardware using linear shift registers.

The correlation performance of the L2C code and the Ipatov sequence are analysed and the resultant plots showing the auto correlation of the L2C code and the Ipatov sequence (327352 samples) are given in Figures 2 and 3. The L2C code assumed here is the L2C CM code alone, which has a code duration of 20ms and a code length of 10230. The autocorrelation performance result shown in Figure 2 for the L2C signal code is represented for this length. For the data set used for acquisition implementation, the 20ms duration result into 327352

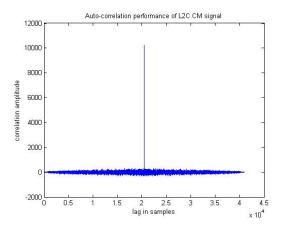


Figure 2. Autocorrelation plot of L2C signal.

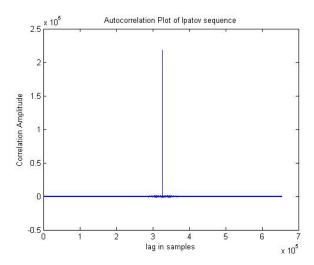


Figure 3. Autocorrelation plot of the Ipatov sequence.

samples of the L2C signal. Hence, the autocorrelation performance for the Ipatov sequence, which is shown in Figure 3 was carried out for 327352 samples.

The correlation amplitude of both the plots clearly shows the increased correlation amplitude of the Ipatov sequence due to the longer length of the same here.

3. Modelling of GPS Signal Compression

The input signal has to be mapped with the compression matrix of required dimensions that is determined by the compression ratio, which in turn is based on the input signal data length and length of the compressed signal data. The elements of the compression matrix are constructed using the ternary sequences such as Ipatov sequence which possess the property of perfect periodic autocorrelation as described above.

The present legacy GPS L1 signal can be represented as

$$S_{L1} = A_p P(t) D(t) \cos(2\pi f_1 t + \phi) + A_c C(t) D(t) \sin(2\pi f_1 t + \phi)$$
 (4)

The civilian C/A signal is the second component in the equation, where S_{L1} is GPS signal at L1 frequency, A_c is amplitude of C/A code, C (t) is +/-1 the phase of C/A code, D(t) is +/-1 the data code, P (t) is P(Y) code, f_1 is L1 frequency and φ is initial phase.

The L2 signal, which has the modernised L2C code, can be written as

$$S_{L2} = A_p P(t) D(t) \cos(2\pi f_2 t + \phi) + A_c \left(D(t) C_{M,0}(t) + C_{0,L}(t)\right) \sin(2\pi f_2 t + \phi)$$
 (5)

The L2C signal is the second component in the equation,

Where S_{L2} is signal at L2 frequency, A_c is amplitude of C code, $C_{M,0}(t)$ is +/- 1 the phase of CM code, $C_{M,0}(t)$ is +/- 1 the phase of CL code, D (t) is +/- 1 the data code, P (t) is P(Y) code, f_2 is L2 frequency and φ is initial phase.

The GPS signal (L1 or L2) are sampled with suitable sampling frequency to have N samples in one block of data (y) that are used during the acquisition of the signal. When such a digitised input GPS signal y of length N has to be compressed to a signal \hat{y} of length M, the linear compression matrix can be written as

$$\hat{y} = Ay \tag{6}$$

Where A is the compression matrix A of size $M \times N$

The correlation properties of the signal vector y is preserved, when linear projection is done by mapping with the matrix such as A to give the output \hat{y} in equation $(6)^4$. As discussed before the compression of the signal can be implemented through deterministic compression matrices, through either the perfect ternary sequences with $a_n \in \{-1, 0, 1\}$ such as Ipatov sequences. The PSD of the input signal vector y are preserved due to the deterministic compression matrix such as A in equation (7) which is formed with sequences of perfect periodic autocorrelation such as Ipatov sequence.

4. Compressed GPS Signal Acquisition using FFT

The compressive sensing has been implemented for GPS signal acquisition for which the GPS raw signal is the input. The architecture used for implementation of compressive sensing for GPS signal acquisition is shown in Figure 4.

Here, the input raw data stream has N number of samples for one data block. The m-sequence is constructed using the GF (313) with 313 elements with suitable polynomial of the order 13. The m-sequence with 313 elements is converted into Ipotav sequence of N elements. From the Ipatov sequence of N elements the linear compression matrix A of equation (7) is constructed for the size $M \times N$. Here, M is the desired compression value. The linear projection of this matrix with the input raw data (N samples) and the locally generated signal data (also of N samples) result in the compressed signal (M samples per data block) of both the input data and the local signal. The circular correlations of these two signals result in the acquired values of the code delay and Doppler of the received GPS signal. An algorithm has been developed to provide the required scaling to get the code delay of the uncompressed signal from the code delay of the compressed signal.

5. Results and Discussion

The implementation method described above was done through Matlab and the results are listed below.

Initially, the frequency response analysis of the GPS signal was done using the FFT of the GPS sisgnal before and after compression. The FFT of the GPS signal before and after compression are given in Figure 5 and 6. The FFT plots cleary showing no major change in the frequency chracteristics of the GPS signal after the compression using the compression matrix.

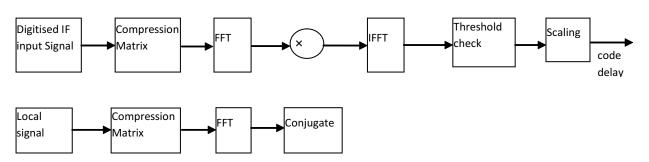


Figure 4. Architecture of the Compressed GPS signal acquisition using FFT.

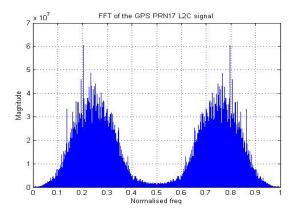


Figure 5. Frequency response plot of the raw GPS signal.

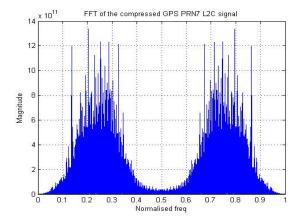


Figure 6. Frequency response plot of the compressed GPS signal.

The architecture of the Compressed GPS signal acquisition using FFT as shown in Figure 4 and described in the previous section was implemented in Matlab for both the L1 C/A signal and the L2C signal and the acquisition loop results obtained are given below. The L1 C/A signal data was generated with the following specifications:

PRN 6, IF frequency is 1.25MHz, random code delay is 750, Doppler frequency is 5 KHz and sampling frequency is 5MHz.

This generated data resulted in a data block size of 5000 samples for 1ms duration. Since the IF frequency is 1.25MHz and C/A code frequency is 1.023MHz, the minimum sampling frequency should be atleast double that of the above values to meet the Nyquist rate. To bring down the data block size below the Nyquist rate, the compression value of 999 samples from 5000 samples is selected for implementation. This compression value (999) was selected since, the value of compressed samples (999) and the original samples (5000) must be co-prime

for the proper implementation⁴. The m-sequence was selected with 3¹³ elements with suitable polynomial of the order 13. Then the generated m sequence was converted to an Ipatov sequence and the compression matrix A with M×N size was implemented. This linear compression matrix was used to compress the data block from 5000 samples per data block to 999 samples per data block. The local signal for the circular correlation was also generated accordingly. Then the GPS L1 C/A acquisition based on circular correlation was implemented and the acquired result of code phase delay is given in Figure 7 without the scaling in the code phase delay. The required scaling in the code phase delay is provided depending on the compression ratio achieved.

The plot in Figure 7 clearly shows that the algorithm developed with the model given above has the capability to acquire the code delay and Doppler correctly. The code delay and Doppler acquired by the model meet the simulated values mentioned before. The acquisition model is also applied for implementation of L2C acquisition also. The data utilized for the L2C acquisition are: PRN 17, IF frequency is 4.134MHz and sampling frequency is 16.3676MHz

This data set specification has resulted in 327352 samples per data block (20ms block). To compress the data below Nyquist rate the linear compression matrix A was constructed for a size of 40999×327352. The compressed sample value of 40999 was selected to ensure that the number of the compressed sample (40999) and the original sample (327352) are co-prime⁴. The m-sequence was constructed with suitable polynomial for an order 13 i.e. of elements. Correspondingly the Ipatov sequence was constructed from the m-sequence and the linear compression

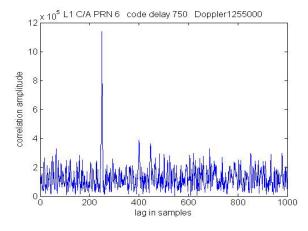


Figure 7. L1 (C/A) PRN 6 acquisition plot for code delay of 750 and Doppler of 5 KHz.

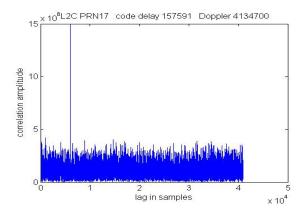


Figure 8. L2C PRN 17 code delay acquisition plot.

matrix A (MxN size) was implemented. Both the input signal and the local signal were then compressed to 40999 samples per data block (compression factor of about 8). The acquisition based on the circular correlation after the compression was implemented and the acquisition result is given in Figure 8 without the scaling in the code phase delay. The required scaling in the code phase delay is provided depending on the compression ratio achieved.

The acquisition plot shows that the acquisition results match with the original code delay and Doppler values which were given for the data set used for acquisition. The correlation amplitude value is very high (15 \times 10 for L2C) compared to all other conventional acquisition implementations (maximum was 8.5 \times 10 4 (L2C) without this sequence due to the longer length of the constructed m-sequence and the Ipatov sequences. This indicates the improvement in the correlation amplitude peak of this optimized method is approximately 6000 times for GPS L1 C/A signal acquisition and 2 \times 10 4 times for the GPS L2C signals compared to that of the conventional acquisition methods.

6. Conclusion

The acquisition algorithms are the most challenging components of a GPS receiver because not only these loops detect the satellites signals by scanning the entire uncertain region but also they need to do it within a short duration. With the proliferation of SDR technology most of the modern receivers use the frequency domain approach because of the reduced computations and improved speeds.

Acquisition of the GPS L2C signal is a challenging process as the Doppler bin size is only 50Hz for the full coherent integration period, and it takes a large number

of bins to search the entire Doppler uncertainty region. Moreover, software receivers use FFT based acquisition and with higher sampling frequencies the data length becomes exhaustive¹. Two factors therefore become important in determining the efficiency of the acquisition of the L2C signal-latency and computational load.

The proposed model was to implement the acquisition of GPS signal using the compressive sensing methods, where the data is compressed using perfect sequences possessing the property of perfect periodic autocorrelation. This was implemented by constructing the m-sequence and Ipatov sequences. The implementation of the optimised GPS signal acquisition for L1 C/A code and the L2C code clearly gives better correlation amplitude, which provides additional benefit in terms of compromise for low SNR conditions. The improvement in the correlation amplitude peak of this optimised method is approximately 6000 times for GPS L1 C/A signal acquisition and 2×10^4 times for the GPS L2C signals compared to that of the conventional acquisition methods.

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